OUR GATE 2010 TOPPERS

ELECTRONICS & COMMUNICATION ENGINEERING TOPPERS

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<tr>
<td>02</td>
<td>SHAH AZEEMUDDIN</td>
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<td>02</td>
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<td>GAURAV NAIR</td>
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<td>MITHUN P</td>
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GATE SYLLABUS

CONTROL SYSTEMS (ECE & EEE)

## CONTROL SYSTEMS CONTENTS

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For

GATE, DRDO & IES

Managing Director
Y.V. Gopala Krishna Murthy

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### CHAPTER 1

#### INTRODUCTION

**System:** A system is an arrangement or combination of different physical components such that it gives the proper output for given input. A kite is an example of a physical system, because it is made up of paper and sticks. A classroom is an example of a physical system.

**Controller:** The meaning of control is to regulate, direct or command a system so that a desired objective is obtained.

**Plant:** It is defined as the portion of a system which is to be controlled or regulated. It is also called a process

**Controller:** It is the element of the system itself, or may be external to the system, it controls the plant or the process.

**Input:** The applied signal or excitation signal that is applied to a control system to get a specified output is called input.

**Output:** The actual response that is obtained from a control system due to the application of the input is termed as output.

**Disturbance:** The signal that has some adverse effect on the value of the output of a system is called disturbance. If a disturbance is produced within the system, it is termed as an internal disturbance; otherwise, it is known as an external disturbance.

**Control System:** It is a arrangement of different physical components such that it gives the desired output for the given input by means of requisite or control either direct or indirect method.

A control system must have (1) input, (2) output, (3) ways to achieve input and output objectives and (4) control action.

Fig. The following shows the cause-and-effect relationship between the input and the output

\[
\text{Reference input } r(t) \quad \text{Control system} \quad \text{Controlled output } C(t)
\]
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INTRODUCTION

Advantages and disadvantages of open-loop system:

Advantages
- These systems are simple in construction and design.
- These systems are economic.
- These systems are easy from maintenance point of view.
- Usually these systems are not much troubled with problems of stability.
- These systems are convenient to use when output is difficult to measure.

Disadvantages
- These systems are not accurate and reliable because their accuracy is dependent on the accuracy of calibration.
- In these systems, inaccurate results are obtained with parameter variations, i.e., internal disturbances.
- Recalibration of the controller is required from time to time for maintaining quality and accuracy.

Closed-loop Control System:

In a closed-loop control system, the output has an effect on control action through a feedback as shown and hence closed-loop control systems are also termed as feedback control systems. The control action is actuated by an error signal (e(t)) which is the difference between the input signal (u(t)) and the output signal (c(t)). This process of comparison between the output and input maintains the output at a desired level through control action process.

Advantages and disadvantages of closed-loop system

Advantages
- These systems accuracy is very high due to correction of any existing error.
- Since these systems sense environmental changes as well as internal disturbances, the errors are modified.
- There is no effect of non-linearities in these systems.
- These systems have high bandwidth, i.e., high operating frequency zone.
- There are facilities of automation in these systems.

Disadvantages
- These systems are complicated in design and, hence, costlier.
- These systems may be unstable.
Comparison of Open-loop and Closed-loop Control Systems:

<table>
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<th>Open - loop C.S.</th>
<th>Closed - loop C.S.</th>
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<tr>
<td>1. The accuracy of an open-loop system depends on the calibration of the input. Any departure from pre-determined calibration affects the output.</td>
<td>1. As the error between the reference input and the output is continuously measured through feedback, the closed-loop system works more accurately.</td>
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<tr>
<td>2. The open - loop system is simple to construct and cheap.</td>
<td>2. The closed - loop system is complicated to construct and costly.</td>
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<tr>
<td>3. The open - loop systems are generally unstable.</td>
<td>3. The closed - loop systems can become unstable under certain conditions.</td>
</tr>
<tr>
<td>4. The operation of open - loop system is affected due to presence of non-linearity's in its elements.</td>
<td>4. In terms of the performance, the closed - loop systems adjust to the effects of non-linearity's present in its elements.</td>
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Open-loop C.S.:

\[ R(s) \rightarrow G(s) \rightarrow C(s) \]

Closed-loop C.S.:

\[ R(s) \rightarrow G(s) \rightarrow C(s) \rightarrow \frac{C(s)}{R(s)} = G(s) \text{ or } C(s) = G(s) R(s) \]

If error signal \( e(s) \) is zero, output is controlled. If error signal is not zero, output is not controlled.

For Positive feedback, error signal = \( x(s) + y(s) \)

For Negative feedback, error signal = \( x(s) - y(s) \)

The purpose of feedback is to reduce the error between the reference input and the system output.

\[ +Ve \text{ feedback: } \sum_{1} \] Non unity FB

\[ -Ve \text{ feedback: } \sum_{1} \] Unity FB

Unity FB: \( H(s) = 1 \)

Non unity FB: \( H(s) = \frac{G(s)}{1 - G(s) H(s)} \)

\[ C(s) = \frac{G(s)}{1 - G(s) H(s)} \]

Where \( G(s) = TF \) without feedback \( (s) \).

\[ TF \text{ of the forward path } H(s) = TF \text{ of the feedback path } \]

\[ C(s) \]

\[ \frac{C(s)}{R(s)} = G(s) \text{ or } C(s) = G(s) R(s) \]

Effect of feedback on Overall gain:

Feedback affects the gain of a non-feedback system by a factor of \( 1 + GH \). The general effect of feedback is that it may increase or decrease the gain. In a practical control system, \( G \) and \( H \) are functions of frequency, so the magnitude of \( 1 + GH \) may be greater than 1 in one frequency range but less than 1 in another. Therefore, feedback could increase the gain of the system in one frequency range but decrease it in another.

Effect of feedback on Sensitivity:

Consider \( G \) as a parameter that may vary. The sensitivity of the gain of the overall system \( M \) to the variation in \( G \) is defined as

\[ S_M = \frac{\delta M}{\delta G} \]

This relation shows that the sensitivity function can be made arbitrarily small by increasing \( GH \), provided that the system remains stable. In an open-loop system, the gain of the system will respond in a one-to-one fashion to the variation in \( G \). In general, the sensitivity of the system gain of a feedback system to parameter variations depends on where the parameter is located.

The effects of feedback are as follows:

(i) Gain is reduced by a factor \( 1 + GH \)

(ii) There is reduction of parameter variation by a factor \( 1 + GH \)

(iii) There is improvement in sensitivity.

(iv) There may be reduction of instability.

For a complicated system, it is easy to find the transfer function of each and every element, and output of a certain block may act as an input to other block or blocks. Therefore, the knowledge of transfer function of each block is not sufficient in this case. The interrelation between the elements is required to find the overall transfer function of the system. There are two methods: (1) Block diagram and (2) Signal flow graph.
CHAPTER 2 BLOCK DIAGRAMS AND SIGNAL FLOW GRAPHS

There are two methods, (1) by using Block diagram or (2) Signal flow graph, to find the overall transfer function of a given complicated control system.

2.1 BLOCK DIAGRAM ALGEBRA

Block diagram reduction techniques

Some of the important rules for block diagram reduction techniques are given below:

1. The block diagram shown below relates the output and input as per the transfer function relation given below:

\[ G(s) \quad C(s) \quad R(s) \quad G(s) \quad C(s) \]  

where \( G(s) \) is known as the transfer function of the system.

2. Take off point:

Application of one input source to two or more systems is represented by a take off point, as shown at point A in the below figure:

\[ R(s) \quad A \quad C_1(s) \quad G_1(s) \quad G_0(s) \quad C_0(s) \]

3. Blocks in cascade:

When several blocks are connected in cascade, the overall equivalent transfer function is determined below:

\[ C(s) \quad R(s) \quad G_1(s) \quad G_2(s) \quad G_3(s) \quad G_0(s) \quad C(s) \]

Therefore, the overall equivalent transfer function is:

\[ \frac{C(s)}{R(s)} = \frac{G_3(s)}{G_0(s) + G_3(s)} \]

4. Summing point:

Summing point represents summing of two or more signal entering in a system. The output of a summing point being the sum of the entering signals.

\[ R_0(s) \quad R_1(s) + R_2(s) \quad Y(s) \]

5. Interchanging summing points: Consecutive summing points can be Interchanged, as this interchange does not alter the output signal.

\[ R_0(s) \quad R_1(s) + R_2(s) \quad Y(s) \]

6. Blocks in parallel:

When one or more blocks are connected in parallel, the overall equivalent transfer function is determined below:

\[ C(s) \quad R(s) \quad G_1(s) \quad G_2(s) \quad G_3(s) \quad G_0(s) \quad C(s) \]

Therefore, the overall equivalent transfer function is:

\[ \frac{C(s)}{R(s)} = \frac{G_0(s) + G_3(s) + G_2(s)}{G_0(s)} \]
The equivalence of above diagram is
\[ R(s) \rightarrow G(s) + G(s) + G(s) \rightarrow C(s) \]

7. Shifting of a take off point from a position before a block to a position after the block is shown below.

8. Shifting of a take off point from a position after a block to a position before the block is shown below.

9. Shifting of a summing point from a position before a block to a position after the block is shown below.

10. Shifting of a summing point from B position after a block to a position A before the block is shown below.

11. Shifting of a take off point from A position before a summing point to a position B after the summing point is shown below.
12. Shifting e take off point from a position after a summing point to a position before the summing point is as shown.

\[ R(s) \xrightarrow{A} B \quad C(s) = \frac{R(s) \ast X(s)}{1 + \frac{R(s) \ast X(s)}{C(s)}} \]

2.2 SIGNAL FLOW GRAPHS

A signal flow graph may be defined as a graphical means of portraying the input-output relationships between the variables of a set of linear algebraic equations.

Consider a linear system described by the set of \( N \) algebraic equations:

\[ y_j = \sum_{k=1}^{N} a_{jk} y_k \quad j = 1, 2, \ldots, N \]

Basic properties of signal flow graphs:

1. A signal flow graph applies only to linear systems.
2. The equations based on which a signal flow graph is drawn must be algebraic equations in the form of \( y = f(x) \) as functions of causes.
3. Nodes are used to represent variables. Normally, the nodes are arranged from left to right, following the sequence of causes and effects through the system.
4. Signals travel along branches only in the direction described by the arrows of the branches.
5. The branch directed from node \( y_i \) to \( y_j \) represents the dependence of the variable \( y_j \) upon \( y_i \), but not the reverse.
6. A signal \( y_i \) traveling along a branch between nodes \( y_i \) and \( y_j \) is multiplied by the gain of the branch, \( a_{ij} \), so that a signal \( a_{ij} y_i \) is delivered at node \( y_j \).

ACE Academy  BLOCK DIAGRAM S AND SIGNAL FLOW GRAPHS

Definitions for Signal Flow Graphs:

Input Node (Source): An input node is a node that has only outgoing branches.
Output Node (Sink): An output node is a node which has only incoming branches.
Path: A path is any collection of a continuous succession of branches traversed in the same direction.
Forward Path: A forward path is a path that starts at an input node and ends at an output node and along which no node is traversed more than once.
Loops: A loop is a path that originates and terminates on the same node and along which no other node is encountered more than once.
Path Gain: The product of the branch gains encountered in traversing a path is called the path gain.
Forward Path Gain: Forward path gain is defined as the path gain of a forward path.
Loop Gain: Loop gain is defined as the path gain of a loop.

Mason Gain Formula:
The general gain formula is

\[ \frac{M}{V_{in}} = \sum_{k=1}^{N} \frac{M_k}{\Delta_k} \]

where

- \( M \) = gain between \( y_0 \) and \( y_m \)
- \( V_{in} \) = output node variable
- \( y_0 \) = input node variable
- \( N \) = total number of forward paths
- \( M_k \) = gain of the \( k \)-th forward path

\[ \Delta = 1 - \sum P_{EA} + \sum P_{AB} - \sum P_{BA} + \cdots \]

- \( P_{EA} \) = (sum of all individual loop gains) + (sum of gain products of all possible combinations of two non-touching loops) - (sum of the gain products of all possible combinations of three non-touching loops) + \cdots

\[ P_{EA} = \text{gain product of the} m^{th} \text{possible combination of} \; r \text{non-touching loops} \]

\[ \Delta_k = \text{the} \Delta \text{for the part of the signal flow graph which is non-touching with the} \; k^{th} \text{forward path} \]
Objective Questions

0. The figure below gives two equivalent block diagrams

\[ X_1 \xrightarrow{G(s)} X_2 \xrightarrow{G(s)} Y_0 \]

The transfer function \( T(s) \) of the block marked 'X' is given by

(a) \( G(s) \)
(b) \( 1 / G(s) \)
(c) \( 1 + G(s) \)
(d) \( 1 + G(s) \)

0. The block diagram below is parallel.
(a) 1
(b) 2
(c) 3
(d) 4

The block diagram shown, the output \( y(t) \) is equal to

\[ u(t) \xrightarrow{G(s)} Y_0 \xrightarrow{X_1} \xrightarrow{X_2} \]

(a) \( U_1(s) + U_2(s) \)
(b) \( U_1(s) G(s) \)
(c) \( G_1(s) U_1(s) + G_2(s) U_2(s) \)
(d) \( G_1(s) + G_2(s) \)

0. The transfer function \( E_1(s) / E_2(s) \) of the RC network shown is given by

\[ E_1(s) \]

(a) \( \frac{1}{RC} \)
(b) \( \frac{1}{RC} + 1 \)
(c) \( RC \)
(d) None

0. The block diagram of a certain system is shown below

\[ U_0(s) \xrightarrow{X_1} \xrightarrow{X_2} \]

The transfer function \( T(s) \) is given by

(a) \( \frac{G(s)}{1 + G(s)} \)
(b) \( \frac{G(s)}{G(s)} \)
(c) \( G(s) \)
(d) \( G(s) \)

0. The closed-loop gain of the system shown below is

\[ R(s) \xrightarrow{X_1} \xrightarrow{X_2} \]

(a) \( -1/3 \)
(b) \( -1 \)
(c) \( 1 \)
(d) \( 1/3 \)

Objective Questions

1. In a signal flow graph, the nodes represent
(a) the system variables
(b) the system parameters
(c) the system parameters
(d) all the above

12. The branch of a signal flow graph represents
(a) the system variable
(b) the function of the variables
(c) the system parameters
(d) none of the above

13. By applying Mason's gain formula, it is possible to get
(a) the ratio of the output variable to input variable only
(b) the system transfer functions between any two variables
(c) the transfer gain of the system
(d) the ratio of any variable to input variable only

14. Two or more loops is a signal flow graph are said to be non-occurring
(a) if they do not have any common branch
(b) if they do not have any common node
(c) if they have common branch
(d) if they do not have any common node

15. The transfer function of the system shown is given by

\[ \frac{X_0(s)}{U(s)} = \frac{10s + 10}{(s + 1)(s + 2)(s + 3)(s + 4)} \]

Key for Objective Questions:
1. a 2. b 3. c 4. a 5. c 6. a 7. d 8. b 9. b 10. b

(a) \( 1 + 2 \times ACE \)
(b) \( 1 + 2 \times ACE \)
(c) \( 1 + 2 \times ACE \)
(d) \( 1 + 2 \times ACE \)
16. Signal flow graph is

(a) Topological representation of a non-linear differential equation.
(b) Schematic graph.
(c) Special type of graph for analysis of modern control system.
(d) Plot between frequency and magnitude in dB.

17. The signal flow graph shown, \( X_2 = T X_1 \), where \( T \) is equal to \( 0.5 \)

\[ X_1 \quad 2 \quad X_2 \]

(a) 2.5  (b) 5  (c) 5.5  (d) 10

18. For the signal flow graph shown in figure, \( X_1 / X_2 = \)

\[ X_1 \quad 10 \quad X_2 \quad 1 \quad X_3 \]

(a) 10  (b) 15  (c) 20  (d) 25

Key for Objective Questions:

11. a 12. b 13. c 14. d 15. e
16. c 17. d 18. e

OBJECTIVE QUESTIONS:

01. Which of the following is/are the example of the open loop control system?
   (A) Metronym
   (B) Field control d.c. motor
   (C) An automatic toaster
   (D) Both 'A' and 'C'

02. The OP of a feedback control system must be a function of

(a) reference input and output
(b) output
(c) reference input
(d) reference input and error signal

03. AC control system has the advantage of

(A) availability of rugged high power amplifiers
(B) smaller frame size of a.c components
(C) both A and B
(D) None

04. The transfer function of a system is the

(a) ramp response
(b) impulse response
(c) square wave response
(d) step response

05. Any physical system which does not automatically correct for variation on its OP is called as

(A) unstable system
(B) closed loop system
(C) closed loop system
(D) None

O6. In a control system the comparator compares the OP response and reference input and actsuates the

(A) primary sensing element
(B) transducer
(C) signal conditioner
(D) control elements

07. Transfer function of a control system depends on

(A) nature of output
(B) nature of input
(C) initial condition of input and output
(D) system parameters only

08. The open loop control system is one in which

(A) OP is independent of control IP
(B) only system parameter have effect on the control output
(C) OP is dependent on control IP
(D) All of these

09. A unit step function on integration results in a

(A) unit parabolic function
(B) unit doublet
(C) step function
(D) ramp function

10. The transfer function of a system is defined as

(a) step response
(b) response due to an exponentially varying input
(c) Laplace transform of the impulse response
(D) All of the above

11. The order of the system is determined by number of

(A) unstable pole terms in denominator
(B) poles at the origin
(C) stable pole of the system
(D) none

12. In a control system an error detector

(A) produces an error signal as actual difference of value and desired value of output
(B) detects the system error
(C) detects the error and signal an alarm
(D) None

13. Potentiometer are used in Control system

(A) to improve stability
(B) to improve frequency response
(C) as an error sensing transducer
(D) to improve time response

14. A control system with excessive noise, I likely to suffer from

(A) loss of gain
(B) vibrations
(C) oscillations
(D) saturation is amplifying stage

15. The type number of a transfer function denotes

(A) the number of poles at infinity
(B) the number of finite poles
(C) the number of zeros at origin
(D) the number of poles at origin

16. For the network given below, what is the transfer function?

\[ \frac{V_o(s)}{V_i(s)} = \frac{1}{sRC + 1} \]

(A) \( \frac{1}{sRC} \)  (B) \( sRC + 1 \)
(C) \( \frac{1}{sRC} \)  (D) \( 1 + sRC \)

17. The system response can be tested better with

(A) signal
(B) exponential decaying
(C) unit impulse input
(D) sinusoidal input

(E) ramp input
18. In a control system, the use of negative feedback
   (A) increases the influence of variations of component parameters on the system performance
   (B) reduces the effects of disturbance and noise signals in the forward path
   (C) increases the instability
   (D) eliminates the sources of instability

19. A signal flow graph is a
   (A) log log graph
   (B) special type of graph to analyze modern control systems
   (C) pole graph
   (D) topological representation of a set of differential equations

20. The signal flow graph for a system is shown below. The number of forward paths is

   (A) 4
   (B) 3
   (C) 1
   (D) 2

2. The impulse response of an infinite release system is \( e^{-t} u(t) \).
   The input must be equal to
   (A) 2 \( e^{-t} u(t) \)
   (B) 2 \( e^{-2t} u(t) \)
   (C) \( e^{-t} u(0) \)
   (D) \( e^{-u} u(0) \)

3. The Laplace transformation of \( f(t) \) is \( \mathcal{L}[f(t)] \).
   Given the \( \mathcal{L}[f(t)] = \frac{1}{s^2 + s + 1} \)
   final value of \( f(t) \) is
   (A) infinity
   (B) zero
   (C) one
   (D) indeterminate

4. As compared to closed loop system, an open loop is
   (A) more stable as well as more accurate
   (B) less stable as well as less accurate
   (C) more stable but less accurate
   (D) less stable but more accurate

5. The impulse response of a system is given by \( e^{-t} u(t) \). Which one of the following is its unit step response?
   (A) \( 1 - e^{-2t} \)
   (B) \( 1 + e^{-t} \)
   (C) \( 2 - e^{-t} \)
   (D) \( 1 - e^{-t} \)

6. Signal flow graph is used to find
   (A) stability of the system
   (B) controllability of the system
   (C) Transfer function of the system
   (D) poles of the system

7. The transfer function of a thermocouple is of the form
   \( G(s) = \frac{K}{s + k} \)
   (A) \( K \)
   (B) \( s \)
   (C) \( s + k \)
   (D) \( s + k + 1 \)

ESU's Question

8. The Laplace transformation of a time function \( f(t) \) is \( \mathcal{L}[f(t)] \).
   (A) \( e^{-t} \)
   (C) \( \frac{1}{s + 5} \)
   (D) \( \frac{1}{s} \)

ESU's Question

9. The state response has utmost importance for the design and analysis of control systems because these are inherently time domain systems where time is the independent variable. During the analysis of response, the variation of output with respect to time can be studied and it is known as time response. To obtain satisfactory performance of the system, the output behavior of the system with respect to time must be within the specified limits. From time response analysis and corresponding results, the stability of system, accuracy of system, and complete evaluation can be studied very easily.

