ALL INDIA 1st RANK 19 TIMES IN GATE
&
ALL INDIA 2nd RANK 10 TIMES IN GATE

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GATE - SYLLABUS

Signals and Systems: Definitions and properties of Laplace transform, continuous-time and discrete-time Fourier series, continuous-time and discrete-time Fourier transforms, DFT and FFT, z-transforms. Sampling theorem, Linear Time-Invariant (LTI) systems: definitions and properties, causality, stability, impulse response, convolution, poles and zeros, parallel and cascade structures, frequency response, group delay, phase delay, Signal transmission through LTI systems.

Plan of the book:

The book begins with chapter 1 by introducing the basic concepts of signal & system models and system classification. This material, which is basic to the remainder of the book, considers the mathematical representation of signals.

Chapter 2 is devoted to the time – domain characterization of C.T. L.T. system & the convolution. To this point the focus is on the time – domain description of signals & systems. Starting with chapter 3, we consider frequency – domain descriptions. We begin the chapter with a consideration of the orthogonal representations of arbitrary signals. The F.S.S & its properties are presented.

Chapter 4 begins with the development of the F.T. equations under which the F.T. axes are presented and its properties are discussed. Applications of F.T. such as sampling theorem, Distortionless transmission are considered.

Chapter 5 deals with both unilateral & bilateral L.T. The concept of transfer function is introduced and other applications of the L.T. such as for the solution of differential equations are discussed.

Chapter 6 considers the Fourier analysis of discrete – time signals. The relation between the continuous – time & discrete – time Fourier transforms of sampled analog signals is derived.

Chapter 7 discusses the Z.T. of D.T. signals. The development follows closely that of chapter 5 for the L.T. properties of the Z.T. are derived & their application in the analysis of discrete – time systems developed. Finally mapping of the f- plane into the z- plane & realization structure is discussed.

Chapter 8 introduces the D.T for analyzing finite length sequences. The properties of the D.T are derived. Two popular fast Fourier transforms (FFT) algorithms for the efficient computation of the D.T are presented. Chapter 9 introduces additional questions of all the above topics discussed.

For any queries or suggestions for further improvement of the book, the readers can write to me at narasingham@gmail.com

M.L. NARASIMHAM
# Signals & Systems

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**Manager of Direct**

Y.V. GOPALA KRISHNA MURTHY

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**Chapter 1: INTRODUCTION**

- Anything that bears information can be considered a signal.
  - E.g.: Speech, Music, sound of an automobile.
- Mathematically, signals are modeled as functions of one or more independent variables.
  - Examples of independent variables used to represent signals are time, frequency, or spatial coordinates.

(A) Continuous - time (CT) or analog signals: Continuous in time and continuous in amplitude. Continuous in amplitude means that the amplitude can assume any value in the continuous range form $-\infty$ to $\infty$.

(B) Discrete - time (DT) Signals: Discretised in time and continuous in amplitude like DT signals arise from sampling CT signals.

(C) Digital signals: Discretised in time and quantised in amplitude.

If the amplitude of a signal can assume only a value from a finite set of numbers, the amplitude is said to be discretised or quantised.
1.1 Elementary Signals:

1. Unit Step Function, \( u(t) \):

\[
u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}
\]

An example of a unit step function is the output of a 1V dc voltage source in series with a switch that is turned on at \( t = 0 \).

2. Rectangular (or Gate) Pulse: \( A \) NOT \( t / 2a \)

The rectangular function is the result of an ON-OFF switching operation of a constant voltage source in an electrical circuit.

3. Signum Function: \( \text{Sgn}(t) \)

\[
\text{Sgn}(t) = \begin{cases} 1, & t > 0 \\ 0, & t = 0 \\ -1, & t < 0 \end{cases}
\]

An example of a ramp function is the linear sweep waveform of a cathode-ray tube.

4. Ramp Function: \( r(t) \)

\[
r(t) = \begin{cases} t, & t > 0 \\ 0, & t = 0 \end{cases}
\]

Example: The ramp function can be used to describe the voltage across a capacitor in an RC circuit.

5. Sampling Function: \( S_a(x) = \frac{\sin x}{x} \)

\[
S_a(x) = \frac{\sin x}{x}
\]

Example: The sampling function is used in signal processing to represent the samples of a continuous-time signal at discrete time intervals.

6. Unit Impulse Function (or Dirac Delta Function, \( \delta(t) \))

Many physical phenomena such as point sources, point charges, or voltage sources acting for very short time can be modeled as delta functions.