Due to the application of an excitation to a system, the response of the system is known as time response and it is a function of time. There are two parts of response of any system: (i) transient response and (ii) steady-state response.

Transient Response:

The part of the time response which goes to zero after a large interval of time is known as transient response. In this case \( L \{ C(t) \} = 0 \). For transient response, we get the following information:
   (a) The time interval after which the system responds taking the instant of application of excitation as reference.
   (b) The total time that it takes to achieve the output for the first time.
   (c) Whether or not the output shoots beyond the desired value and how much.
   (d) Whether or not the output oscillates about its final value.
   (e) The time that it takes to settle to the final value.

Steady State Response:

The part of the response that remains even after the transient has died out is said to be steady-state response. From steady-state response, we get the following information:
   (a) The time that output takes to reach the steady state.
   (b) Whether or not any error exists between the desired and the actual value.
   (c) Whether this error is constant, zero, or infinite.

The total response of a system is the sum of transient response and steady-state response:

\( C(t) = C_d(t) + C_s(t) \)

Figure shows the transient and steady-state responses along with steady-state error.
3.1 Transient analysis

(1) Step function: A step function is described as sudden application of input signal as illustrated in figure.

\[ r(t) = A \cdot u(t) \]

Where:
- \( r(t) \) is the output
- \( A \) is the magnitude of the input
- \( u(t) \) is the unit step function

In the Laplace transform form, \( R(s) = \frac{A}{s} \).

Step function displacement function:

(2) Ramp function: The ramp function starts at a value of zero and increases linearly with time. Mathematically,

\[ r(t) = At \cdot u(t); \text{ for } t \geq 0 \]

In the Laplace transform form, \( R(s) = \frac{A}{s^2} \).

Ramp function is also called velocity function.

(3) Parabolic function: A parabolic function is described by a gradual application of input in comparison with many functions as illustrated in figure.

\[ r(t) = \frac{At^2}{2} \cdot u(t); \text{ for } t \geq 0 \]

If \( A = 1 \), then \( r(t) = \frac{t^2}{2} \) and the parabolic function is called unit parabolic function and the corresponding Laplace transform is \( R(s) = \frac{A}{s^3} \).

Parabolic function is also called acceleration function.

(4) Impulse function: A unit impulse is defined as a signal which has zero value everywhere except at \( t = 0 \), where its magnitude is infinite. It is generally called the \( \delta \) function and has the following property:

\[ \delta (t) = 0; \text{ for } t \neq 0 \]

Unit impulse function = \( \delta(t) \) (unit step function)

Hence, the Laplace transform of unit impulse function is derived from the Laplace transform of unit step function as follows:

\[ \frac{1}{s} \cdot (\text{unit impulse function}) + \frac{1}{s} = 1 \]

Time response of a First order Control System:

A first order control system is one where the highest power of \( s \) in the denominator of its transfer function equals 1. Thus, a first order control system is expressed by a transfer function given below:

\[ C(s) = \frac{1}{s + \alpha} \]

The block diagram representation of the above equation is shown in the below figure.

The transfer function of a first order control system subjected to unit step input function:

The output for the system is expressed as:

\[ C(s) = \frac{1}{s + \alpha} \]

As the input is a unit step function, \( r(t) = 1 \) and \( R(s) = \frac{1}{s} \).

Therefore, substituting in Eq. (1):

\[ C(s) = \frac{1}{s + \alpha} \cdot \frac{1}{s} \]

Breaking R.H.S into partial fractions:

\[ C(s) = \frac{1}{s + \alpha} \cdot \frac{1}{s} \]

\[ C(s) \] is:

Taking Laplace transform on both sides:

\[ e(t) \] is:

The error is given by:

\[ e(t) = r(t) - c(t) = 1 - (1 - e^{-\alpha t}) = e^{-\alpha t} \]

The steady state error is:

\[ \lim_{t \to \infty} e(t) = 0. \]

The time response to the above equation is shown in the figure.
Time response of a first order control system subjected to unit ramp input function:

The output for the system is expressed as

\[ C(s) = R(s) \frac{1}{sT + 1} \]

As the input is a unit ramp function is \( r(t) = t \) and \( R(s) = 1/s^2 \).

Therefore, \( C(s) = \frac{1}{s^2T + T} \)

By breaking R.H.S. into partial fractions,

\[ C(s) = \frac{1}{sT + 1} + \frac{1}{s + 1/T} \]

\[ C(s) = \frac{1}{s^2} + \frac{1}{sT + 1} \]

Taking inverse Laplace transform on both sides,

\[ e(t) = e^{-Tt} \left[ 1 - \frac{e^{-Tt}}{s^2} \right] \]

The error is given by

\[ e(t) = r(t) - e(t) = t - (1 - T e^{-t/T}) \]

The steady state error is \( e_{ss} = \lim_{t \to \infty} (1 - T e^{-t/T}) = T \)

The time response in relation to the above equation is shown in the figure.

Time response of a first order control system subjected to unit impulse input function:

The output for the system is expressed as

\[ C(s) = R(s) \frac{1}{sT + 1} \]

As the input to the system is a unit impulse function, its Laplace transform is 1, i.e., \( R(s) = 1 \), therefore,

\[ C(s) = \frac{1}{sT + 1} \]

Taking inverse Laplace transform on both sides of Eq(2)

\[ e(t) = (1/T) e^{-Tt} \]

A second order control system is one wherein the highest power of \( s \) in the denominator of its transfer function equals 2.

A general expression for the T.F. of a second order control system is given by

\[ C(s) = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2} \]

The block diagram representation of the transfer function given above is shown in the figure.

Characteristics of a second order control system:

The general expression for the T.F. of a second order control system is given by

\[ C(s) = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2} \]

The characteristic equation of a second order control system is given by

\[ s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \]

The equation of roots of transfer function of various values of \( \zeta \) (keeping \( \omega_n \) fixed) and the corresponding time response for a second order control system is shown in below figure.

Fig: Location of roots of the characteristic equation and corresponding time response.

From above figure, it is inferred that the change over from underdamped to overdamped response takes place at \( \zeta = 1 \). The value of \( \zeta \) from the location of roots is calculated as

\[ \zeta = \cos \theta \]
Time response of a second order control system subjected to unit step input function:

\[ C(s) = \frac{a_1 s^2}{s^2 + 2\zeta a_1 s + a_0^2} \]

As the input is a unit step function

Therefore, substituting in above equation

\[ C(s) = \frac{1}{s} \cdot \frac{a_1 s^2}{s^2 + 2\zeta a_1 s + a_0^2} \]

The solution for the above equation

\[ e(t) = 1 - \frac{\exp(-\zeta t \sqrt{a_1})}{\sqrt{1 - \zeta^2}} \sin \left( a_0 \sqrt{1 - \zeta^2} t + \tan^{-1} \left( \frac{\sqrt{1 - \zeta^2}}{\zeta} \right) \right) \]

The time response expression is given by the above equation for values of \( \zeta < 1 \) is,

- exponentially decaying oscillations having a frequency \( a_0 \sqrt{1 - \zeta^2} \) and the time constant of exponential decay is \( (1/\zeta) a_0 \).

Where

- \( a_0 \) is called natural frequency of oscillations.
- \( \zeta \) is called damping ratio.
- \( a_0 \sqrt{1 - \zeta^2} \) is called damped frequency of oscillations.
- \( a_0 \) affects the damping and called damping ratio.
- \( a_0 \) is called damping factor or actual damping or damping coefficients.

\[ e(t) = \frac{\exp(-\zeta t \sqrt{a_1})}{\sqrt{1 - \zeta^2}} \sin \left( a_0 \sqrt{1 - \zeta^2} t + \tan^{-1} \left( \frac{\sqrt{1 - \zeta^2}}{\zeta} \right) \right) \]

\[ e(t) \]

\[ c(t) = \frac{\exp(-\zeta t \sqrt{a_1})}{\sqrt{1 - \zeta^2}} \sin \left( a_0 \sqrt{1 - \zeta^2} t + \tan^{-1} \left( \frac{\sqrt{1 - \zeta^2}}{\zeta} \right) \right) \]

\[ t \]

\[ e(t) \]

\[ c(t) \]

\[ t \]

\[ e(t) \]

Fig. Time response of a second order control system (\( \zeta < 1 \), under damped case) subjected to unit step input function.

The time response of a second order control system is influenced by its damping ratio (\( \zeta \)). The cases for the values of damping ratio are

- (a) \( \zeta \leq 1 \)
- (b) \( \zeta = 0 \)
- (c) \( 0 < \zeta < 1 \)
- (d) \( \zeta = 1 \)
- (e) \( \zeta > 1 \)

The time response of an underdamped control system exhibits damped oscillations prior to reaching steady state. The specifications pertaining to time response during transient period are shown in the following figure.

- Delay Time: \( t_d \)
- Time required for the response to rise from zero to 50% of the final value.

\[ t_d = 1.3 \frac{a_0}{\omega_n} \]
(2) The rise time : $t_r$

The rise time is the time needed for the response to reach from 10 to 90% or 0 to 100
% of the desired value of the output at the very first instant. Usually 0 – 100% basis is used
for underdamped systems and 10 to 90% for overdamped system.

$$t_r = \frac{X - \phi}{\alpha_1 \sqrt{1 - \zeta^2}}$$

where $\phi = \tan^{-1} \left( \frac{\sqrt{1 - \zeta^2}}{\zeta} \right)$

(3) peak time : $t_p$

It is the time required for required for the response to rise from peaks of the response
$\zeta = \frac{R}{a_2}$

(4) Maximum overshoot : $M_o$

It is given by the normalized difference between time response peak to steady state O/P

Percentage $M_o = \frac{C(t_{\text{peak}}) - C(\infty)}{C(\infty)} \times 100$

$\%M_o = \exp \left( \frac{-Cz_1}{C(z_1 - 1)} \right) \times 100$

A graph relating $M_o$ and $\zeta$ is plotted below in figure.

(5) The settling time : $t_s$

For 2% tolerance band, the settling time is given by

$$t_s = \frac{4}{\zeta a_1}$$

On 5% basis the settling time for a second order control system is given by

$$t_s = \frac{3}{\zeta a_1}$$

An expression for the time response of a second order control system having

$\zeta = 1$ (critically damped) when subjected to a unit step input function is:

$$c(t) = \exp \left( -\zeta a_1 t \right) \left( 1 + a_1 t \right)$$

The time response in equation $E_x(11)$ is plotted in the below figure. The response is called
critically damped response.

Time response of a second order C.S. ($\zeta > 1$, over-damped ) subjected to a unit step function.

An expression for the time response of a second order control system having

$\zeta > 1$ (Overdamped) when subjected to unit step input function is derived hereunder:

The output for the system is given by

$$C(s) = \frac{\alpha_2}{s^2 + 2\zeta a_1 s + a_1^2}$$

As the input is a unit step function $r(t) = 1$ and $R(t) = 1\, /\, s$, therefore,

$$\alpha_2 = \frac{\alpha_1^2}{s^2 + 2\zeta a_1 s + a_1^2}$$

It can also be written as $C(s) = \frac{1}{s^2 + 2\zeta a_1 s + a_1^2}$

$$= \frac{1}{s^2 + 2\zeta a_1 s + a_1^2}$$

Expanding R.H.S of above equation into partial fractions,

$$C(s) = \frac{1}{s} - \frac{1}{s} \frac{1}{s + \sqrt{\zeta^2 - 1} a_1} \frac{1}{s + (\zeta - \sqrt{\zeta^2 - 1}) a_1}$$

Taking inverse Laplace transform on both sides

$$c(t) = 1 - \frac{\exp \left[ (\zeta - \sqrt{\zeta^2 - 1}) a_1 t \right]}{2 \sqrt{\zeta^2 - 1} (\zeta - \sqrt{\zeta^2 - 1})} + \frac{\exp \left[ (\zeta + \sqrt{\zeta^2 - 1}) a_1 t \right]}{2 \sqrt{\zeta^2 - 1} (\zeta + \sqrt{\zeta^2 - 1})}$$
OBJECTIVE QUESTIONS

01. The radial distance between a pole and the origin gives
   a) damping frequency of oscillation
   b) undamped frequency of oscillation
   c) time constant
   d) natural frequency of oscillation.

02. For a type 1, second order control system, when there is an increase of 25%
     in its natural frequency, the steady-state error to unit step input is
     a) increased by 20% of its value.
     b) equal to 2 \( \zeta / \omega_n \), where \( \zeta = \) damping factor.
     c) decreased by 21%.
     d) decreased effectively by 20%.

03. In a type 1, second order system, first peak overshoot occurs at a time equal to
   \( \frac{\pi \omega_n}{\sqrt{1 - \zeta^2}} \).

04. Type number of a system gets decreased if
   a) an integrator is included in the system.
   b) an integrator is included in the forward path.
   c) a controller with in parallel path.
   d) a differentiator is included in the forward path.

05. When the pole of a system is moved towards the imaginary axis, then
   a) settling time decreases.
   b) settling time increases by 50% of initial value.
   c) steady-state error is reduced to zero.
   d) settling time of the system increases.

06. The damping factor of a second-order system whose response to unit step
     input is showing sustained oscillations is
     a) \( \zeta = 1 \)
     b) \( \zeta > 1 \)
     c) \( \zeta = 0 \)
     d) \( \zeta > 0 \)

07. The transient response of a system with feedback compared to that without
     feedback
     a) decays slowly.
     b) rises slowly.
     c) rises more quickly.
     d) decays more quickly.

08. The settling time for the system
     \[ G(s) = \frac{5}{s^2 + 5s + 2} \]
     is approximately equal to the output settles within \( \pm 2\% \) for a unit step input
     a) 0.8
     b) 1.2
     c) 2.0
     d) 1.6

09. The type of the system whose transfer function is given by
     \[ G(s) = \frac{s + 3}{s^2 + 4s + 2} \]
     a) 3
     b) 2
     c) 5
     d) 1

10. Physically the damping ratio \( \zeta \) represents the
    a) energy available for transfer.
    b) energy available for exchange.
    c) ratio of energy available for exchange to that available for transfer.
    d) ratio of energy lost to the energy available for exchange.

11. The static acceleration constant of a type 2 system is
    a) \( \infty \)
    b) \( 0 \)
    c) cannot be found out
    d) finite
3.2 Steady state Analysis:

The steady state part of time response reveals the accuracy of a control system. Steady state error is observed if the actual output does not exactly match with the input.

\[ e(t) = r(t) - e(t) \]

Steady state error, \( e_{ss} = \text{Lt} \to e(t) \)

Using final value theorem,

\[ e_{ss} = \text{Lt} \to s E(s) \]

\[ C(s) = E(s) G(s) \Rightarrow E(s) = \frac{C(s)}{G(s)} \]

\[ e_{ss} = \text{Lt} \to s \frac{R(s)}{1 + G(s)} \]

The open-loop transfer function, the type indicates the number of poles at the origin and the order indicates the total number of poles. The type of the system determines steady state response and the order of the system determines transient response.

**Standard test signals used in Steady state response:**

1. Step input signal:

\[ r(t) = A \tau(t) \]

\[ R(s) = \frac{A}{s} \]

2. Ramp input signal:

\[ r(t) = At \]

\[ R(s) = \frac{A}{s^2} \]

Key for Objective Questions:

1. rise time
2. peak time
3. setting time
4. peak overshoot

(3) Parabolic input signal:

\[ r(t) = \frac{At^2}{2} \]

\[ R(s) = \frac{A}{s^3} \]

For step input:

\[ e_{ss} = \text{Lt} \to \frac{R(s)}{1 + G(s)} \]

\[ = \text{Lt} \to s \frac{\frac{A}{s^2}}{1 + G(s)} \]

\[ = \frac{\text{Lt} \to s \frac{A}{s^2}}{1 + \text{Lt} \to s \frac{1}{s + G(s)}} = \frac{A}{1 + K_p} \]

where \( K_p = \text{Lt} \to G(s) = \text{Position error constant} \)

For ramp input:

\[ e_{ss} = \text{Lt} \to s \frac{R(s)}{1 + G(s)} \]

\[ = \text{Lt} \to s \frac{A \tau}{1 + G(s)} \]

\[ = \frac{\text{Lt} \to s \frac{A}{s^2} \tau}{s \frac{1}{s + G(s)}} = \frac{A}{1 + K_v} \]

where \( K_v = \text{Lt} \to s \frac{1}{s + G(s)} = \text{Velocity error constant} \)

For parabolic input:

\[ e_{ss} = \text{Lt} \to s \frac{R(s)}{1 + G(s)} \]

\[ = \text{Lt} \to s \frac{A \tau^2}{1 + G(s)} \]

\[ = \frac{\text{Lt} \to s \frac{A}{s} \tau^2}{s \frac{1}{s + G(s)}} = \frac{A}{1 + K_a} \]

where \( K_a = \text{Lt} \to s^2 \frac{1}{s + G(s)} = \text{Acceleration error constant} \)

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OBJECTIVE QUESTIONS

01. The presence of non-linearities in a control system tends to introduce
   a) transient error  b) instability  
   c) static error  d) steady-state error

02. The static acceleration constant of a type 2 system is
   a) infinite  b) zero  
   c) cannot be found  d) finite

03. The transfer function of the system which will have more steady state error for step input is
   a) \( \frac{80}{(s+1)(s+2)(s+3)} \) 
   b) \( \frac{120}{(s+1)(s+3)} \) 
   c) \( \frac{60}{(s+0.5)(s+3)(s+5.5)} \) 
   d) \( \frac{120}{(s+1)(s+4)(s+15)} \)

04. The presence or absence of steady state error for any given system depends upon
   a) presence or absence of pole at the infinity.  
   b) presence or absence of poles and zeros at the origin.  
   c) absence or presence of zeros at the origin.  
   d) absence or presence of pole at the origin.

05. When the gain "k" of a system is increased, the steady-state error of the system
   a) increases  
   b) remains unchanged  
   c) may increase or decrease  
   d) decreases.

06. The plant is represented by the transfer function. The system is given a degenerative feedback. The effective of the feedback is to shift the pole
   a) positively to \( s = \alpha + k \) and reduce the time constant to \( \alpha + 1/k \) 
   b) negatively to \( s = -\alpha + k \) and increase the time constant to \( \alpha + 1/k \) 
   c) positively to \( s = -(\alpha + k) \) and reduce the time constant to \( 1/\alpha + 1/k \) 
   d) negatively to \( s = -(\alpha + k) \) and decrease the time constant to \( 1/\alpha + 1/k \).

07. In a system with input \( R(s) \) and output \( C(s) \), the transfer functions of the plant and the feedback system are given by \( G(s) \) and \( H(s) \) respectively. The system has got a negative feedback. Then the error signal is given by the expression:
   a) \( E(s) = \frac{G(s)R(s)}{1 + G(s)H(s)} \) 
   b) \( E(s) = \frac{C(s)G(s)}{1 + G(s)H(s)} \) 
   c) \( E(s) = \frac{1}{1 + G(s)H(s)} \) 
   d) \( E(s) = \frac{G(s)}{1 + G(s)H(s)} \)

08. The static error constants depend on
   a) the order of the system 
   b) the type of the system 
   c) both type and order of the system 
   d) None of the above.

Key:

ACE Academy  Time Domain Analysis

07. A unity feed back system has transfer function \( G(s) = \frac{2}{(s+3)} \), what?
   a) natural frequency \( \omega_n = 2 \)  
   b) peaks frequency \( \omega_p = 3 \) 
   c) damping ratio \( \zeta = 0.5 \)  
   d) damping ratio \( \zeta = 0.9 \)

08. The system whose characteristic equation \( s^3 + 6s^2 + 5 = 0 \) has the following roots
   a) \( -3, -2, 1 \) 
   b) \( -3, -2, 1 \) 
   c) \( -3, -2, 0 \) 
   d) \( 2+3, 2-3, 2 \)

09. The overshoot of the system having the transfer function 25s^2+25 for a unit step input applied would be
   a) 20%  
   b) 30% 
   c) 35%  
   d) 100%

10. Potentiometers are used in control system.
    a) to improve stability  
    b) to improve frequency response  
    c) as error sensing transducers  
    d) to improve time response

11. The position and velocity errors of a type 2 system are
    a) zero, constant  
    b) constant, constant  
    c) zero, zero  
    d) constant, infinity

12. Bandwidth is used as means of specifying performance of a control system related to
    a) the speed of response  
    b) the constant gain  
    c) relative stability of the system  
    d) all of the above
13. The value of steady-state error to the type 1 system, when the input signal is a step of magnitude 2, will be
(A) 0.5  (B) 2.5  (C) 1.54  (D) zero

14. Which of the following system is generally preferred
(A) critically damped  (B) under damped  (C) over damped  (D) oscillatory

15. As the system type becomes higher steady state error
(A) remains constant  (B) increases  (C) is eliminated  (D) none of the above

16. The steady state acceleration error for a type 1 system is
(A) between zero and unity  (B) zero  (C) infinite  (D) unity

17. For a second order differential equation of the damping ratio is 1, the
(A) the poles are equal, negative and real  (B) both poles are negative and real  (C) the poles are in right half of the plane  (D) the poles are in a left half of the plane

18. The position and velocity errors of a type 2 systems
(A) zero, infinity  (B) zero, zero  (C) zero, constant  (D) constant, zero

19. With the feedback system, the transient response
(A) rises slowly  (B) rises quickly  (C) decays slowly  (D) decays rapidly

20. Error constants of a system are number of
(A) steady state response  (B) steady response as well as transient state response  (C) relative stability  (D) transient state response

21. Static error coefficients are used in a measure of the effectiveness of closed loop systems for specified
(A) velocity input signal  (B) acceleration input signal  (C) position input signal  (D) all of the above

22. The transient response of a system is mainly due to
(A) friction  (B) stored energy  (C) internal forces  (D) inertia forces

23. For any given closed loop system, all the coefficients are always zero
(A) the all the coefficients can have zero value  (B) only two of the static error coefficients has a finite non zero value  (C) all of the above  (D) none of the above

24. A unity feedback system has open loop transfer function
\[ G(s) = \frac{1}{(s+1)^2} \]. The pole of the closed loop system is located on the real axis in the S-plane is
(A) -2  (B) -1  (C) 2  (D) -5

25. The output of a linear system for a unit step input is given by \( e(t) \). The transfer function is given by
\[ \frac{28(s+1)^2}{(s+5)^2} \]
\[ \frac{1}{(s+1)} \]
\[ \frac{(s+5)^2}{(s+1)^2} \]

26. The steady-state error coefficient for a system is given by \( K_p = 0 \), \( K_v = \) finite constant, \( K_i = \infty \). The system is
(A) Type - 1 system  (B) Type - 2 system  (C) Type - 3 system  (D) Type - 0 system

27. In the case of second order differential equation when the damping ratio is less than 1 then
(A) poles will be complex conjugate  (B) poles will be complex conjugate & negative  (C) poles will be equal, negative and real  (D) poles will be positive

28. The error signal produced in a control system is a constant. The output of P action will be
(A) linear  (B) infinity  (C) constant  (D) zero

29. The transient response of the system depends on
(A) input  (B) output  (C) system (D) none

30. The steady state response of the system depends on
(A) input  (B) output  (C) system  (D) input & output

31. The normal range of damping ratio for a control system is
(A) 0.5 to 1.0  (B) 0.5 to 2.0  (C) 0.5 to 0.5  (D) 0.5 to 2.5

32. The study state error due to a ramp input for a type two system is equal to
(A) zero  (B) infinite  (C) constant  (D) data is insufficient

33. Given the transfer function
\[ G(s) = \frac{1}{s^2 + 13.2 s + 121} \]

01. The study state error due to a ramp input for a type two system is equal to
(A) zero  (B) infinite  (C) constant  (D) data is insufficient

02. Given the transfer function
\[ G(s) = \frac{1}{s^2 + 13.2 s + 121} \]

03. Consider the following statements with a reference to a system with velocity error constant \( K_v = 1000 \)

1. The system is stable
2. The system is of type 1
3. The test signal used is a step input
Which of these statements are correct?
(A) 1 & 2  (B) 1 & 3  (C) 2 & 3  (D) 1, 2, & 3

04. The close loop transfer function of control system is given
\[ G(s) = \frac{1}{s^2 + 13.2 s + 121} \]

For the input \( e(t) \) = unit, the steady state value of \( e(t) \) is equal to
(A) \( \frac{1}{2} \)  (B) 1  (C) \( \frac{1}{2} \)  (D) \( \frac{1}{2} \) sin \( t \)
05. The transient response of a system is mainly due to:
(A) internal forces
(B) Stored energy
(C) Friction
(D) Inertial forces

06. A system is critically damped. Now if the gain of the system is increased, the system will behave as:
(A) over damped
(B) Underdamped
(C) Oscillatory
(D) Critically damped

07. Consider a system with the transfer function
\[ G(s) = \frac{K}{s^2 + 3s + 2} \]
where  \( K \) is the gain of the system in radians/s^2. For this system to be critically damped, the value of \( K \) should be:
(A) 1
(B) 2
(C) 3
(D) 4

09. The impulse response of a system is given by \( e^{-2t} u(t) \). Its transfer function is:
(A) \( \frac{1}{s + 2} \)
(B) \( \frac{1}{s^2 + 2s} \)
(C) \( \frac{1}{s + 2s} \)
(D) \( \frac{1}{s + 2} \)

10. The impulse response of a system is given by \( (s + 2)^2 \). Which one of the following is its unit step response?
(A) \( -e^{-2t} \)
(B) \( 1 - e^{-2t} \)
(C) \( -e^{-2t} \)
(D) \( 1 - e^{-2t} \)

11. A system is represented by
\[ \dot{y} - 2y = 4t u(t) \]
The ramp component in the forced response will be:
(A) \( 4t u(t) \)
(B) \( 2t u(t) \)
(C) \( 3t u(t) \)
(D) \( 4t u(t) \)

12. Which one of the following is the steady state error of a step input applied to a unity feedback system with the open loop transfer function
\[ G(s) = \frac{10}{s^2 + 14 + 50} \]
when the value of \( K \) is
(A) 2/5
(B) 3/5
(C) 1/5
(D) 1/6

13. The unit step response of a particular control system is given by \( e(t) = 1 - 10e^{-t} \), then its transfer function is:
(A) \( 1/s + 1 \)
(B) \( 1/s^2 + 1 \)
(C) \( 1 - 5e^{-s}/s + s + 1 \)
(D) \( 1 - 5e^{-s}/s + s^2 + 1 \)

14. The open loop transfer function
\[ G(s) = \frac{1}{s(s + 1)} \]
an unity feedback control system is The system is subjected to an input \( 10 \) sin \( t \). The steady state error will be:
(A) zero (not defined)
(B) 1
(C) \( 2 \) sin \( t \) - \( s/4 \)
(D) \( 2 \) sin \( t \) + \( s/4 \)

15. A second order system has the damping ratio \( \zeta \) and undamped natural frequency of oscillation \( \omega_n \), the settling time at 2% tolerance bandwidth of the system is:
(A) \( \zeta \omega_n \)
(B) \( \frac{4}{\zeta} \omega_n \)
(C) \( \frac{4}{\zeta} \omega_n \)
(D) \( \frac{4}{\zeta} \omega_n \)

16. The response \( e(t) \) to a system is described by the differential equation
\[ \frac{d^2 e(t)}{dt^2} + 4\frac{de(t)}{dt} + 5e(t) = 0 \]
The system response is:
(A) un damped
(B) under damped
(C) critically damped
(D) oscillatory

17. The steady state error of a stable type unity feedback system for a unit step function is:
(A) 0
(B) \( \frac{1}{1 + K} \)
(C) 1
(D) \( \frac{1}{K} \)

18. What is the steady state error for a unity feedback control system having \( G(s) = \frac{1}{s(s + 1)} \) due to unit ramp input?
(A) 1
(B) 0.5
(C) 0.25
(D) 0.5

19. When the time period of observation is large, the type of the error is:
(A) Transient error
(B) Steady state error
(C) Half power error
(D) Position error constant

20. For which of the following input, the error series using dynamic error coefficients doesn't converge:
(A) Step input
(B) Ramp input
(C) Acceleration (parabolic) input
(D) Sinusoidal input

JTO KEYS:
01. C 02. C 03. D 04. D 05. B
31. A

PSU KEYS:
01. A 02. B 03. A 04. D 05. B
06. B 07. C 08. D 09. B 10. A
Concept of stability:

Any system is called a stable system if the output of the system is bounded for a bounded input. Any signal is called bounded if the max. and min. value are finite.

Stability of any system depends only on the location of poles but not on the location of zeros.

1. If the poles are located in left side of s-plane, then the system is stable.
2. If the roots are located on imaginary axis including the origin (except repeated roots), the system is stable.
3. If the poles are located in right half of s-plane, then the system is unstable.

At poles is approaches origin, stability decreases.

When roots are located on imaginary axis, then the system is marginally stable.

The poles which are close to the origin are called dominant poles.

The systems are classified as:

1. Absolutely stable systems
2. Unstable systems
3. Conditionally stable systems

When variable parameter is varied from 0 to \( \infty \), if the poles are located on left side and it is always stable, then it is absolutely stable.

When variable parameter is varied and a system is stable for values 0 to \( \infty \) at some point onwards there is zero pole(s) in right side then it is called conditionally stable.

Techniques used to calculate stability are:

1. Routh–Hurwitz criterion
2. Root locus
3. Routh plot
4. Nyquist plot
5. Nichols chart

Relative Stability Analysis:

Once a system is shown to be stable, we proceed to determine its relative stability quantitatively by finding the settling time of the dominant roots of its characteristic equation. The settling time is inversely proportional to the real part of the dominant roots, the relative stability can be specified by requiring that all the roots of the characteristic equation be more negative than a certain value, i.e., all the roots must lie to the left of the line \( s = -\alpha \) (\( \alpha > 0 \)). The characteristic equation of the system under study is then modified by shifting the origin of the s-plane to \( s = -\alpha \), i.e., by the substitution \( s = \sigma + j\omega \).

If the new characteristic equation in \( \sigma \) satisfies the Routh criterion, it implies that all the roots of the original characteristic equation are more \( \sigma \)-negative than \( -\alpha \).

4.4 ROU\textsc{th}–HUR\textsc{witz} CR\textsc{iterion}:

The Routh–Hurwitz criterion represents a method of determining the location of poles of a polynomial with constant real coefficients with respect to the left half and the right half of the s-plane, without actually solving for the poles.

Consider the characteristic equation of a linear time-invariant system is of the form

\[
F(s) = a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \ldots + a_1 s + a_0 = 0
\]

In order that there be no roots of the last equation with positive real parts, it is necessary but not sufficient that:

1. All the coefficients of the polynomial have the same sign.
2. None of the coefficients vanishes.

The necessary and sufficient condition that all roots of above equation lie in the left half of the s-plane is that the Polynomial's Hurwitz determinants must all be positive.

The roots of the polynomials are all in the left half of the s-plane if all the elements of the first column of the Routh tabulation are of the same sign. If there are changes of signs in the elements of the first column, the number of sign changes indicates the number of roots with positive real parts.

The following difficulties may occur occasionally when carrying out the Routh test:

1. The first element in any one row of the Routh tabulation is zero, but the other elements are not.
2. The elements in one row of the Routh tabulation are all zero.
Example: Consider the following equation:
\[ s^4 + 1.5s^3 + 2s^2 + 4s^2 + 5s + 10 = 0 \]

The Routh array is given below:

| s^4 | 1 | 5 | K |
| s^3 | 5 | 4 | 12 |
| s^2 | 21/5 | K |
| s^1 | \[ \frac{(445 - 5K)}{21/5} \] |
| s^0 | K |

The Routh array for this equation is

\[ s^4 + 5s^3 + 3s^2 + 4s + K = 0 \]

Since for a stable system, the signs of elements of the first column of the Routh array should be all positive, the condition of system stability requires that

\[ K > 0 \]

and

\[ \frac{(445 - 5K)}{21/5} > 0 \]

Therefore for stability, K should be lie in the range

\[ \frac{445}{21} > K > 0 \]

Special Cases:

Difficulty 1: When the first term is in any row of the Routh array is zero while rest of the row has at least one nonzero term.

Because of this zero term, the terms in the next row become infinite and Routh's test breaks down. The following method can be used to overcome this difficulty.

Example: Determine the stability of a closed-loop control system whose characteristic equation is

\[ s^3 + s^2 + 2s + 2s + 15s + 10 = 0 \]

The Routh array is formed below:

| s^4 | 1 | 5 | 2 |
| s^3 | 1 | 10 |
| s^2 | 0 | 10 |

while forming the Routh array as above, the third element in the first column is zero and thus the Routh criterion fails at this stage. The difficulty is solved if zero in the third row of the first column is replaced by a symbol "c" and Routh array is formed as follow:

| s^4 | 1 | 2 | 11 |
| s^3 | 0 | 10 |
| s^2 | c | 10 |
| s^1 | \[ \lim_{s \to \infty} \left( \frac{2c-1}{s} \right) = 0 \] |
| s^0 | \[ \lim_{s \to \infty} \left( 1 - \frac{10s^2}{2s-1} \right) = 0 \] |

The limits of the fourth and fifth element in the first column as \( s \to \infty \) from positive side are \( -\infty \) and +1 respectively indicating two sign changes, therefore, the system is unstable and the number of roots with positive real parts of the characteristic equation is 2.

Difficulty 2: When all the elements in any one row of the Routh array are zero.

This condition indicates there are symmetrically located roots in the s-plane. Because of a zero row in the array, the Routh's test breaks down. This situation is overcome by replacing the row of zeros in the Routh array by a row of coefficients of the polynomial generated by taking the first derivative of the auxiliary polynomial. The following example illustrates the procedure.

Example: Determine the stability of a system having following characteristic equation:

\[ s^4 + s^3 + 3s^2 + 2s^2 + 4s + 8 = 0 \]

The Routh table is formed below:

| s^4 | 1 | 5 | 2 |
| s^3 | 1 | 10 |
| s^2 | 0 | 0 |
| s^1 | AUXILIARY EQUATION |
| s^0 | 0 | 0 |
It is observed in this example that all the elements in the fourth row vanish and the application of Routh’s criterion fails. This situation occurs when the array has two consecutive rows having the same ratio of corresponding elements.

This difficulty faced is overcome by forming an auxiliary equation using elements of the last but one vanishing row. The derivative of this auxiliary equation is taken w.r.t. ‘s’ and the coefficients of the differentiated equation are taken in the denominator of the following row:

The auxiliary equation is

\[ \Delta(s) = 2s^4 + 6s^3 - 8 \]

And

\[ 4\Delta(s)/4s = 8s^2 + 12s - 0 \]

The coefficients of the fourth row are thus 8, 12 and 0. The modified Routh array is given by:

| \( s^4 \) | 3 | 5 | 2 | -8 |
| \( s^3 \) | 1 | 3 | -4 | 0 |
| \( s^2 \) | 2 | 6 | -8 | 0 |
| \( s^1 \) | 8 | 12 | 0 | 0 |
| \( s^0 \) | 3 | -8 | 0 | 0 |

coefficients of differentiated A.E.

| \( s^3 \) | 100/3 | 0 | 0 | 0 |

Sign change

| \( s^2 \) | -8 | 0 | 0 | 0 |

As there is one sign change in the first column, the system has one root with positive real part indicating that the system is unstable.

**OBJECTIVE QUESTIONS**

Q1. The transfer function of a system is

\[ G(s) = \frac{K}{s^3 + 3s^2 + 5s + 2} \]

For the system to be absolutely stable,

a) \( a_3, a_2, a_1, a_0 > 0 \) and \( a_3 - a_2 + a_1 - a_0 < 0 \)

b) \( a_3, a_2, a_1, a_0 > 0 \) and \( a_3 a_2 - a_2 a_1 + a_1 a_0 < 0 \)

c) \( a_3, a_2, a_1, a_0 > 0 \) and \( a_3 a_2 a_1 - a_2 a_1 a_0 = 0 \)

d) \( a_3 a_2 a_1 > 0 \) and \( a_3 a_2 a_1 < 0 \)

Q2. Routh’s array for a system is given below:

| \( s^3 \) | 1 | 3 | 5 |
| \( s^2 \) | 1 | 2 | 0 |
| \( s^1 \) | 1 | 5 |
| \( s^0 \) | -3 | 0 |

The system is

a) stable
b) unstable
c) marginally stable
d) conditionally stable

Q3. The number of sign changes in the entries in the first column of Routh’s array denotes

a) the number of zeros of the system in the RHP
b) the number of roots of characteristic polynomial in RHP
c) the number of open-loop poles in RHP
d) the number of open-loop zeros in RHP

Q4. Construct a characteristic equation

\[ s^4 + 3s^3 + 5s^2 + 2s + K = 0 \]

The condition for stability is

a) \( K > 5 \)
b) \( K > -10 \)
c) \( K < -4 \)
d) \( -10 < K < -4 \)

Q5. The characteristic equation of a unity feedback system is given by

\[ s^4 + 3s^2 + 4s + 1 = 0 \]

a) The system has one pole in the RHP plane
b) The system has no poles in the RHP plane
c) The system is asymptotically stable
d) The system exhibits oscillatory behavior

Q6. An electromechanical closed-loop control system has the following characteristic equation

\[ s^4 + 4s^3 + (K-2)s^2 + 8 = 0 \]

where \( K \) is the forward gain of the system. The condition for closed-loop stability is

a) \( K = 0.528 \)
b) \( K = 2 \)
c) \( K = 2.528 \)

d) \( K = 5 \)

Q7. The case in the Routh table in which a particular row elements are zero, show that

a) a differentiation has to be carried out
b) a is a special case in Routh array
c) whether the system is stable or not
d) some roots are distributed symmetrically about the origin.

Q8. The system is represented by its transfer function has some poles lying on the imaginary axis, it is

a) intrinsically stable
b) conditionally stable
c) unstable
d) marginally stable

Q9. In the first column of the Routh array, an element was found to be zero. The first column element above this zero and below this zero has the same sign. This condition indicates that the system

a) is stable
b) is unstable
c) has all the roots in LHP except one
d) has some roots on the jω-axis
4.2 ROOT LOCUS TECHNIQUE

It is the graphical representation of the roots of the characteristic equation, then the variable parameter is varied from 0 to ∞.

1) Root Locus (RL) \( (K \to 0 \text{ to } \infty) \)
2) Complementary RL \( (K \to 0 \text{ to } \infty) \)
3) Complete RL \( (K \to \infty \text{ to } \infty) \)
4) Root contour \( \text{(Multiple parameter variation)} \)

Concept of Root locus:

It is not possible to plot the root locus if there is no variable parameter in characteristic equation.

Classification of stable systems:

1) Undamped system \( \text{(roots on imaginary axis i.e., real part = 0)} \)
2) Under damped system \( \text{(imaginary but real part is negative)} \)
3) Critically damped \( \text{(roots are real and same)} \)
4) Over damped system \( \text{(roots are real and different)} \)
Rules for the construction of Root Locus:

- The root locus is always symmetrical with respect to the real axis.
- The root locus always starts (K=0) from the open-loop poles and terminates (K=∞) on either finite open-loop zeros or infinity. This statement is valid only if P = Z.
- The number of separate branches of the root locus equals either the number of open-loop poles or number of open-loop zeros whichever is greater.
  \[ N = P, \text{if } P > Z \]
  \[ N = Z, \text{if } Z > P \]
- A section of root locus lies on the real axis if the total number of open-loop poles and zeros to the right of the section is odd.
- The value of 'K' at any point on the root locus can be calculated by using the magnitude criterion:
  \[ K = \text{Product of poles magnitude (or length)} / \text{Product of zeros magnitude (or length)} \]
- If P ≠ Z, none of the branches terminate at 'z' or some of the branches will start from 'z'.
  If P > Z, (P - Z) branches will terminate at 'z'.
  If Z > P, (Z - P) branches will start from 'z'.
- Whenever any branch will terminate at 'z' means that a zero is located at 'z'.
- Whenever any branch is start from 'z' means that a pole is located at 'z'.
- The angle of asymptotes:
  If P ≥ Z, (P - Z) branches will terminate at 'z' along straight line asymptotes whose angles are:
  \[ (2n + 1)180° / (P - Z) \]
  If Z ≥ P, (Z - P) branches will start from 'z' along straight line asymptotes whose angles are:
  \[ (2n + 1)180° / (Z - P) \]

Stability:

Centroid: The asymptotes meet the real axis at centroid.

\[ \frac{\text{Sum of real parts of poles} - \text{Sum of real parts of zeros}}{P - Z} \]

- Intersection points with imaginary axis: The value of 'K' and the point at which the Root locus branch crosses the imaginary axis is determined by applying the criterion to the characteristic equation. The roots at the intersection point are imaginary.

- Break-away point and break-in point:
  Break-away point is calculated when root locus lies between two poles.
  Break-in point is calculated when root locus lies between two zeros.

\[ \frac{dK}{ds} = 0 \]

Procedure:

a) From the characteristic equation (C.E.)
b) Rewrite the characteristic equation in the form of K = f(s)
c) \[ \frac{dK}{ds} = 0 \]
d) The root of dK/ds = 0 gives the valid and invalid break point
e) The valid break point which must be on root locus branch

- Angle of arrival:
  It is applied when there are complex zeros.
  \[ \theta_a = 180° + \phi \]
  where \( \phi = \angle \) poles - \( \angle \) zeros

- Angle of departure:
  It is applied when there are complex poles.
  \[ \theta_d = 180° - \phi \]
Complementary Root locus:

In this the magnitude criteria remains same but angle criteria changes.

i.e., \( \angle \text{zeros} - \angle \text{poles} = \) even multiples of \( \pi \)

1) Asymptotic angles = \( \frac{24 \pi}{P - Z} \)

2) Angle of departure = \( 180^\circ - \phi \)

where \( \phi = \angle \text{poles} - \angle \text{zeros} \)

3) Angle of arrival = \( 180^\circ + \phi \)

4) A point on the real axis lies in the complementary RL, if the number of poles and zeros to the right side of any point is an even number.

Example: Sketch the complete root locus for the system having

\[ G(s) H(s) = \frac{K (s + 5)}{(s^2 + 4s + 20)} \]

Sol: Step 1: Number of poles \( P = 2 \), \( Z = 1 \), \( N = P - Z \)

One branch has to terminate at finite zero \( s = -5 \) while \( P - Z = 1 \) branch has to terminate at \( \infty \).

Starting points of branches are, \(-2 \pm j4\).

Step 2: Pole-zero plot of the system is shown below.

Step 3: Angle of asymptotes

\[ \theta = \frac{(s + 1) 180^\circ}{P - Z}, \quad q = 0 \]

Step 4: Centroid.

As there is one branch approaching \( \infty \) and one asymptote exists, centroid is not required.

Step 5: Breakaway point.

\[ 1 + G(s) H(s) = 0 \]

\[ \frac{\frac{dK}{ds}}{s^2 + 4s + 20 + K(s + 5)} = 0 \]

\( K = 0 \quad \Rightarrow \quad s(s + 10) = 0 \)

\( s = 0 \) and \( s = -10 \) are breakaway points. But \( s = 0 \) cannot be breakaway point.

Hence \( s = -10 \) valid breakaway point.

Step 6: Intersection with imaginary axis.

Characteristic equation,

\[ s^3 + 4s^2 + 20 + K(s + 5) = 0 \]

\[ s^3 + s(K + 4)(s + 20 + 5K) = 0 \]

Routh's Array can be formed as below:

\[ \begin{array}{c|ccc}
    s^3 & 1 & 20 + 5K & \\
    s^2 & K + 4 & 0 & \\
    s^1 & 20 + 5K & & \\
    s^0 & & & \\
\end{array} \]

\( K_{\text{crit}} = -4 \) makes \( s^1 \) row as row of zeros.

But as it is negative, there is no intersection of root locus with imaginary axis.

Step 7: Angle of departure.

\[ \phi = 90^\circ, \quad \phi_0 = \tan^{-1}(4/3) = 53.13^\circ \]

\[ \phi = 180^\circ - \phi = 143.13^\circ \]

at \( -2 + j4 \) pole.

\[ \phi_0 = -143.13^\circ \]

at \( -2 - j4 \) pole.
Step 9: Prediction of stability

For all ranges of $K$, i.e., $0 < K < \infty$, both the poles are always in left half of $s$-plane.
So system is inherently stable.

Example: Sketch the complete root locus of system having

$$G(s)H(s) = \frac{K}{s(s + 1)(s + 2)(s + 3)}$$

Sol.: Step 1: $P = 4$, $Z = 0$ & $N = 4$ i.e., four branches in the root locus.

Step 2: All four branches start from open-loop poles and terminates at $\infty$.

Step 3: Angle of asymptotes = \frac{2\pi}{4} = 45^\circ, 135^\circ, 225^\circ, 315^\circ

Step 4: Centroid = \frac{a - 1 - 2 - 3}{4} = -1.5

Step 5: Breakaway point

$$K = -s^2 - 6s - 11$$

$$\frac{dK}{ds} = 0 \implies s = -1.5, -0.381, -2.619$$

Here, $-1.5$ lies in the root locus and $-0.381, -2.619$ lies in for complementary root locus.

Step 6: Intersection of root locus imaginary axis.

Characteristic Equation

$$c^4 + 6c^3 + 11c^2 + 6c + K = 0$$

$$c^4$$ 1 11
$$c^3$$ 6 6
$$c^2$$ 10 K
$$c$$ \frac{60 - 6K}{10}

Auxiliary equation:

$$10s^2 + K = 0$$

At $K = 10$, $s^2 = -1, s = \pm j$

Step 7: Complete root locus.

Step 8: For $0 < K < 10$, system is absolutely stable. At $K = 10$, system is marginally stable oscillating with 1 rad/sec. For $K > 10$, system is unstable.
Complementary Root Locus:

Step 1: \( P = 4, Z = 0 \) \& \( N = 4 \) (i.e., four branches in the root locus.)

Step 2: All four branches start from open-loop poles and terminate at \( \infty \).

Step 3: Angle of asymptotes \( \theta = \) \( 180^\circ \) \( \div 4 \) \( = 45^\circ, 90^\circ, 135^\circ, 180^\circ \)

Step 4: Centroid \( = \) \( 0 \) \( \div 4 \) \( = -1.5 \)

Step 5: Breakaway point

\[ \frac{dK}{ds} = 0 \Rightarrow s = -1.5, -0.381, -2.619 \]

Here, -1.5 lies in the root locus and -0.381, -2.619 lies in the complementary root locus.

RL of system with transportation lag:

\( r(t) \) \( \rightarrow \) System \( \rightarrow \) \( \alpha (-T) \)

\( r(0) \) \( \rightarrow \) \( \alpha (0) \)

Transfer function \( \frac{L[\text{output}]}{L[\text{input}]} = \frac{C(s)e^{-Ts}}{R(s)} \)

Root Locus Plots for Typical Transfer Functions:

<table>
<thead>
<tr>
<th>G(s)</th>
<th>Root Locus</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \frac{K}{sT_1 + 1} )</td>
<td>( \begin{array}{c} \text{Root locus} \ \begin{array}{c} \alpha \rightarrow s \rightarrow j\infty \end{array} \end{array} )</td>
</tr>
<tr>
<td>2. ( \frac{K}{(sT_1 + 1)(sT_2 + 1)} )</td>
<td>( \begin{array}{c} \text{Root locus} \ \begin{array}{c} \alpha \rightarrow s \rightarrow j\infty \end{array} \end{array} )</td>
</tr>
<tr>
<td>3. ( \frac{K}{(sT_1 + 1)(sT_2 + 1)(sT_3 + 1)} )</td>
<td>( \begin{array}{c} \text{Root locus} \ \begin{array}{c} \alpha \rightarrow s \rightarrow j\infty \end{array} \end{array} )</td>
</tr>
<tr>
<td>4. ( \frac{K}{s} )</td>
<td>( \begin{array}{c} \text{Root locus} \ \begin{array}{c} \alpha \rightarrow s \rightarrow j\infty \end{array} \end{array} )</td>
</tr>
<tr>
<td>5. ( \frac{K}{s(T_1 + 1)} )</td>
<td>( \begin{array}{c} \text{Root locus} \ \begin{array}{c} \alpha \rightarrow s \rightarrow j\infty \end{array} \end{array} )</td>
</tr>
<tr>
<td>6. ( \frac{K}{s(T_1 + 1)(sT_2 + 1)} )</td>
<td>( \begin{array}{c} \text{Root locus} \ \begin{array}{c} \alpha \rightarrow s \rightarrow j\infty \end{array} \end{array} )</td>
</tr>
</tbody>
</table>
OBJECTIVE QUESTIONS

1. The root locus plot is shown alongside. What is the transfer function?

\[ \frac{K(sT_s + 1)}{s(sT_s + 1)(sT_s + 1)} \]

(a) \( \frac{K}{s(s + 2)} \)  (b) \( \frac{K}{s} \)  (c) \( K(s + 1) \)  (d) \( \frac{K}{s(s + 1)} \)

02. The asymptotes and the break point coincide at \( s = -2 \). The transfer function can be

(a) \( \frac{K(s + 1)(s + 2)}{s(s + 1)(s + 2)} \)  (b) \( \frac{K(s + 1)(s + 2)}{s(s + 1)} \)  (c) \( \frac{K(s + 1)}{s(s + 2)} \)  (d) \( \frac{K}{s + 2} \)

03. The transfer function is

\[ \frac{K}{s(s + 1)(s + 2)(s + 3)} \]

The break point will lie between

(a) \( 0 \) and \(-1\)  (b) \(-1 \) and \(-2\)  (c) \(-2 \) and \(-3\)  (d) beyond \(-3\)

04. A unity feedback system has an open loop transfer function \( G(s) = \frac{K}{s(s + 4)(s + 13)} \).

The centroid of the asymptotes of the root locus plot lies at

a) \(-4\)  b) \(-4(3)\)  c) \(+13\)  d) \(-10\)

05. The open-loop transfer function of an unity feedback system is given by

\[ G(s) = \frac{K}{s(s + 2)} \]

The number of asymptotes of the root locus plot that tend to infinity is given by

a) 3  b) 1  c) 2  d) 4

06. The open-loop transfer function of an unity feedback system is given by

\[ G(s) = \frac{K}{s(s + 1)(s + 2)} \]

The breakaway point of the root locus plot is given by

(\(s = -1\) and \(-2\))

a) \(-0.423\)  b) \(-0.523\)  c) \(-0.700\)  d) \(-0.5\)

07. A unity feedback system has an open loop transfer function

\[ G(s) = \frac{K}{s(s + 4)(s + 13)} \]

The angle of asymptotes are given by

a) 45\(^\circ\), 135\(^\circ\), 225\(^\circ\), 315\(^\circ\)  b) 0\(^\circ\), 180\(^\circ\), 360\(^\circ\)  c) 90\(^\circ\), 180\(^\circ\), 270\(^\circ\)  d) 45\(^\circ\), 90\(^\circ\), 135\(^\circ\)

08. The open-loop transfer function of a feedback system is

\[ G(s) = \frac{K}{s(s + 4)(s + 10)} \]

The four branches of root-locus originate at

a) \(-2, -1, -1 + j4, -1 - j4\)  b) \(-1, -3, -3 + j4, -3 - j4\)  c) \(-4, -2, -2 + j4, -2 - j4\)  d) \(0, -2, -1 + j4, -1 - j4\)
19. The break away points are obtained by
   a)Putting 1 + GH(jn) = 0 and solving for n
   b)Putting GH(jn) = 0 and solving for n
   c) Differentiating 1 + GH(jn) with respect to x and equating dG/dxn = 0
   d) Differentiating 1 + GH(jn) with respect to x and equating dG/dxn = 0
20. The pole is used
   a) To draw the root locus only
   b) To close the root locus and determine it's limits of variable parameters
   c) To find the closed loop roots only
   d) To find the damping ratio only

Key for Objective Questions:
  1. d, g, d, d, b, e, b, b, c, d, a
  19. c, 20, b
  20. JTO PREVIOUS QUESTION:

Previous PSU's Question
01. The root of a system has three asymptotes. The system can have:
   a) Four poles and one zero
   b) Three poles
   c) Five poles and two zeros
   d) All of these
02. Root locus diagram can be used to determine
   a) Conditional stability
   b) Absolute stability
   c) Relative stability
   d) None
03. The values of K for which the system is stable are given by
   a) K = 8
   b) K = 7
   c) K > 7
   d) None
04. The transfer function of a closed loop system is shown in the figure
   a) Marginally stable
   b) Conditionally stable
   c) Unstable
   d) Stable

Previous PSU's Question
01. The characteristic equation of a system is given by $3s^3 + 10s^2 + 5s + 2 = 0$. The system has
   a) Stable
   b) Marginal stable
   c) Instable
   d) Data is insufficient
02. A control system has
   \[ G(s)H(s) = \frac{K}{s(s + 4)(s^2 + 4s + 20)} \]
   for $(9 < K< c)$.
   What is the number of breakaway points in the root locus diagram?
   a) One
   b) Two
   c) Three
   d) Four
03. By using a suitable set of the transfer function parameter K, the system shown in
   the figure is made to oscillate continuously at a frequency
   a) 1 rad/sec
   b) 2 rad/sec
   c) 4 rad/sec
   d) 4 rad/sec
09. The root locus plot of the system having the loop transfer function
\[ G(s) = \frac{K}{s(s+3)(s+1)(s+3)} \]
has
(A) one breakaway point
(B) three real breakaway points
(C) only one breakaway point
(D) one real and two complex breakaway points

10. The open-loop transfer function of the unity feedback control system is
\[ G(s) = \frac{K}{s(s+3)(s+1)} \]
with the imaginary axis in
(A) 2
(B) 4
(C) 6
(D) 8

05. The value of 'K' for which the unity feedback system
\[ G(s) = \frac{K}{s(s+3)(s+1)} \]
crosses the imaginary axis is
(A) 2
(B) 4
(C) 6
(D) 8

06. The root locus of the system
\[ G(s)H(s) = \frac{K}{s(s+3)(s+1)} \]
has the break-away point located at
(A) -5.5, 0
(B) -2.5, 0
(C) -0.5, 0
(D) -0.784, 0

07. The transfer function of the system is
\[ \frac{2s^2 + 6s + 5}{s(s+1)(s+2)} \]
the characteristic equation of the system is
(A) \[ 2s^2 + 5s + 5 = 0 \]
(B) \[ 2s^2 + 6s + 5 = 0 \]
(C) \[ 2s^2 + 5s + 5 = 0 \]
(D) \[ 2s^2 + 5s + 5 = 0 \]

08. Which one of the following techniques is utilized to determine the actual point at which the root locus crosses the imaginary axis?
(A) Nyquist technique
(B) Routh–Hurwitz criterion
(C) Nichols criterion
(D) Bode technique

KEYS:

JTO:
(1) D (2) C (3) D (4) C (5) B

PSU:
(1) C (2) C (3) C (4) D (5) D

CHAPTER 5
FREQUENCY RESPONSE ANALYSIS
The various frequency response analysis techniques are:
1) Bode plot
2) Pole plot
3) Nyquist plot
4) M & N circles
5) Nichols chart

1) Bode Plots:
It is used to display the frequency response of an open-loop and closed-loop system. The representation of the logarithm of |G(jω)| and phase angle of G(jω), both plotted against frequency in logarithmic scale. These plots are called Bode plots.

-Bode Plot of first order system:
Let the Transfer Function
\[ \frac{1}{1 + Ts} \]

\[ \text{subs. } s = j \omega \]

\[ \text{TF. } \frac{1}{1 + j \omega T} \]

\[ M = \frac{1}{\sqrt{1 + (\omega T)^2}} \]
\[ \phi = -\tan^{-1}(\omega T) \]

\[ M = 20 \log \left( \frac{1}{\sqrt{1 + (\omega T)^2}} \right) \]
\[ \omega \ll 1/T \]
\[ \omega = 1/T \]
\[ M\% = 10 \log M \]
\[ M\% = 10 \log (1 + \omega T)^2 \]
\[ \omega = 0 \]
\[ = -20 \log \omega T \]

Therefore, the error at the corner frequency \( \omega = 1/T \) is:
-10 log 2 + 10 log 1 = -3 dB

The error at frequency \( (\omega = 1/2T) \) one octave below the corner frequency is:-10 log (1 + 1) + 10 log 1 = -1 dB
Bode Plot of second order system:

\[ \text{T.F.} = \frac{\omega_n^2}{s^2 + 2\xi \omega_n s + \omega_n^2} \]

\[ \text{substituting } s = j\omega \]

\[ \text{T.F.} = \frac{\omega_n^2}{-\omega^2 + j2\xi\omega\omega_n + \omega_n^2} \]

Divide with \( \omega_n^2 \):

\[ \frac{1}{(1 - \mu)^2} \frac{\omega_n^2}{2\mu} \]

\[ M = \frac{1}{\sqrt{(1 - \mu)^2 + (2\mu)^2}} \quad \Phi = -\tan^{-1} \left( \frac{2\mu}{1 - \mu^2} \right) \]

\[ M_{\infty} = -10 \log \left( (1 - \mu)^2 + 4 \xi^2 \mu^2 \right) \]

Case 1) When \( \mu < 1 \Rightarrow \omega/a_n < 1 \Rightarrow \omega < a_n \)

\[ M_{\infty} = -10 \log 1 = 0 \text{ dB} \]

Case 2) When \( \mu > 1 \Rightarrow \omega/a_n > 1 \Rightarrow \omega > a_n \)

\[ M_{\infty} = -10 \log \mu \Rightarrow -40 \log \mu \]

The error between the usual magnitude and the asymptotic approximation is as given below:

For \( 0 < \mu < 1 \), the error is

\[ -10 \log \left( (1 - \mu)^2 + 4 \xi^2 \mu^2 \right) + 10 \log 1 \]

and for \( \mu < 0 \), the error is

\[ -10 \log \left( (1 - \mu)^2 + 4 \xi^2 \mu^2 \right) + 40 \log \mu \]

Bode Plots for Typical Transfer Functions:

<table>
<thead>
<tr>
<th>G(s)</th>
<th>Bode Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( K ) ( sT_1 + 1 )</td>
<td><img src="image1" alt="Bode Plot 1" /></td>
</tr>
<tr>
<td>2. ( K ) ( \frac{1}{s(T_1 + 1)(T_2 + 1)} )</td>
<td><img src="image2" alt="Bode Plot 2" /></td>
</tr>
<tr>
<td>3. ( K ) ( \frac{1}{s(T_1 + (T_2 + 1))} )</td>
<td><img src="image3" alt="Bode Plot 3" /></td>
</tr>
<tr>
<td>4. ( K ) ( \frac{1}{s} )</td>
<td><img src="image4" alt="Bode Plot 4" /></td>
</tr>
</tbody>
</table>
### Control Systems

#### 2) Polar Plot:

The sinusoidal transfer function $G(j\omega)$ is a complex function and is given by

$$
G(j\omega) = \text{Re}[G(j\omega)] + j\omega \text{Im}[G(j\omega)]
$$

or

$$
G(j\omega) = |G(j\omega)|e^{j\phi}
$$

from above equation, it is seen that $G(j\omega)$ may be represented as a phasor of magnitude $M$ and phase angle $\phi$. As the input frequency $\omega$ is varied from 0 to $\infty$, the magnitude $M$ and phase angle $\phi$ change and hence the tip of the phasor $G(j\omega)$ traces a locus in the complex plane. The locus thus obtained is known as the polar plot.

When a transfer function consists of $P$ poles and $Z$ zeros, and it doesn't consist poles at origin then the pole plot starts from 0° with some magnitude and terminates at $-90^\circ \times (P-Z)$ with zero magnitude.

When a transfer function consists of poles at origin, then the pole plot starts from $-90^\circ \times$ no. of poles at origin with "$\omega^\circ" magnitude and ends at $-90^\circ \times (P-Z)$ with zero magnitude.

#### 2) Nyquist Stability Criteria:

It is used to determine the stability of a closed-loop system using polar plots. This concept is derived from complex analysis using 'Principle of Argument'.

Let $G(s) = \frac{(s + Z_1)(s + Z_2)}{(s + P_1)(s + P_2)}$  \quad (1)

Characteristic Equation $1 + G(j\omega) = 0$

$$
1 + G(j\omega) = 1 + \frac{(s + Z_1)(s + Z_2)}{(s + P_1)(s + P_2)} = \frac{(s + P_1)(s + P_2) + (s + Z_1)(s + Z_2)}{(s + P_1)(s + P_2)} \quad \rightarrow (2)
$$

### Frequency Response Analysis

#### 5. $\frac{K}{s(T_1 + 1)}$

- $20$ dB/dec
- $40$ dB/sec

#### 6. $\frac{K}{s(T_1 + 1)(T_2 + 1)}$

- $20$ dB/dec
- $40$ dB/sec

#### 7. $\frac{K(T_1 + 1)}{s(T_1 + 1)(T_2 + 1)}$

- $20$ dB/dec
- $40$ dB/sec

#### 8. $\frac{K}{s^2}$

- $20$ dB/dec
- $40$ dB/sec

#### 9. $\frac{K(T_1 + 1)}{s^2(T_1 + 1)}$

- $20$ dB/dec
- $40$ dB/sec

#### 10. $\frac{K}{s^2}$

- $20$ dB/dec
- $40$ dB/sec

#### 11. $\frac{K(T_1 + 1)}{s^2}$

- $20$ dB/dec
- $40$ dB/sec
From (1) and (7), the open-loop poles and CE poles are same.

$$C.E. = \frac{(s + Z_1)(s + Z_2)}{(s + P_1)(s + P_2)} \rightarrow (1)$$

Overall transfer function $$= \frac{G(s)}{1 + G(s)} = \frac{(s + Z_2)(s + Z_1)}{(s + Z_2)(s + Z_1)} \rightarrow (4)$$

From (3) and (4), the C.E. zeros and closed-loop poles are same.

---For the closed-loop system to be stable, the zeros of the C.E. should not be located on the right half of the s-plane.

Using Principle of Argument

$$Q(s) = 1 + G(s)$$

Consider a contour as shown which covers the entire right half of the s-plane. If each and every point is along the boundary of the contour is substituted in C.E. according to the principle of argument,

The no. of encirclements with respect to origin, $$N = Z - P$$

where Z and P are the zeros and poles of the C.E. located inside the contour or located in right half of the s-plane.

For the closed-loop system to be stable, $$Z = 0$$.

---For the open-loop system to be stable, $$P = 0$$, then $$N = Z$$.

In $$N = Z - P$$, Z becomes 0 only if $$N = 0$$ [C.E. contour shouldn’t encircle the origin]

If the open-loop system is stable, the closed-loop system will be stable only if the Nyquist contour doesn’t encircle origin.

---For the open-loop system to be unstable, $$P \neq 0$$.

If the open-loop system is unstable, the closed-loop system will be stable only if the Nyquist contour encircles origin in clockwise direction. The number of encirclements should be equal to the number of open-loop poles located inside the contour.

Nyquist Plots for Typical Transfer Functions:

<table>
<thead>
<tr>
<th>G(s)</th>
<th>Nyquist Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $$\frac{K}{sT_1 + 1}$$</td>
<td><img src="image1" alt="Nyquist Plot 1" /></td>
</tr>
<tr>
<td>2. $$\frac{K}{(sT_1 + 1)(sT_2 + 1)}$$</td>
<td><img src="image2" alt="Nyquist Plot 2" /></td>
</tr>
<tr>
<td>3. $$\frac{K}{(sT_1 + 1)(sT_2 + 1)(sT_3 + 1)}$$</td>
<td><img src="image3" alt="Nyquist Plot 3" /></td>
</tr>
<tr>
<td>4. $$\frac{K}{s}$$</td>
<td><img src="image4" alt="Nyquist Plot 4" /></td>
</tr>
<tr>
<td>5. $$\frac{K}{s(T_1 + 1)}$$</td>
<td><img src="image5" alt="Nyquist Plot 5" /></td>
</tr>
<tr>
<td>6. $$\frac{K}{s(T_1 + 1)(sT_2 + 1)}$$</td>
<td><img src="image6" alt="Nyquist Plot 6" /></td>
</tr>
</tbody>
</table>
4. M & N Circles

Constant Magnitude Loci : M-Circles

M-circles are used to determine the magnitude response of a closed-loop system using open-loop transfer function.

It is applicable only for unity feedback systems. The open-loop transfer function \( G(j\omega) \) of a unity feedback control system is a complex quantity and can be expressed as

\[
G(j\omega) = \frac{C(j\omega)}{R(j\omega)} = \frac{G(j\omega)}{1 + G(j\omega)}
\]

\[ x + jy \quad |z| = \frac{M}{1} \]

\[
M = \frac{x + jy}{\sqrt{x^2 + y^2}}
\]

On squaring both sides and simplifying following equation is obtained:

\[
(1 - M^2)x^2 - 2M^2xy + (1 - M^2)y^2 = M^2
\]

or

\[
x^2 = \frac{2M^2}{(1 - M^2)}
\]

\[
y^2 = \frac{1}{(1 - M^2)}
\]

Add \( \frac{M^2}{1 - M^2} \) to both sides,

\[
x^2 = \frac{2M^2}{(1 - M^2)} + \left( \frac{M^2}{1 - M^2} \right) + y^2 = \frac{M^2}{1 - M^2} + \left( \frac{M^2}{1 - M^2} \right)
\]

or

\[
\left( \frac{x}{1 - M^2} \right)^2 + \left( \frac{y}{1 - M^2} \right)^2 = \frac{1}{1 - M^2}
\]

For different values of \( M \), above Eq. represents a family of circles with centres at \( x = \left( \frac{M^2}{1 - M^2}, y = 0 \right) \) and radius \( \frac{M}{1 - M^2} \). On a particular circle the value of \( M \) (magnitude of closed-loop transfer function) is constant, therefore, these circles are called M-circles.

The centres and radii of M-circles for different values of \( M \) are given in the following table and M-circles are drawn in the following figure.
Control Systems

<table>
<thead>
<tr>
<th>M'</th>
<th>centre $x - M'^2/1 - M'^2, y = 0$</th>
<th>Radius $r = M'/1 - M'^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.33</td>
<td>0.67</td>
</tr>
<tr>
<td>1.0</td>
<td>=</td>
<td>=</td>
</tr>
<tr>
<td>1.5</td>
<td>-3.27</td>
<td>2.73</td>
</tr>
<tr>
<td>1.6</td>
<td>-1.64</td>
<td>1.03</td>
</tr>
<tr>
<td>2.0</td>
<td>-1.33</td>
<td>0.67</td>
</tr>
</tbody>
</table>

In G (jω) plane the Nyquist plot is superimposed on M-circle and the points of intersection that give the magnitude of $C(j\omega)R(j\omega)$ at different values of $M'$.

**Constant Phase Angle Loci - Notation:**

$N = \omega$-values are used to determine the phase response of a closed-loop system using open-loop transfer function.

The phase angle of the closed-loop transfer function of a unity feedback system is given by

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left[\frac{y}{(1 + x)}\right]$$

or

$$\phi = \tan^{-1}\left[\frac{\tan^{-1}(y/x) - \tan^{-1}(y/(1 + x))}{1 + \tan^{-1}(y/x)\cdot\tan^{-1}(y/(1 + x))}\right]$$

or

$$\phi = \frac{y}{x^2 + x^2 + y^2}$$

**Substituting** $\phi = N\pi$ in above equation

$$N = \frac{y}{x^2 + x^2 + y^2}$$

or

$$x^2 + x^2 + y^2 - (y/N) = 0$$

Add \(1 + \frac{1}{4N^2}\) on both sides

$$\left(\frac{x^2 + x^2 + y^2}{4N^2}\right) + \left(\frac{y}{4N}\right) + \frac{1}{4} = \frac{1}{4}$$

or

$$\left(\frac{x + 1}{2}\right)^2 + \left(\frac{y + 1}{2N}\right)^2 + \frac{1}{4} = \frac{1}{4}$$

For different values of $N$, above equation represents a family of circles with centre at $x = -\frac{1}{2}, y = 1/2N$ and radius $\frac{1}{2\sqrt{N}}$.

On a particular circle, the value of $N$ or the value of phase angle of the close-loop transfer function is constant. Therefore, these circles are called $N$-circles.

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$N = \tan \phi$</th>
<th>centre $x = -\frac{1}{2}, y = 1/2N$</th>
<th>Radius $R = \sqrt{1/4 + 1/4N^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-90°</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-60°</td>
<td>1.132</td>
<td>-0.289</td>
<td>0.577</td>
</tr>
<tr>
<td>-50°</td>
<td>-1.99</td>
<td>0.864</td>
<td>1.0</td>
</tr>
<tr>
<td>-30°</td>
<td>-0.577</td>
<td>0.864</td>
<td>1.0</td>
</tr>
<tr>
<td>-10°</td>
<td>-0.179</td>
<td>2.84</td>
<td>2.38</td>
</tr>
<tr>
<td>0°</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10°</td>
<td>0.176</td>
<td>2.84</td>
<td>2.38</td>
</tr>
<tr>
<td>30°</td>
<td>0.377</td>
<td>0.864</td>
<td>1.0</td>
</tr>
<tr>
<td>50°</td>
<td>1.19</td>
<td>0.864</td>
<td>1.0</td>
</tr>
<tr>
<td>60°</td>
<td>1.752</td>
<td>0.289</td>
<td>0.577</td>
</tr>
<tr>
<td>90°</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
5) Nichols Chart

The transformation of constant - M and constant - N circles to log-magnitude and phase angle coordinates and the resulting chart is known as the Nichols chart.

OBJECTIVE QUESTIONS

01. In a polar plot, the curve was found to cross the negative real axis at -1.2. Then
   a) The gain margin is 1.2 and the system is stable.
   b) The gain margin is 0.83 and the system is unstable.
   c) The gain margin is 1.2 and the system is unstable.
   d) The gain margin is 0.83 and the system is stable.

02. The polar plot of a closed-loop system with a transfer function \( \frac{G}{1+GH} \) is shown alongside. It can be stated that
   a) The finite zero is closer to the origin than the finite pole.
   b) The finite-zero is closer to the origin than the zero is.
   c) The system is non-minimum phase.
   d) The system is non-minimum phase.

03. The polar plot of a transfer function is shown alongside. It can be stated that
   a) a = b
   b) b = 1 GHz
   c) b = 1 GHz
   d) b = 1 GHz

04. The polar plot is drawn.
   a) Decline versus angle
   b) Decline versus angle
   c) Decline versus angle
   d) Magnitude and phase angle incorporated in the x-y plane.
05. For a transfer function with pure transportation lag, the polar plot is:
   a) a semi-circle with centre at (-1, 0) and radius 1 in the clockwise direction.
   b) an anti-clockwise semi-circle with centre at (-1, 0) and radius 1.
   c) a circle with origin as the centre and radius 1.
   d) polar plot does not exist.

06. By substituting \( z = \frac{1}{s} \), the frequency response plot gives:
   a) transient response of the system
   b) steady state response of the system
   c) initially transient and then steady state response
   d) none of the above

07. The polar plot of a system with transfer function
   \[ G(s) = \frac{K}{s(s + 1)} \]
   for \( s = j\omega \) and \( s = -j\omega \) will be:
   a) in the first quadrant
   b) in the second quadrant
   c) in the third quadrant
   d) in the fourth quadrant

08. Which of the following transfer functions is a non-minimum phase transfer function?
   a) \( (s + 2)(s + 3) \)
   b) \( (s + 2)(s + 4) \)
   c) \( \frac{s}{s + 1} \)
   d) \( \frac{s}{s + 3} \)

09. In the \( G(j\omega) \) plane, the constant phase angle loci are:
   a) straight lines passing through the origin.
   b) straight lines passing through (-1, 0) point.
   c) straight lines passing through (1, 0) point.
   d) some pass through (-1, 0), some pass through (1, 0) and some pass through origin.

10. The inverse polar plot is the plot of the following sinusoidal transfer function:
    a) \( G(1/j\omega) \)
    b) \( \frac{1}{G(j\omega)} \)
    c) \( G(1/j\omega) \)
    d) None

Key:
1. d  2. c  3. a  4. d  5. c  6. b  7. a  8. d  9. a  10. b

**OBJECTIVE QUESTIONS**

01. A stable feedback control system has open-loop transfer function with a pole at RHP and zero at LHP. The corresponding Nyquist plot will:
    a) encircle (-1, 0) point in counter clockwise direction once.
    b) have anti-clockwise encirclement of the (-1, 0) point once.
    c) encircle (-1, 0) point as many times as the number of LHP poles of the closed-loop transfer function.
    d) no encircle (-1, 0) point at all.

02. Nyquist plot can be used:
    a) to find the closed-loop poles in the right half plane.
    b) to determine the stability only.
    c) to find the open-loop poles in the right half plane.
    d) to find the number of closed-loop poles in the left half plane.

04. Nyquist plot of a system is shown alongside. What is the type of the system?

06. A unit feedback system has an open-loop transfer function:

\[ G(j\omega) = \frac{K}{j(0.2\omega + 1)(0.5\omega + 1)} \]

The phase cross-over frequency of the Nyquist plot is given by:

a) 1 rad/sec  b) 10 rad/sec  c) 100 rad/sec  d) 1000 rad/sec

08. The Nyquist plot (for positive frequencies) of the open-loop transfer function is shown in figure. The gain margin is:

-2  0  2

09. The Nyquist plot given figure corresponds to the system whose transfer function is:

\[ \frac{1}{s^2 + 2s + 1} \]

a) stable  b) unstable  c) marginally stable  d) conditionally stable
10. If the gain margin of a certain feedback system is given as 20 dB, the Nyquist plot will cross the negative real axis at the point:
a) $s = -0.05$
b) $s = -0.2$
c) $s = -0.1$
d) $s = 0.01$

Key:
a, b, c, d, a, b, c, d, a, b, c, d

Objective Questions:
01. If the phase $\phi$ which is the angle between radial line connecting a pole and origin is equal to $45^\circ$, then the peak overshoot is
a) 0.5%  
b) 1.2% 

c) 3%  
d) 4.32% 

02. The denominator bandwidth for a particular value of $\zeta$ and damping factor is $\zeta$ is
a) $\omega_0 \sqrt{1 - \zeta^2}$  
b) $(\omega_0 + \zeta)^{1/2}$

c) $\omega_0 (1 - 2\zeta^2 + \sqrt{4\zeta^4 + 4\zeta^2})$  
d) $\omega_0 (1 - 2\zeta^2 - \sqrt{4\zeta^4 + 4\zeta^2})^{1/2}$

03. The resonant and damping frequency of a certain system was found to be 7.07 rad/s and 1.666 rad/s respectively. The real co-ordinate of the dominant pole is:
a) -1, 1.8  
b) -1, 0  
c) -6.5  
d) -3.5

04. Large bandwidth corresponds to
a) small rise time and suppresses noise
b) small rise time and increases noise
c) high rise time and suppresses noise
d) high rise time and increases noise

05. The resonant peak of a certain second order system is given by
a) $M_0 = \exp(-\omega_0/\sqrt{1 - \zeta^2})$
b) $M_0 = \omega_0 / \zeta$

c) $M_0 = \frac{\zeta}{\sqrt{1 - \zeta^2}}$

d) $M_0 = \frac{\sqrt{1 - \zeta^2}}{\zeta}$

06. The centre and radius of a constant $M$ circles are given by
a) $x = \frac{M}{1 - M}$, $y = 0$, $r = \frac{M}{1 - M}$

b) $x = \frac{M}{M - 1}$, $y = \frac{-M}{M - 1}$, $r = \frac{M}{M - 1}$

c) $x = \frac{M^2}{(M - 1)^2}$, $y = 0$, $r = \frac{M}{M - 1}$

d) $x = \frac{M^2}{(M - 1)^2}$, $y = 0$, $r = \frac{M}{M - 1}$

07. The centre and radius of a constant $N$ circles are given by
a) $x = -(U/2)$, $y = (U/2N)$, $r = \frac{N(N+1)}{2N}$

b) $x = -(U/2)$, $y = (U/2N)$, $r = \frac{N(N+1)}{2N}$

c) $x = -(U/2)$, $y = -(U/2N)$, $r = \frac{N(N+1)}{2N}$

d) $x = -(U/2)$, $y = -(U/2N)$, $r = \frac{N(N+1)}{2N}$

08. In the $G^\infty(\omega)$ plane, the constant $M$ circles has the following centre and radius
a) $(0, y) = (-1, 0)$, $r = M$

b) $(0, y) = (-1, 0)$, $r = M$

c) $(0, y) = (1, 0)$, $r = M$

d) $(0, y) = (1, 0)$, $r = M$

09. Undamped natural frequency $\omega_n$ and resonance frequency $\omega_r$ of a unity feedback system with open-loop transfer function (M. Koppil)

\[
\begin{align*}
G(s) &= \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \\
\omega_r &= \frac{\omega_n}{\sqrt{1 - \zeta^2}} \\
\omega_n &= \sqrt{\frac{1}{s^2 + 2\xi\omega_s s + \omega_n^2}} \quad (s = 2\zeta\omega_n) \\
\end{align*}
\]

are related as
a) $\omega_r = \omega_n$  
b) $\omega_r > \omega_n$  
c) $\omega_r < \omega_n$  
d) None

10. Undamped natural frequency $\omega_n$ and bandwidth $\omega_b$ of a unity feedback system with open-loop transfer function

\[
G(s) = \frac{1}{s + 2\zeta\omega_n s + \omega_n^2}
\]

are related as
a) $\omega_b = \omega_n$  
b) $\omega_b > \omega_n$  
c) $\omega_b < \omega_n$  
d) None

Key:
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, c

JTO PREVIOUS QUESTION:
01. The bode plot is applicable to network
(A) all phase  
(B) maximum phase  
(C) minimum phase
(D) none

02. Nyquist criterion is used to find which of the following
(A) Relative stability  
(B) Absolute stability  
(C) Both A and B

03. The frequency range specification required to satisfactorily describe the system responsible is?
(A) resonance  
(C) voltage  
(D) all of the above

04. Bandwidth of a control system is used as a means of specifying its performance relating to?
(A) stability of the system  
(B) speed of the system

(C) constant gain of the system  
(D) none

05. can be extended to systems which are time – varying?
(A) Root locus design  
(B) Bode – Nyquist stability methods

(C) State model representations  
(D) Transfer functions

06. When gain K of the loop transfer function is varied from zero to infinity the closed loop system
(A) stability is improved  
(B) always become unstable

(C) Stability is not allowed  
(D) may become unstable

07. Frequency domain analysis is preferred when dealing with systems having input is
(A) ramp and periodic  
(B) sinusoidal with fixed frequency

(C) sinusoidal with variable frequency  
(D) non – sinusoidal with lagging power factor

08. For gain K log – magnitude curve in Bode plot is
(A) negative or positive depending upon the value of K

(B) Convex

(C) negative  
(D) positive

09. What is the type of the system, for the Nyquist plot of a system shown below?
(A) 1  
(B) 0

(C) 2  
(D) 3

10. What is the type of the system, for the Nyquist plot of a system shown below?
10. Which of the following statements is not true for Nyquist criterion?

(A) h indicates the degree of stability of a stable system
(B) It provides some amount of information about absolute stability as the Routh criterion
(C) Either A or B
(D) None

11. A minimum phase function has m finite poles and n finite zeros. Its phase angle at infinity is

(A) m radians
(B) (m - n) m/2 radians
(C) (m - n) m/2 radians
(D) none

12. A transfer function which has one or more zeros in the R.H.S. plane is known as

(A) all phase
(B) minimum phase
(C) non-minimum phase
(D) none of the above

PREVIOUS PSU'S QUESTION:

1. If the Nyquist plot cuts the negative real axis at a distance of 0.4. The gain margin of the system is

(A) 0.4
(B) -0.4
(C) 0.4
(D) 2.5

2. A minimum phase unity feedback system has a Bode plot with a constant slope of -20 db/decade for all frequencies. What is the value of the maximum phase margin for the system?

(A) 0°
(B) 90°
(C) 90°
(D) 180°

05. A minimum phase unity feedback system has a Bode plot with a constant slope of -20 db/decade for all frequencies. What is the value of the maximum phase margin for the system?

(A) 0°
(B) 30°
(C) 60°
(D) 180°

06. The ideal plot is valid for

(A) minimum phase network
(B) all phase network
(C) non-minimum phase network
(D) none of the above

05. The initial slope of the bode plot gives an indication of

(A) type of the system
(B) nature of the system sine response
(C) system stability
(D) gain margin

06. If the magnitude of the polar plot at phase crossover is 'a', the gain margin is

(A) a
(B) 0
(C) a/114
(D) none

07. In the bode plot of a unity feedback control system, the value of phase of G(jw) at the gain crossover frequency is 125°. The phase margin of the system is

(A) 125°
(B) 55°
(C) 55°
(D) 125°

08. Nichols's chart is useful for detailed study and analysis of

(A) closed loop frequency response
(B) open loop frequency response
(C) closed loop and open loop frequency response
(D) none of the above

10. Nyquist plot shows in the given figure is for a type

(A) zero-system
(B) one system
(C) two system
(D) three system

11. The open-loop transfer function of a unity feedback control system is given as

\[ G(s) = \frac{1}{(1 + j \Omega_1)(1 + j \Omega_2)} \]

The phase crossover frequency and the gain margin are respectively

(A) \( \angle \frac{1}{\Omega_2} \) and \( \angle \frac{1}{\Omega_1} \)
(B) \( \frac{1}{\Omega_1} + \frac{1}{\Omega_2} \) and \( \frac{1}{\Omega_1} + \frac{1}{\Omega_2} \)
(C) \( \frac{1}{\Omega_1} + \frac{1}{\Omega_2} \) and \( \frac{1}{\Omega_1} + \frac{1}{\Omega_2} \)
(D) \( \frac{1}{\Omega_1} + \frac{1}{\Omega_2} \) and \( \frac{1}{\Omega_1} + \frac{1}{\Omega_2} \)

12. The polar plot of a transfer function passes through the critical point (-1, j0).

Gain margin is

(A) zero
(B) -1 db
(C) 1 db
(D) infinity
GAIN MARGIN AND PHASE MARGIN

Gain cross-over frequency: The frequency at which the magnitude equals 0 dB
Phase cross-over frequency: The frequency at which the phase angle equals -180°.

Gain margin \( G.M = \log_{10} \left( \frac{|G(j\omega)|}{|G(j\omega)|} \right) \) (in linear)

The gain margin is a factor by which the gain of a stable system is allowed to increase before the system reaches instability.

The gain margin in dB is

\[ G.M = 20 \log_{10} \left( \frac{1}{|G(j\omega)|} \right) \text{ dB} \]

Procedure to calculate Gain margin:
1. Calculate Phase cross-over frequency
   a) by equating phase equation to 180° or
   b) by equating imaginary part to zero
2. Calculate the magnitude at phase cross-over frequency and is equal to 'n'.
3. Gain margin is equal to 20 log (1/n) dB.
   For stable systems, if \( |G(j\omega)| |H(j\omega)| > 1 \), the gain margin in dB is positive.
   For marginally stable systems, if \( |G(j\omega)| |H(j\omega)| = 1 \), the gain margin is 0 dB in zero.
4. For unstable systems, if \( |G(j\omega)| |H(j\omega)| > 1 \), the gain margin is still negative and the gain is to be reduced to make the system stable.

Phase margin:
The phase margin of a stable system is the amount of additional phase lag required to bring the system to the point of instability. The phase margin is given by P.M. = 180° + \angle G(j\omega)

Procedure for calculation of P.M.:
1. Calculate 'n\(0\)' by equating magnitude equation to '1'.
2. Calculate the phase at \( \omega_0 \).
3. P.M. = 180° + \angle G(j\omega).
4. P.M. is positive, the system is stable.
5. P.M. is negative, the system is unsteady.
6. P.M. = 0, the system is marginally stable.

JTO PREVIOUS QUESTION

01. Gain margin is the amount of angle to make the system.
   (A) Oscillatory 
   (B) unstable 
   (C) exponential 
   (D) stable

02. A system with gain margin close to unity or a phase margin close to zero is
   (A) relatively stable
   (B) highly stable
   (C) oscillatory
   (D) none

03. The phase shift of the second order system with transfer function 1/s is
   (A) -90° 
   (B) 90° 
   (C) 180° 
   (D) -180°

04. A minimum phase unity feedback system has a Bode plot with a constant slope of \( 0 \) in decades for all frequencies. What is the value of the maximum phase margin for the system?
   (A) 0° 
   (B) 90° 
   (C) 90° 
   (D) 180°

05. The gain margin of a unity negative feedback system having forward transfer function is \( K/(s(T+1)) \)
   (A) infinity 
   (B) KT 
   (C) 1 
   (D) Zero

06. If the magnitude of the polar plot at phase crossover is 'a', the gain margin is
   (A) a \( -\) 90° 
   (B) 90° 
   (C) a 
   (D) 1/\( a \)

07. For the transfer function \( G(s)H(s) = \frac{\frac{1}{s+1}}{s+0.5} \), the phase crossover frequency is
   (A) 0.5 rad/sec
   (B) 0.707 rad/sec
   (C) 1.77 rad/sec
   (D) 2 rad/sec

08. The open loop transfer function of a feedback control system is
   \( G(s)H(s) = \frac{\frac{1}{s+1}}{(s+1)^2} \)
   The gain margin of the system is
   (A) 2 
   (B) 4 
   (C) 8 
   (D) 16

09. The gain margin (in dB) of a system having the loop transfer function
   \( G(s)H(s) = \frac{\frac{1}{s}}{s+1} \)
   (A) 0 
   (B) 3 
   (C) 6 
   (D) \( \infty \)

10. If the gain of the loop system is doubled, the gain margin of the system is
    (A) not affected
    (B) doubled
    (C) halved
    (D) one fourth of original value

11. The forward path transfer function of an unity feedback system is given by
    \( G(s) = \frac{K}{s} \)
    What is the phase margin of this system?
    (A) \( -\) 90°
    (B) 0°
    (C) 90°
    (D) 180°
CHAPTER 6

DESIGN OF CONTROL SYSTEMS

4.1 COMPENSATORS

Lag compensator:

A compensator having the characteristic of a lag network is called a lag compensator. Lag compensation results in a large improvement in steady-state performance but results in a slower response due to increased bandwidth. Lag compensator is essentially a low-pass filter and at high frequency noise signals are attenuated.

Transfer function of lag compensator, \( G_c(s) = \frac{s + \frac{1}{T}}{s + \frac{1}{\tau}} \)

Pole-zero plot of lag compensator.

NOTE:

Minimum phase transfer function: Transfer function have no poles and zeros in the R.H.S of s-plane.

Non-minimum phase transfer function: Transfer function having at least one pole or zero in the R.H.S of s-plane.

If pass transfer function: Transfer function have symmetric pole and zero about the imaginary axis in s-plane.
Lead compensator:
A compensator having the characteristics of a lead network is called a lead compensator. The lead compensation increases the bandwidth, which improves the speed of the response and also reduces the amount of overshoot. Lead compensation appreciably improves the transient response.
A lead compensator is basically a high-pass filter and so it amplifies high-frequency noise signals.

Transfer function of a lead compensator:
\[ G_c(s) = \frac{s + \alpha}{s + (\alpha T)} \]

Frequency response of a lead compensator:
Consider the general form of lead compensator,
\[ G_c(s) = \frac{s(1 + \alpha T)}{s + (1 + \alpha T)} \]

The expression for maximum phase lead \( \phi_m \) in terms of \( \alpha \) and \( \alpha \) is in terms of \( \phi_m \) are given below:
\[ \phi_m = \tan^{-1} \left( \frac{1 - \alpha}{2\alpha} \right) \]
\[ \alpha = \frac{1 - \tan \phi_m}{1 + \tan \phi_m} \]

Lag - Lead Compensator:
A compensator having the characteristics of lag-lead network is called a lag-lead compensator. A lag-lead compensator improves both transient and steady-state response.

The transfer function of lag-lead compensator
\[ G_c(s) = \frac{(s + 1/\beta T)}{(s + 1/\alpha T)} \]
where \( \beta > 1 \) and \( 0 < \alpha < 1 \)

Frequency of maximum phase lead
\[ \phi_m = \sqrt{\alpha_1 \alpha_2} = \sqrt{1/(\alpha T)} \]
\[ \frac{1}{\sqrt{\alpha}} \]
OBJECTIVE QUESTIONS

01. The lead compensator introduces:
   a) Phase lead in the system
   b) Attenuation in the system
   c) Amplification in the system
   d) Initially phase lead and then phase lag in the system.

02. The lead compensator typically:
   a) Improves the steady state error
   b) Improves the transient response
   c) Improves both steady state and transient response equally
   d) None of the above

03. The lag compensator:
   a) Improves both steady state and transient response
   b) Improves steady state only
   c) Improves transient only
   d) None of the above

04. The lag-lead compensator:
   a) Improves steady state but reduces speed of response.
   b) Improves transient response but no effect on steady state
   c) Improves both steady state and transient response
   d) None of the above

05. The lag network achieves the desired result through its:
   a) Attenuation property at high frequencies
   b) Amplification property at low frequencies
   c) Phase property
   d) None of the above

06. A lag network for compensation normally consists of:
   a) R only
   b) R and C elements
   c) R and L elements
   d) R, L and C elements

07. The transfer function is $1 + \frac{1}{s^2}$.

08. A network has a pole at $s = -1$ and a zero at $s = 2$. If this network is excited by a sinusoidal input, the output:
   a) Falls the output
   b) Lags the input
   c) Is in phase with input
   d) Decay exponentially to zero

09. The pole-zero plot given below is that of:

JTO PREVIOUS QUESTIONS

01. The high cut off frequency function of the is:

02. The transportation delays occurring in distributed systems are largely mental to stability because they produce:
   a) A phase lag
   b) Transient
   c) Attenuation
   d) Both attenuation and a phase lag

03. Phase lag network does which of the following?
   (A) Increases system stability
   (B) Decreases bandwidth
   (C) System control policy
   (D) Maximization control

04. Adding pole in a system causes which of the following?
   (A) Lead compensation
   (B) Lag
   (C) Lag-compensation
   (D) None

05. Lead compensator in a feedback system:
   (A) Increases the system error constant in some extent
   (B) Speeds up the transient response
   (C) Increases the margin of stability
   (D) All of the above

06. The zero-pole plot given below is that of:

07. Which of the following increases the steady state accuracy?
   (A) Phase - lead compensator
   (B) Phase - lag compensation
   (C) Differentiator
   (D) Integrator

08. A phase lag compensator will:
   (A) Improve the speed of response
   (B) Increase overshoot
   (C) Increase relative stability
   (D) Increase bandwidth

09. The bandwidth of a control system can be increased by:
   (A) Phase lead compensator
   (B) Phase lag compensator
   (C) Phase lag - lead compensator
   (D) All of these

PREVIOUS PSQ'S QUESTIONS

01. Indicate which of the following transfer functions represents phase lead compensator?

02. Which of the following is correct expression for the transfer function of an Electrical RC phase lag compensating network?

03. A system has the transfer function:

04. The transfer function is:

05. A lag network for compensation normally consists of:

ACE Academy — Control Systems
6.4 CONTROLLERS

(1) Proportional Controller:
\[ G_p(s) = K_p \]
It is used to vary the transient response of a system. Proportional controller is usually a amplifier with gain, \( K_p \).
One cannot determine the steady state response by changing \( K_p \). Steady state response depends on the type of the system.

(2) Integral Controller:
\[ G_i(s) = Ki/s \]
It is used to decrease the steady state error by increasing the type of the system.
Disadvantages: Stability decreases

(3) Derivative Controller:
\[ G_d(s) = K_d \]
It is used to increase the stability of the system. Stability of any system is increased by adding zeros.
Disadvantages: Steady state error increases, since type of the system decreases.

(4) Proportional + Integral (PI) Controller:
\[ G_{PI}(s) = K_p + K_i/s \]
It is used to decrease the steady state error without effecting stability, since a pole at origin and a zero is added.

(5) Proportional + Derivative (PD) Controller:
\[ G_{PD}(s) = K_p + K_d s \]
It is used to increase the stability without effecting steady state error. Since type is not changed and a zero is added.

(6) Proportional + Integral + Derivative (PID) Controller:
\[ G_{PID}(s) = K_p + K_i/s + K_d s \]
It is used to decrease the steady state error and to increase the stability. Since poles at origin and two zeros are added, one zero compensates the pole and other zero will increase the stability.
**OBJECTIVE QUESTIONS**

01. An integral controller is used to improve the transient response of a first-order system. If

\[ G(s) = \frac{1}{1 + Ts} \]

and the system is operated in closed-loop with unity feedback, what is the value of \( T_w \) if the integral controller transfer function is 10% to provide damping ratio of 0.7?

a) 0.5  b) 1  c) 2  d) 4

02. A step response of a system is given below. \( T_o \) represents the delay due to transportation lag and \( T_p \) is the rise time of the system. As a thumb rule, the system is easily controllable if

\[ T_p / T_o < 5 \]

a) less than 1  b) less than 3  c) greater than 10  d) equal to 6

03. A system has open-loop transfer given by

\[ (1 + 5s)(1 + 0.5s) \]

The performance of this system \( K(s) + T(s) \)

is made finite with a controller of the form. The system with controller is operated in closed-loop with unity feedback. In order to increase the speed of response:

a) \( T_w = 1 \)

b) \( T_w = 0.5 \) and \( T_p = 1 \)

c) \( T_w = 1 \) and \( T_p = 1 \)

d) \( T_w = 0.5 \) and \( T_p < 0.5 \)

04. If stability error for step input and speed of response are the criteria for design what controller would you recommend?

a) P controller  b) PD controller  c) PI controller  d) PID controller  e) PD controller

05. An ON-OFF controller is a

a) P controller  b) I controller  c) D controller  d) PID controller

06. The term 'reset control' refers to

a) proportional control  b) integral control  c) derivative control  d) PI controller

07. The pole-zero plot given below is that of a fan

\[ \text{Pole} \rightarrow R_c \]

a) Integrator  b) PD controller  c) PID controller  d) Lead-lag compensating network

08. The log magnitude plot of a system is given below:

\[ \text{Magnitude} \rightarrow \log s \]

The system is

a) Integrator  b) P controller  c) PI controller  d) Propotional controller

09. The phase angle versus frequency plot is shown below:

\[ \text{Phase} \rightarrow \log s \]

The network is

a) P controller  b) PD controller  c) Lead-lag network  d) PI controller  e) PD controller

10. The input-output relationship of a network is given above. The network is a

\[ \text{Output} \rightarrow \text{Input} \]

a) Integrator  b) PD controller  c) PI controller  d) Proportional controller

Key for Objective Questions:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30

JTO PREVIOUS QUESTION:

01. The use of PI controllers

\( A \) reduces oscillations

\( B \) causes the system to be stable

\( C \) increases the phase margin

\( D \) reduces the steady state error

02. A PD controller is used to eliminate a time constant. Compared to the uncompensated system, the compensated system has

a) a higher time constant  b) reduced overshoot  c) increase noise amplification  d) high gain  e) lower transient time

03. The industrial control systems having the best steady state accuracy are

a) a derivative controller  b) an integral controller  c) a PID controller  d) an oscillatory controller

04. In industrial control systems, which of the following methods is most commonly used in designing system for meeting performance specifications?

\( A \) the transfer function is first determined and then either a lead compensation or lag compensation is implemented.

\( B \) The transfer function is first determined and then either a lead compensation or lag compensation is implemented.

\( C \) The transfer function is first determined and then either a lead compensation or lag compensation is implemented.

\( D \) The transfer function is first determined and then either a lead compensation or lag compensation is implemented.

JTO KEYS:

State variable analysis

Advantages:
1) Analysis is done by considering initial conditions.
2) More accurate than transfer function.
3) Analyze of multi-input, multi-output systems are less complex.

Output equations:
\[ y(0) = c_0 x_0(0) + c_0 x_0(0) + d_0 u_0(0) + d_0 u_0(0) \]
\[ y(0) = c_0 x_0(0) + c_0 x_0(0) + d_0 u_0(0) + d_0 u_0(0) \]

By representing these in matrix form,
\[
\begin{pmatrix}
\dot{x}(0) \\
y(0)
\end{pmatrix} =
\begin{bmatrix}
c_0 & c_0 \\
0 & 0
\end{bmatrix}
\begin{pmatrix}
x(0) \\
u(0)
\end{pmatrix} +
\begin{bmatrix}
d_0 & d_0 \\
d_0 & d_0
\end{bmatrix}
\begin{pmatrix}
x(0) \\
u(0)
\end{pmatrix}
\]
\[
x(0) = C x(0) + D u(0)
\]

State transition matrix:

\[
\frac{d x(t)}{dt} = A x(t)
\]

Let \( A \) be an \( n \times n \) matrix that represents the state transition matrix; then it must satisfy the equation
\[
\frac{d \psi(t)}{dt} = A \psi(t) \tag{1}
\]

Let \( x(0) \) denote the initial state at \( t = 0 \); then \( \psi(t) \) is also defined by the matrix equation
\[
x(t) = \phi(t) x(0)
\]

which is the solution of the homogeneous state equation for \( t > 0 \).

One way of determining \( \phi(t) \) is by taking the Laplace Transform on both sides of Eq. (1): we have
\[
S X(s) - x(0) = A X(s)
\]

Solving for \( X(s) \) from the last equation, we get
\[
x(s) = \left( s I - A \right)^{-1} x(0)
\]

where it is assumed that the matrix \((s I - A)\) is nonsingular. Taking the inverse Laplace transform on both sides,
\[
x(t) = \mathcal{L}^{-1} \left( s I - A \right)^{-1} x(0) \quad t > 0
\]

Properties of the State Transition Matrix:

The state transition matrix \( \psi(t) \) possesses the following properties:
1. \( \psi(0) = I \) the identity matrix.
2. \( \dot{\psi}(t) = \psi(t) \)
3. \( \psi(t_2 - t_1) \psi(t_1 - t_0) = \phi(t_2 - t_0) \quad \text{for any } t_0, t_1, t_2 \)
4. \( \dot{\psi}(t)^2 = \psi(t) \)

Characteristic Equation, Eigen values, and Eigen vectors:

The characteristic equation can be expressed as
\[
|s I - A| = 0
\]
The roots of the characteristic equation are often referred to as the eigenvalues of the matrix A.

The n x 1 nonzero vector \( \lambda_i P_i \) that satisfies the matrix equation

\[
(\lambda_i I - A) P_i = 0
\]

where \( \lambda_i \) is the eigenvalue of A, is called the eigenvector of A associated with the eigenvalue \( \lambda_i \).

Controllability of linear systems:

Consider a linear time-invariant system described by the following dynamic equations:

\[
\dot{x}(t) = Ax(t) + Bu(t)
\]

\[
y(t) = Cx(t) + Du(t)
\]

where \( x(t) \) is an n x 1 state vector,
\( u(t) \) is a p x 1 input vector,
\( y(t) \) is a q x 1 output vector,
\( A \) is an n x n coefficient matrix,
\( B \) is a n x p coefficient matrix,
\( C \) is a q x n coefficient matrix,
\( D \) is a q x p coefficient matrix.

The state \( x(t) \) is said to be controllable at \( t = t_0 \) if there exists a piecewise continuous input \( u(t) \) that will drive the state to any final state \( x(t_0) \) for a finite time \( t_0 - t \geq 0 \). If every state \( x(t_0) \) of the system is said to be completely state controllable or simply state controllable.

The following shows that the condition of controllability depends on the coefficient matrices A and B of the system. The theorem also gives one way of testing state controllability:

For the system to be completely state controllable, it is necessary and sufficient that the following \( n \times n \) matrix has a rank of \( n \):

\[
S = [B \ AB \ A^2B \ \ldots \ \ldots \ \ldots \ A^{n-1}B]
\]

Since the matrices A and B are involved, sometimes we say that the pair \( [A, B] \) is controllable, which implies that 'S' is of rank 'n'.

Observability of Linear Systems:

Given a linear time-invariant system is said to be observable if any given input \( u(t) \), there exists a finite time \( t \leq t_0 \) such that the knowledge of \( y(t) \) for \( t \leq t \leq t_0 \); the matrices A, B, C, D:

and the output \( y(t) \) for \( t_0 \leq t \) are sufficient to determine \( x(t_0) \). If every state of the system is observable for a finite time, we say that the system is completely observable, or simply observable.

The following shows that the condition of observability depends on the coefficient matrices A and D of the system. The theorem also gives one method of testing observability:

For the system to be completely observable, it is necessary and sufficient that the following \( n \times p \) matrix has a rank of \( p \):

\[
V = [C^T \ A^TC^T \ (A^T)^2C^T \ \ldots \ \ldots \ \ldots \ (A^T)^{n-1}C^T]
\]

The condition is also referred to as the pair \( [A, C] \) being observable. In particular, if the system has only one output, C is an \( 1 \times n \) matrix; \( V \) is an \( n \times n \) square matrix. Then the system is completely observable if \( V \) is nonsingular.

Example: Obtain the time response of the system given below:

\[
\dot{\hat{x}} = Ax
\]

where

\[
A = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} \quad \text{given} \ u(t) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T
\]

and

\[
y = [1 \ -1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}
\]

SOL: The time response is given by:

\[
x(t) = e^{(t - t_0)A}x(t_0)
\]

\[
x(t) = \left(e^{(t - t_0)A}\right)x(t_0)
\]

\[
(A - t_0) = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}^T
\]

\[
(A - t_0)^{-1} = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}^T
\]

\[
(A - t_0)^{-1} = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}^T
\]

\[
(A - t_0)^{-1} = \begin{bmatrix} (t - t_0)^{-1} & 0 \\ 0 & (t - t_0)^{-1} \end{bmatrix}
\]

Since

\[
(A - t_0)^{-1}
\]
\[
\Phi(t) = \begin{bmatrix}
\frac{1}{s^2 + 2} & \frac{1}{s^4 + 2} \\

\frac{1}{s^2 + 2} & \frac{1}{s^4 + 2}
\end{bmatrix}
\]

The state transition matrix \(\Phi(t)\) is

\[
\Phi(t) = \Phi^{-1} \Phi(t)
\]

\[
= \begin{bmatrix}
\cos \sqrt{2} t & \left(\frac{1}{\sqrt{2}}\right) \sin \sqrt{2} t \\

-\sqrt{2} \sin \sqrt{2} t & \cos \sqrt{2} t
\end{bmatrix}
\]

\[
x(0) = \Phi(0) x(0)
\]

\[
x(0) = \begin{bmatrix}
\cos \sqrt{2} t \\

-\sqrt{2} \sin \sqrt{2} t
\end{bmatrix} + \begin{bmatrix}
\frac{1}{\sqrt{2}} \sin \sqrt{2} t \\

\cos \sqrt{2} t
\end{bmatrix} \begin{bmatrix} 1 \end{bmatrix}
\]

\[
= \begin{bmatrix}
\cos \sqrt{2} t + \left(\frac{1}{\sqrt{2}}\right) \sin \sqrt{2} t \\

\sqrt{2} \sin \sqrt{2} t + \cos \sqrt{2} t
\end{bmatrix}
\]

\[
y = x_1 - x_2
\]

\[
y = (3\sqrt{2}) \sin \sqrt{2} t
\]

---

**OBJECTIVE QUESTIONS**

01. Given a state variable model

\[
k = A x + b u
\]

\[
y = c x + d u
\]

Under this transformation \(x = P \xi\), \(P\) is a nonsingular matrix, the model becomes

\[
k = \tilde{A} \xi + \tilde{b} u
\]

\[
y = \tilde{c} \xi + \tilde{d} u
\]

a) \(\tilde{A} = P^{-1} A P\); \(\tilde{b} = P^{-1} b\); \(\tilde{c} = c P\)

b) \(\tilde{A} = P^{-1} A P\); \(\tilde{b} = P^{-1} b\); \(\tilde{c} = c P\)

c) \(\tilde{A} = P^{-1} A P\); \(\tilde{b} = P b\); \(\tilde{c} = c P\)

d) \(\tilde{A} = P^{-1} A P\); \(\tilde{b} = P b\); \(\tilde{c} = c P^{-1}\)

02. A state variable formulation of a system is given by the equations

\[
\begin{bmatrix}
x_0 \\

x_1
\end{bmatrix} = \begin{bmatrix}
-1 & 0 \\
0 & 1
\end{bmatrix} \begin{bmatrix}
x_1 \\

x_0
\end{bmatrix} + \begin{bmatrix} 1 \end{bmatrix} u
\]

\[
y = [1, \beta] x
\]

The transfer function of the system is:

a) \(\frac{1}{(s + 1)(s + 3)}\)

b) \(\frac{1}{s + 1}\)

c) \(\frac{1}{s + 3}\)

d) None

03. A state variable formulation of a system is given by the equations

\[
\begin{bmatrix}
x_0 \\

x_1
\end{bmatrix} = \begin{bmatrix}
-1 & 0 \\
0 & 3
\end{bmatrix} \begin{bmatrix}
x_1 \\

x_0
\end{bmatrix} + \begin{bmatrix} 1 \end{bmatrix} u
\]

\[
x(0) - x_0(0) = 0
\]

\[
y = [1 \ 0] x
\]

The response \(y(t)\) to unit step input is:

a) \(1 + e^{-t}\)

b) \((1/3)(1 - e^{-3t})\)

\(\beta\) None of the answers is correct

04. The Eigen values of the matrix

\[
A = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & -3 & -4
\end{bmatrix}
\]

a) \(0, -1, -3\)

b) \(0, -3, -4\)

c) \(0, 0, -4\)

d) None
05. Given the system
\[
\begin{bmatrix}
0 & 0 & -20 \\
0 & 1 & -24 \\
0 & 1 & -9
\end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} a
\]
\[
y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x
\]
The characteristic equation of the system is
a) \( s^3 + 20 s^2 + 24 s + 9 = 0 \)
b) \( s^3 + 9 s^2 + 24 s + 26 = 0 \)
c) None of the answers is correct.

06. A state variable model of a system is given by
\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\
-2 & -1 & 0 \\
1 & 0 & 0
\end{bmatrix} \begin{bmatrix} x_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} + \begin{bmatrix} 0 \\
0 \\
1
\end{bmatrix} u
\]
\[
y = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\
\dot{x}_2
\end{bmatrix}
\]
The system is
a) controllable and observable
b) controllable but unobservable
c) observable and uncontrollable
d) uncontrollable and unobservable

07. The transfer function
\( G(s) = (sI - A)^{-1} b \)
of the system is
\[
\dot{x} = Ax + bu
\]
\[
y = c^T x + du
\]
has pole-zero cancellation. The system is
a) uncontrollable and unobservable
b) observable but uncontrollable
c) controllable but unobservable
d) may be any one of a), b), and c)

08. The transfer function
\( G(s) = (sI - A)^{-1} b \)
of the system is
\[
\dot{x} = Ax + bu
\]
\[
y = c^T x + du
\]
has no pole-zero cancellation. The system is
a) controllable and observable
b) observable but uncontrollable
c) uncontrollable but unobservable
d) may be any one of a), b), and c)

09. Consider the system
\[
A = \begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad c = [0, 1]
\]
The transfer function of the system has pole-zero cancellation. The system is
a) controllable and observable
b) uncontrollable and unobservable
c) uncontrollable but unobservable
d) observable but uncontrollable

10. For all values of \( t \), the matrix exponential \( e^{At} \) is non-singular for
a) singular A
b) non-singular A
c) all A
d) nothing can be said, in general, about non-singularity of \( e^{At} \) for a given A.

Key:
1. a 2. b 3. c 4. a 5. b 6. a 7. d 8. a 9. d 10. c
01. The transfer function of the system shown is

\[ \frac{V(s)}{R(s)} = \frac{1}{(s + 1)(s + 2)} \]

(a) \( \frac{1}{1 + s} \)  
(b) \( \frac{1}{1 + 2s} \)  
(c) \( \frac{s}{1 + s} \)  
(d) \( \frac{s}{1 + s} \)

02. The performance specifications for a unity feedback control system having an open loop transfer function

\[ G(s) = K\left( \frac{s}{s + 1}\right)(s + 2) \]

are

(a) Velocity error coefficient \( K_v > 10 \text{ sec}^{-1} \)
(b) Stable closed-loop operation
(c) Value of \( K \), satisfying above specifications, is
(a) \( K > 6 \)  
(b) \( 6 < K < 10 \)  
(c) \( K > 10 \)  
(d) None of the above

03. A system has a complex conjugate root pair of multiplicity two or more in its characteristic equation. The impulse response of the system will be

(a) a sinusoidal oscillation which decays exponentially; the system is therefore stable
(b) a sinusoidal oscillation with time multiplier, the system is therefore unstable
(c) a sinusoidal oscillation which rises exponentially with time, the system is therefore unstable
(d) a dc term and harmonic oscillation; the system therefore becomes limitingly stable

04. The unit step response of a second order linear system with zero initial state is given by

\[ \phi(t) = 1 + 3.25 e^{-1.3} \sin(3.33t) \]

where \( t \geq 0 \). The damping ratio and the undamped natural frequency of oscillation of the system are respectively

(a) 0.6 & 10 rad/s  
(b) 0.6 & 12.5 rad/s  
(c) 0.8 & 10 rad/s  
(d) 0.8 & 12.5 rad/s

05. Laplace Transform of a unit ramp starting at \( t = a \) is

(a) \( 1/(a^2) \)  
(b) \( 1/2 \)  
(c) \( 1/(a^2) \)  
(d) \( 1/(a^2) \)

06. The open loop transfer function of a system is

\[ G(s) = K\left(1 + 2s + 3s^2\right) \]

The phase margin frequency \( \omega_p \) is

(a) \( \sqrt{2} \)  
(b) 1  
(c) \( \sqrt{2} \)  
(d) 0

07. An open loop transfer function is given by

\[ G(s) = Ks(s^2 + 2s + 2) \]

It has

(a) one zero at infinity
(b) two zeros at infinity
(c) three zeros at infinity
(d) four zeros at infinity

08. Which of the following system is unstable?

(a) \( \frac{(s+1)(s+2)}{(s+3)(s+4)} \)  
(b) \( \frac{(s+1)(s+2)}{(s+3)(s+4)} \)  
(c) \( \frac{(s+1)}{(s+2)} \)  
(d) \( \frac{(s+1)(s+2)}{(s+3)(s+4)} \)

09. The open loop transfer function of a unity feedback control system is given by

\[ G(s) = K\left(1 + s\right) \]

If the gain is increased to infinity, then the damping ratio will tend to become

(a) \( \sqrt{2} \)  
(b) 1  
(c) 0  
(d) \( \sqrt{2} \)

10. The characteristic equation of a closed loop system is given by

\[ s^2 + 6s + 11s + 6 + s + K = 0 \]

Stable closed-loop behavior can be obtained when gain \( K \) is such that

(a) \( K > 10 \)  
(b) \( K < 10 \)  
(c) \( K < 10 \)  
(d) None of the above

11. The maximum phase shift that can be obtained using a lead compensator with transfer function

\[ G(s) = \left(1 + 0.1s\right)/\left(1 + 0.05s\right) \]

is equal to

(a) \( 90^\circ \)  
(b) \( 45^\circ \)  
(c) \( 0^\circ \)  
(d) \( 0^\circ \)

12. Consider a system shown. If the system is disturbed so that \( y(0) = 1 \), then \( y(t) \) for a unit step input will be

(a) \( 1 + t \)  
(b) \( 1 - t \)  
(c) \( 1 + 2t \)  
(d) \( 1 - 2t \)

13. The closed loop transfer function of a control system is given by

\[ \frac{y(s)}{r(s)} = \frac{2}{s(s^2 + 2s + 2)} \]

For the input \( r(t) = u(t) \), the steady state value of \( y(\infty) \) is equal to

(a) \( 1 \)  
(b) \( 2 \)  
(c) \( 2 \)  
(d) None of the above

14. The impulse response of an initially relaxed 2nd order system is \( e^{-2t} u(t) \). To produce a response of \( e^{-2t} u(t) \), the input must be equal to

(a) \( 2 e^{-2t} u(t) \)  
(b) \( \frac{1}{2} e^{-2t} u(t) \)  
(c) \( e^{-2t} u(t) \)  
(d) \( e^{-2t} u(t) \)

15. The closed loop transfer function of a control system is given by

\[ G(s) = \frac{2(s+2)}{(s+2)(s+1)} \]

For a unit step input the output is

(a) \( 2e^{-2t} + 2e^{-t} \)  
(b) \( 2e^{-2t} + 2e^{-t} \)  
(c) \( 2e^{-2t} + 2e^{-t} \)  
(d) \( 2e^{-2t} + 2e^{-t} \)

16. The Laplace transformation of \( f(t) \) is

\[ F(s) = K\left[\frac{2e^{2s} + 4}{s^2 + 2s + 2}\right] \]

The final value of \( f(t) \) is

(a) Infinity  
(b) 0  
(c) None of the above

17. For \( M > 1 \), the constant \( M \), circle corresponding to the magnitude \( M \) of the closed loop transfer function of a linear system lie in the \( G \)-plane and to the

(a) right of the \( M = 2 \) line  
(b) left of the \( M = 1 \) line  
(c) upper side of the \( G \)-plane line  
(d) lower side of the \( G \)-plane line

18. The position and acceleration error coefficients for the open loop transfer function

\[ G(s) = K\left(s^2 + 10\right) \]

are respectively

(a) zero and infinity  
(b) infinity and zero  
(c) \( K/1000 \) and zero  
(d) infinity and \( K/(1000) \)

19. The position and velocity error coefficients for the system of transfer function

\[ G(s) = K\left(1 + 0.1s\right) \]

are respectively

(a) \( K > 0 \)  
(b) \( K = 0 \)  
(c) \( K \)  
(d) None of the above

20. The phase shift at \( \omega = 0 \) and \( \omega = \infty \) for

(a) 90° and 0°  
(b) 180° and 180°  
(c) -90° and 70°  
(d) None of the above

21. A system has the transfer function

\[ H(s) = 
\]

It is known as

(a) low pass system  
(b) high pass system  
(c) all pass system  
(d) none of the above
22. The transfer function of a control system is given as
\[ T(s) = \frac{K}{(s^2 + 2s + K)} \]
where \( K \) is the gain of the system in rad/deg/s. For this system to be critically damped, the value of \( K \) should be
(a) 1
(b) 2
(c) 3
(d) 4

23. A second order differential equation is given by. The natural frequency is rad/sec and damping ratio are respectively
(a) 1.5
(b) 5
(c) 1
(d) 47

24. The transfer function of a system is
\[ 10/(1 + s) \]
when operated as a unity feedback system, the steady state error is a unit step input will be
(a) zero
(b) 1/11
(c) 10
(d) infinity

25. By Routh's stability criterion, it is possible to find the roots in RHP of the vertical line passing through (n, 0) by setting to the characteristic polynomial
(a) \( s^2 + \frac{\lambda + a}{\lambda} \)
(b) \( s^2 + \frac{\lambda - a}{\lambda} \)
(c) \( s^2 + \frac{\lambda - a}{1+a} \)
(d) \( s^2 + \frac{\lambda + a}{1+a} \)

26. If the gain of the open loop system is doubled, the gain margin
(a) is not affected
(b) gets doubled
(c) becomes half
(d) becomes one fourth

27. Given \( G(s) = \frac{1+18s}{s^2+2s+2} \). The system with this transfer function is operated in a closed loop with unity feedback. The closed loop system is
(a) stable
(b) unstable
(c) marginally stable
(d) conditionally stable

KEY FOR GRAND TEST
01.b 02.d 03.b 04.a 05.d
06.b 07.e 08.b 09.c 10.a
11.b 12.c 13.a 14.e 15.a
16.d 17.b 18.d 19.a 20.d
21.e 22.d 23.d 24.b 25.a
26.c 27.a 28.c 29.c 30.a

“All The Best”

OBJECTIVE QUESTIONS : REVISION AND PRACTICE SET
01. For a second-order system with the closed-loop transfer function
\[ T(s) = \frac{3}{s^2 + 6s + 9} \]
the settling time for 2% error, in seconds, is
(a) 1.5
(b) 2.0
(c) 3.0
(d) 4.0

02. The gain margin (in dB) of a system having the loop transfer function
\[ G(s) = \frac{1}{s+1} \]
is
(a) 0
(b) 3
(c) 6
(d) 12

03. The system mode described by the state equation:
\[ \dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \]
where \( x = [x_1 x_2]^T \)
is
(a) controllable and observable
(b) controllable, but not observable
(c) observable, but not controllable
(d) neither controllable nor observable

04. The phase margin (in degrees) of a system having the loop transfer function
\[ G(s)H(s) = \frac{2\sqrt{2}}{s+1} \]
is
(a) 45
(b) 60
(c) 80
(d) 100

05. An amplifier with negative feedback has two left half plane poles in its open-loop transfer function. The amplifier
(a) will always be unstable at high frequency
(b) will be stable for all frequency
(c) may be unstable, depending on the feedback factor
(d) will oscillate at low frequency

06. A system described by the transfer function
\[ H(s) = \frac{1}{s^2 + 6s + 9} \]
is stable.

The constraints on \( \alpha \) and \( k \) are
(a) \( \alpha > 0, \alpha < 3 \)
(b) \( \alpha < 0, \alpha > 3 \)
(c) \( \alpha < 0, \alpha < 0 \)
(d) \( \alpha > 0, \alpha > 0 \)

07. The equivalent of the block diagram in Fig-1 is given in
(a) G(s)H(s)C
(b) G(s)H(s)D
(c) G(s)H(s)A
(d) G(s)H(s)B
08. If the characteristic equation of a closed-loop system is $s^2 + 2s + 2 = 0$, then the system is
(a) overdamped
(b) critically damped
(c) underdamped
(d) undamped

09. The root-locus diagram for a closed-loop feedback system is shown in figure. The system is overdamped.

10. The Nyquist plot for the open-loop transfer function $G(s)$ of a unity negative feedback system is shown in figure. If $G(s)$ has no pole in the right-half of $s$-plane, the number of roots of the system characteristics equation in the right-half of $s$-plane is

11. An electrical system and its signal-flow graph representation are shown in figure(a) and (b), respectively. The values of $G_1$ and $H$, respectively, are

14. Figure shows the Nyquist plot of the open-loop transfer function $G(zH(z))$ of a system. If $G_0$ is $H_0$, the closed-loop system is

15. A PD controller is used to compensate a system. Compared to the uncompensated system, the compensated system has
(a) a higher type number
(b) reduced damping
(c) lower noise amplification
(d) larger transient overshoot

16. The gain margin for the system with an open-loop transfer function
\[
G(s)H(s) = \frac{2(s + 1)}{s^2 + 2s + 1}
\]
is
(a) $\frac{4}{9}$
(b) $\frac{4}{13}$
(c) $\frac{9}{4}$
(d) $\frac{13}{4}$

17. Given the transfer function $G(z)$, the point of intersection of the asymptotes of the root loci with the real axis is

18. A linear system is equivalently represented by two sets of state equations.

\[
\begin{align*}
\begin{array}{c}
X = AX + BU \quad \text{and} \\
Y = CW + DU
\end{array}
\end{align*}
\]
The eigenvalues of the representations are also computed as $[s]$ and $[a]$. Which one of the following statements is true?

(a) $[s] = [a]$ and $X = W$
(b) $[s] = [a]$ and $X = V$
(c) $[s] = [a]$ and $X = W$
(d) $[s] = [a]$ and $X = W$

19. Which one of the following pole diagrams corresponds to a lag network?
20. Despite the presence of negative feedback, control systems still have problems of instability because the 
(a) components used have nonlinearities 
(b) dynamic equations of the subsystem are not known exactly 
(c) mathematical analysis involves approximations 
(d) system has large negative phase margin at high frequencies. 

21. The open-loop transfer function of a unity-gain feedback control system is given by 
\[ G(s) = \frac{K}{s + (s+2)} \] 

The gain margin of the system in dB is given by 
(a) 0 (b) 1 (c) 20 (d) \( \infty \) 

22. In the system shown below, \( x(t) = \text{unit}(t) \). In steady-state the response \( y(t) \) will be 
\[ y(t) = \frac{1}{s+1} y(0) \] 
(a) \( 1 \) \( e^{-t} \) 
(b) \( \frac{1}{\sqrt{2}} \) \( e^{-t/\sqrt{2}} \) 
(c) \( \frac{1}{\sqrt{2}} \) \( e^{-t/\sqrt{2}} \) 
(d) stable 

23. If the closed-loop transfer function of a control system is given as 
\[ T(s) = \frac{-3}{s + 2(s+3)} \] 
then it is 
(a) an unstable system 
(b) an uncontrollable system 
(c) a minimum phase system 
(d) non-minimum phase system 

24. If the Laplace transform of a signal \( y(t) \) is 
\[ Y(s) = \frac{1}{s^2 + 1} \] 
then its final value is 
(a) \( 1 \) 
(b) 0 
(c) 1 
(d) unbounded 

25. The pole-zero plot given below corresponds to 
(a) Low pass filter 
(b) High pass filter 
(c) Band pass filter 
(d) notch filter 

26. Step responses of a set of three second-order underdamped systems all have the same percentage overshoot. Which of the following diagrams represents the poles of 3 systems? 

(a) 
(b) 
(c) 
(d) 

27. Which of the following points is NOT on the root locus of a system with the open-loop transfer function 
\[ G(s)H(s) = \frac{1}{s(s+1)(s+3)} \] 
(a) \( s = -\sqrt{2} \) 
(b) \( s = -1.5 \) 
(c) \( s = -3 \) 
(d) \( s = \infty \) 

28. The phase margin of a system with the open-loop transfer function 
\[ G(s)H(s) = \frac{1}{(s+1)(s+2)} \] 
(a) \( 90^\circ \) 
(b) \( 63.4^\circ \) 
(c) \( 90^\circ \) 
(d) \( 0^\circ \) 

29. The transfer function \( Y(s)/X(s) \) of a system described by the state equations 
\[ \dot{x} = Ax + Bu \] 
where \( A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \) and \( B = \begin{bmatrix} p \\ q \end{bmatrix} \) is 
\[ 0.5(s-2) \] 
(a) \( s+1 \) 
(b) \( 1+s \) 
(c) \( 0.5(s+2) \) 
(d) \( 1(s-2) \) 

30. Consider the system 
\[ \dot{x} = Ax + Bu \] 
with \( A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \) and \( B = \begin{bmatrix} p \\ q \end{bmatrix} \) 
where \( p \) and \( q \) are arbitrary real numbers. Which of the following statements about the controllability of the system is true? 
(a) The system is completely controllable for any nonzero values of \( p \) and \( q \). 
(b) Only \( p=0 \) and \( q=0 \) result in controllability. 
(c) The system is uncontrollable for all values of \( p \) and \( q \). 
(d) We cannot conclude about controllability from the given data.
Two Mark Questions

01. If the closed-loop transfer function \( T(s) \) of a unity negative feedback system is given by

\[
T(s) = \frac{K}{s(s+1)(s+2)
\]

then the steady state error for a unit ramp input is

(a) \( \frac{K}{K+1} \)  
(b) \( \frac{K}{K+2} \)  
(c) \( \frac{K}{K+3} \)  
(d) \( \frac{K}{K+4} \)

02. Consider the points \( n = -3, -j4 \) and \( m = -3, -j5 \) in the s-plane. Then, for a system with the open-loop transfer function

\[
G(s)H(s) = \frac{K}{(s+1)(s+2)}
\]

(a) \( n \) is on the root locus, but not \( m \)  
(b) \( m \) is on the root locus, but not \( n \)  
(c) both \( n \) and \( m \) are on the root locus  
(d) neither \( n \), nor \( m \) is on the root locus

03. For the system described by the state equation

\[
\begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
0.5 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} u(t)
\]

if the control signal \( u(t) \) is given by \( x = [-0.5, -1, -2] \), then the eigen values of the closed-loop system will be

(a) \( 0, -1, -2 \)  
(b) \( 0, -1, -3 \)  
(c) \( -1, -2, -3 \)  
(d) \( 0, -1, -1 \)

04. Consider a system with the transfer function \( G(s) = \frac{K}{s^2 + 6s + 9} \) its damping ratio will be 0.5 when the value of \( K \) is

(a) 2/6  
(b) 3  
(c) 1/6  
(d) 6

05. The system shown in the figure remains stable when \( GATE-2002 \)

\[
\begin{array}{c}
(a) k < -1 \\
(b) 1 < k < 3 \\
(c) k > 3
\end{array}
\]

06. The transfer function of a system is

\[
G(s) = \frac{100}{(s+1)(s+100)}
\]

For a unit-step input to the system the approximate settling time for 2% criterion is

(a) 100 sec  
(b) 4 sec  
(c) 1 sec  
(d) 0.01 sec

07. The characteristic polynomial of a system \( G(s) = s^2 + 4s + 2s + 1 \). The system is

(a) stable  
(b) marginally stable  
(c) unstable  
(d) oscillatory

08. The system with the open-loop transfer function \( G(s)H(s) = \frac{1}{s^2 + 3s + 1} \), has a gain margin of

(a) -6 dB  
(b) 0 dB  
(c) 3.5 dB  
(d) 6 dB

09. The root locus of the system

\[
G(s)H(s) = \frac{K}{s(s + 3)(s + 2)}
\]

has the break - away point located at \( GATE-2003 \)

(a) (-0.5, 0)  
(b) (-2.5, 0)  
(c) (-4, 0)  
(d) (-0.75, 0)

10. The signal flow graph of a system is shown in figure. The transfer function \( G(s) \) of the system is

\[
\frac{R(s)}{E(s)} = \frac{1}{s^2 + 4s + 4}
\]

With \( e(t) \) as the unit-step function, the response \( r(t) \) of the system is represented by \( GATE-2003 \)

11. The approximate Bode magnitude plot of a minimum-phase system is shown in figure. The transfer function of the system is

\[
G(s) = \frac{2(s + 2)(s + 2)}{(s + 2)^2 + (s + 3)^2}
\]

12. A second-order system has the transfer function

\[
\frac{G(s)}{R(s)} = \frac{1}{s^2 + 4s + 4}
\]

With \( e(t) \) as the unit-step function, the response \( r(t) \) of the system is represented by \( GATE-2003 \)
16. A control system having the transfer function \( H(s) = \frac{1}{s + 1} \) is excited with 100. The time at which the output reaches 99% of its steady state value is... GATE-2004
(a) 2.7 sec (b) 2.2 sec (c) 2.5 sec (d) 2.1 sec

17. A system has poles at 0.01, 1 Hz and 8 Hz. Zeros at 5 Hz, 100 Hz and 500 Hz. The approximate phase of the system response at 20 Hz is...
(a) -90° (b) 90° (c) 0° (d) -180°

18. Consider the signal flow graph shown in figure. The gain \( \frac{X_2}{X_1} \) is...
(a) 1 (b) 4 (c) 4 (d) 1

19. The open-loop transfer function of a unity feedback system is...
(a) \( K \left( s^2 + 1 \right) \) (b) \( \frac{K}{s^2 + 2} \) (c) \( K \left( s^2 + 1 \right) \) (d) \( \frac{K}{s^2 + 1} \)

20. For the polynomial \( P(s) = s^3 + 2s^2 + 3s + 4 \), the number of roots which lie in the right half of the s-plane is...
(a) 4 (b) 2 (c) 3 (d) 1

21. The state variable equations of a system are...

\[ \begin{align*}
\dot{x}_1 &= -3x_1 + x_2 + u \\
\dot{x}_2 &= 2x_1 + y = x_1 + u
\end{align*} \]

The system is...
(a) uncontrollable but not observable (b) observable but not controllable (c) neither controllable nor observable (d) controllable and observable

22. A system described by the following differential equation...

\[ \frac{dy}{dt} + 2y = x(t) \] is initially at rest. For input \( x(t) = 2u(t) \), the output \( y(t) \) is...
(a) \( 0 - 2e^{-2t} + e^{-2t} \) (b) \( 0 - 2e^{-2t} + e^{-2t} \) (c) \( 0 - 2e^{-2t} + e^{-2t} \) (d) \( 0 - 2e^{-2t} + e^{-2t} \)

23. Given \( A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \) and the state transition matrix \( e^{At} \) is given by...
(a) \( 0 \) (b) \( e^t \) (c) \( e^{2t} \) (d) \( e^{1t} \)

24. The polar diagram of a conditionally stable system for open loop gain \( K = 1 \) is shown in the figure. The open-loop transfer function of the system is known to be stable. The closed loop system is stable for...
(a) \( K < 5 \) (b) \( K < \frac{1}{2} \) (c) \( K < 1 \) (d) \( K < 5 \)

25. In the derivation of expression for peak present overshoot, \( M_d = \exp \left( -\frac{s}{\xi} \right) \) which and \( \xi = \xi \) of the following conditions is NOT required?
(a) System is linear and time-invariant (b) System transfer function has a pair of complex conjugate poles and no zeroes (c) There is no transportation delay in the system (d) The system has zero initial conditions.

26. A ramp input applied to an unity feedback system results in 9% steady state error. The type number and zero frequency gain of the system are...
(a) \( 1 \) and \( 20 \) (b) \( 0 \) and \( 20 \) (c) \( 1 \) and \( 20 \) (d) \( 1 \) and \( 20 \)
27. A double integrator plant,

\[ G(s) = \frac{k}{s^2}, \quad H(s) = 1 \]

is to be compensated to achieve the damping ratio \( \zeta = 0.5 \), and an undamped natural frequency, \( \omega_n = 5 \) rad/s. Which one of the following compensators \( G_c(s) \) will be suitable?

(4) \( \frac{3}{s + 9} \)

(3) \( \frac{6}{s + 8.33} \)

(2) \( \frac{s}{s + 3} \)

(1) \( \frac{s + 3}{s + 6} \)

28. An unity feedback system is given as, \( G(s) = \frac{1}{(s + 2)(s + 3)} \). Indicate the correct root locus diagram.

(4) \( \frac{s}{s + 3} \)

(3) \( \frac{6}{s + 8.33} \)

(2) \( \frac{s + 3}{s + 6} \)

(1) \( \frac{s}{s + 9} \)

29. The gain and phase crossover frequencies in rad/sec are, respectively.

(4) 0.632 and 1.26

(3) 0.632 and 0.483

(2) 0.483 and 0.632

(1) 1.26 and 0.632

30. Based on the above results, the gain and phase margins of the system will be.

(4) 7.09 dB and \( -87.5^\circ \)

(3) 7.09 dB and \( -87.5^\circ \)

(2) -7.09 dB and \( -87.5^\circ \)

(1) -7.09 dB and \( 87.5^\circ \)

31. Consider two transfer functions

\[ G_0(s) = \frac{1}{s + 5 + a} \quad \text{and} \quad G_1(s) = \frac{s}{s + 5 + b} \]

The 3-dB bandwidths of their frequency responses are, respectively.

(4) \( \sqrt{a^2 + 4b} \)

(3) \( \sqrt{a^2 + 4b} \)

(2) \( \sqrt{a^2 - 4b} \)

(1) \( \sqrt{a^2 + 4b} \)

32. The unit step response of a system starting from rest is given by \( G(s) = \frac{1}{1 - e^{-Tt}} \) for \( t \geq 0 \).

The transfer function of the system is.

(4) \( \frac{1}{s + 2} \)

(3) \( \frac{2}{s + 3} \)

(2) \( \frac{2}{s + 4} \)

(1) \( \frac{1}{s + 2} \)

33. The Nyquist plot of \( G(s) \) at \( j\omega \) for a closed-loop control system, passes through \((-1, \omega_c)\) point in the G/H plane. The gain margin of the system in db is equal to.

GATE-2006

(a) infinte

(b) greater than zero

(c) less than zero

(d) zero

34. The positive values of \( K_c \) and \( K_p \) so that the system shown in the figure below oscillates at a frequency of 2 rad/sec are respectively.

GATE-2006

(4) \( K_c = 1.05, \ K_p = 0.25 \)

(3) \( K_c = 1.0, \ K_p = 0.75 \)

(2) \( K_c = 1, \ K_p = 2 \)

(1) \( K_c = 0.75, \ K_p = 1 \)

35. The unit impulse response of a system is \( 0(t-w)^2 \), \( w > 0 \).

For this system, the steady-state value of the output for unit step input is equal to.

GATE-2006

(4) \(-1\)

(3) \(-1\)

(2) \(-1\)

(1) \(-1\)

36. The transfer function of a phase-lead compensator is given by

\[ G_c(s) = \frac{1}{s + T} \]

where \( T > 0 \).

The maximum phase-shift provided by such a compensator is.

GATE-2006

(a) \( \pi/2 \)

(b) \( \pi \)

(c) \( \pi/4 \)

(d) \( \pi/6 \)

37. A linear system is described by the following state equation

\[ x(t) = AX(t) + BU(t) \]

where \( X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \).

The state-transition matrix of the system is.

GATE-2007

(a) \( \begin{bmatrix} \cos \omega t & \sin \omega t \\ -\sin \omega t & \cos \omega t \end{bmatrix} \)

(b) \( \begin{bmatrix} \cos \omega t & -\sin \omega t \\ -\sin \omega t & \cos \omega t \end{bmatrix} \)

(c) \( \begin{bmatrix} -\cos \omega t & \sin \omega t \\ -\sin \omega t & -\cos \omega t \end{bmatrix} \)

(d) \( \begin{bmatrix} \cos \omega t & \sin \omega t \\ -\sin \omega t & -\cos \omega t \end{bmatrix} \)
42. The transfer function of a plant is

\[ T(s) = \frac{5}{s^2 + 5s + 6} \]

The second-order approximation of \( T(s) \) using dominant pole concept is GATE-2007

(a) \( \frac{1}{s^2 + 5s + 10} \)  
(b) \( \frac{4}{s^2 + 5s + 6} \)  
(c) \( s^2 + 5s + 1 \)  
(d) \( s^2 + s + 1 \)

43. The open-loop transfer function of a plant is given as \( G(s) = \frac{1}{s^2 + s + 1} \). If the plant is operated in a unity feedback configuration, then the lead compensator \( 1 + K \) s that can stabilize this control system is GATE-2007

(a) \( 10(s^2 + 1) \)  
(b) \( 10(s + 6) \)  
(c) \( 10(s + 2) \)  
(d) \( 10(s + 6) \)

44. A unity feedback control system has an open-loop transfer function

\[ G(s) = \frac{K}{s^2 + 7s + 12} \]

The gain K for which \( s = -1 \) will lie on the root locus of this system is GATE-2007

(a) 4  
(b) 5.5  
(c) 6  
(d) 10

45. The asymptotic Bode plot of a transfer function is as shown in the figure. The transfer function \( G(s) \) corresponding to this Bode plot is GATE-2007

(a) \( 20 \log |G(s)| \) dB  
(b) \( 10 \log |G(s)| \) dB  
(c) \( 20 \log |G(s)| \) dB  
(d) \( 10 \log |G(s)| \) dB

46. The state space representation of a separately excited DC servo motor dynamics is given as GATE-2007

\[ \begin{align*}
\dot{x} &= A x + B u \\
\dot{\theta} &= C x + D u
\end{align*} \]

where \( x \) is the speed of the motor, \( i \) is the armature current and \( u \) is the armature voltage. The transfer function \( H(s) \) of the motor is GATE-2007

(a) \( \frac{1}{s^2 + 1} \)  
(b) \( \frac{1}{s^2 + 11s + 11} \)  
(c) \( \frac{1}{s^2 + 11s + 11} \)  
(d) \( \frac{1}{s^2 + s + 1} \)

47. The eigenvalue and eigenvector pairs \( (\lambda, \mathbf{v}) \) for the system are GATE-2007

(a) \( -1 \) and \( \begin{bmatrix} 1 \\ 1 \end{bmatrix} \)  
(b) \( -2 \) and \( \begin{bmatrix} 1 \\ 1 \end{bmatrix} \)  
(c) \( -1 \) and \( \begin{bmatrix} 1 \\ 1 \end{bmatrix} \)  
(d) \( -2 \) and \( \begin{bmatrix} 1 \\ 1 \end{bmatrix} \)

48. Consider a linear system whose state space representation \( \dot{x}(t) = Ax(t) \). If the initial state vector of the system is \( x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \), then the system response is

\[ x(t) = e^{At} \]  
If the initial state vector of the system changes to \( x(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \), then the system response becomes

\[ x(t) = e^{At} \]

49. A linear, time-invariant, causal continuous-time system has a rational transfer function with simple poles at \( s = -2 \) and \( s = -4 \), and one simple zero at \( s = -1 \). A unit step input \( u(t) \) is applied at the input of the system. At steady state, the output has constant value of 1. The impulse response of unit system is GATE-2008

(a) \( 2e^{-2(t-1)} + 2e^{-4(t-1)} \)  
(b) \( -4e^{-(t-2)} + 2e^{-(t-4)} \)  
(c) \( -4e^{-(t-2)} + 2e^{-(t-4)}u(t) \)  
(d) \( -0.5e^{-(t-2)} + 1.5e^{-(t-4)}u(t) \)

50. Group I lists a set of four transfer functions. Group II gives a list of possible step responses \( y(t) \). Match the step responses with the corresponding transfer functions. 

Group I

\[ \begin{align*}
& G(s) = \frac{25}{s^2 + 25} \\
& G(s) = \frac{36}{s^2 + 20s + 36} \\
& G(s) = \frac{36}{s^2 + 12s + 36} \\
& G(s) = \frac{49}{s^2 + 7s + 49}
\end{align*} \]

Group II

(a) \( 1 \)  
(b) \( 1 \)  
(c) \( 1 \)  
(d) \( 1 \)
51. A certain system has transfer function
\[ G(s) = \frac{s+5}{s^2 + 4s + 4}, \]
where s is a parameter. Consider the standard negative unity feedback configuration as shown below.

Which of the following statements is true?

- The closed loop system is never stable for any value of s.
- For some positive values of s, the closed loop system is stable, but not for all positive values.
- For all positive values of s, the closed loop system is unstable.
- The closed loop system is stable for all values of s, both positive and negative.

\[ \text{GATE-2005} \]

52. The number of open right half plane poles of
\[ G(s) = \frac{s^2 + 2s + 2}{s^4 + 3s^3 + 6s^2 + 5s + 2} \]

is (a) 0 (b) 1 (c) 2 (d) 3

\[ \text{GATE-2005} \]

53. The magnitude of frequency response of an underdamped second order system is 5 at 0 rad/sec and peaks to 10 at \( \sqrt{2} \) rad/sec. The transfer function of the system is GATE 2008
\[ (a) \frac{s^2 + 10s + 100}{s^2 + 5s + 75} \]
\[ (b) \frac{s^2 + 12s + 144}{s^2 + 25s + 225} \]
\[ (c) \frac{s^2 + 5s + 25}{s^2 + 5s + 25} \]
\[ (d) \frac{s^2 + 10s + 100}{s^2 + 5s + 75} \]

54. A signal flow graph of a system is given below.

The set of equations that correspond to this signal flow graph is GATE 2005
\[ \begin{align*}
\frac{dx_1}{dt} &= -\alpha x_1 + \beta v_1 \\
\frac{dx_2}{dt} &= -\alpha x_2 + \beta v_1 \\
\frac{dx_3}{dt} &= -\alpha x_3 + \beta v_1 \\
\frac{dx_4}{dt} &= -\alpha x_4 + \beta v_1 \\
\frac{dx_5}{dt} &= -\alpha x_5 + \beta v_1 \\
\frac{dx_6}{dt} &= -\alpha x_6 + \beta v_1
\end{align*} \]

55. The unit step response of an underdamped second order system has steady-state value of 2. Which one of the following transfer functions has these properties?
\[ (a) \frac{-2.24}{s^2 + 2.59s + 1.12} \]
\[ (b) \frac{-3.82}{s^2 + 1.92s + 1.91} \]
\[ (c) \frac{-2.24}{s^2 + 3.59s + 1.12} \]
\[ (d) \frac{-3.82}{s^2 + 4.91s + 1.91} \]

56. Group I gives two possible choices for the impedance Z in the diagram. The circuit elements in Z satisfy the condition R1C1 > R2C2. The transfer function \( V_o / V_i \) represents a kind of controller. Match the impedances in Group I with the types of controller in Group II.

Group I
(a) R2, C2 (b) R1, C1
(c) C1, R2 (d) C2, R1

Group II
1. PI controller
2. Lead compensator
3. Lag compensator

\[ \text{Correct answer: (b) R1, C1 (c) C1, R2} \]

57. The feedback configuration and the pole-zero locations of
\[ G(s) = \frac{e^{-c}}{s^2 + 2s + 2} \]
are shown below. The root locus for negative values of \( c \), i.e., for \( c < 0 \), has breakaway-break-in points and angle of departure at point P (with respect to the positive real axis) equal to

\[ \text{Correct answer: (c) 110°} \]

58. Which of the following statements is true?

(a) G(s) is an all-pass filter
(b) G(s) has zero in the right-half plane,
(c) G(s) is the impedance of a positive network
(d) G(s) is marginally stable

59. The gain and phase margin of G(s) for the closed loop stability are
\( 6 \) dB and \( 10^\circ \), \( 6 \) dB and \( 10^\circ \), \( 3 \) dB and \( 10^\circ \), \( 3 \) dB and \( 10^\circ \)

60. How many roots with positive real parts do the equation \( x^3 + 3x + 1 = 0 \) have?

(a) Zero (b) One (c) Two (d) Three
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