7. Sampling Property:

\[
x(t) \ast (t - b) = x(t) \delta(t - b)
\]

Example: For \( x(t) = 2t + 3 \) and \( b = 3 \), the convolution becomes:

\[
x(t) \ast (t - 3) = 2(t + 3) + 3(t - 3)
\]

8. Sifting Property:

\[
\int_{-\infty}^{\infty} x(t) \delta(t - b) \, dt = x(b)
\]

Example: For \( x(t) = t^2 \) and \( b = 2 \), the integral becomes:

\[
\int_{-\infty}^{\infty} t^2 \delta(t - 2) \, dt = 3 + 2^2 = 12
\]

P1.1.1: Find the value of the following integrals:

(A) \[ \int_{-\infty}^{\infty} (3 + 2t) \delta(t - 4) \, dt = 0 \]

(B) \[ \int_{-\infty}^{\infty} (t \cos \pi t) \delta(t - 1) \, dt = \frac{1}{2} \]

(C) \[ \int_{0}^{\infty} \cos \pi t \delta(t - 1) \, dt = 0 \]

(D) \[ \int_{0}^{\infty} \delta(t - 2) \, dt = 1 \]

(E) \[ \int_{0}^{\infty} \delta(t) \, dt = 1 \]

9. Differentiation:

\[
n(t) = \frac{d}{dt} \left( x(t) \right)
\]

Example: The derivative of a ramp function is a step function.

\[
x(t) = t \Rightarrow \frac{d}{dt} x(t) = \delta(t)
\]
1.2. TRANSFORMATIONS OF THE INDEPENDENT VARIABLE:

Eq. (1):

To get \( x(0) \) replot \( x(0) \)

About \( t = 0 \)

\[ x(t) = \begin{cases} 1 & -1 < t < 2 \\ 0 & \text{otherwise} \end{cases} \]

Eq. (2):

Sketch the waveform

\[ x(t) = \begin{cases} 1 & -1 < t < 2 \\ 0 & \text{otherwise} \end{cases} \]

(i) Compress \( x(t) \) by a factor of 3 to obtain \( x(3t) \)

(ii) Time-reverse \( x(t) \) to obtain \( x(-t) \)

(iii) \( x(-3t-1) \) Shift \( x(-3t) \) left by 2 units.

\( x(2t) = \begin{cases} 1 & -1 < 2t < 2 \\ -1/2 & t < 1 \end{cases} \)
1.3. Classification of Signals:

1. Energy and Power Signals:

- A signal \( x(t) \) (or \( x[n] \)) is called an energy signal if its total energy has a non-zero finite value, \( E_x < \infty \).
- A signal is called a power signal if it has non-zero finite power, \( 0 < P_x < \infty \).
- A signal can't be both an energy and power signal simultaneously.
- The term instantaneous power is reserved for the true rate of change of energy in a system. In most cases, when the term power is used it refers to average power, \( P_x \), the average rate of energy utilization, a constant quantity independent of time.

\[
E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt
\]

\[
P_{ave} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt
\]

\[
P_{ave} = \frac{1}{N} \sum_{n=-N}^{N} |x[n]|^2
\]
(3) Periodic and Non-Periodic (or Aperiodic) Signals:

A periodic function is one which has been repeating an exact pattern for an infinite time and will continue to repeat that exact pattern for an infinite time.

A signal is periodic if \( g(0) = g(t + T) \) for any integer \( T \) = period of \( g \) function.

![Image of periodic signal](image)

- The sum of harmonic signals \( y(t) = a_1 \sin(2\pi f_1 t) + a_2 \sin(2\pi f_2 t) + \ldots \) is periodic with overall period \( T = \text{LCM}(T_1, T_2, T_3, \ldots) \).

- A discrete signal \( x(n) \) is periodic if \( x[n] = x[n + N] \), where \( N \rightarrow \) period of \( x[n] \).

- For finding the fundamental period of discrete signals, find the equation of \( x[n] \) for \( n \rightarrow \) integer.

### 3.8. Determine which of the following signals are periodic, if periodic find the fundamental period?

(a) \( x(t) = \cos(12\pi t) + \sin(12\pi t) \)

(b) \( x(t) = \sin(2\pi t) + \cos(4\pi t) \)

(c) \( x(t) = e^{2t} \)

(d) \( x(t) = \cos(2\pi t) + \sin(2\pi t) \)

(e) \( x(t) = \cos(2\pi t + \pi) \)

(f) \( x(t) = \cos(2\pi t + \pi/2) \)

(g) \( x(t) = e^{2t} \cdot \cos(2\pi t) \)

(h) \( x(t) = e^{-2t} \cdot \sin(2\pi t) \)

(i) \( x(t) = e^{-t} \cdot \cos(2\pi t) \)

(j) \( x(t) = e^{-t} \cdot \sin(2\pi t) \)

(k) \( x(n) = u(n) - u(-n) \)

(l) \( x(n) = -x(-n) \)

(m) \( x(n) = \sum_{k=-\infty}^{\infty} \delta(n - k) \cdot \delta(n - k - 1) \)

### 3.9. A signal \( x(t) = 2 \cos(350t + 3\pi) \) is sampled at 200Hz. Find the fundamental period of discrete signal?

5. Deterministic and Random Signals:

- If the value of a signal can be predicted for all time (t 0 or n) in advance with any error, it is a deterministic signal. Eg: \( x(t) = e^{2t} \), \( x(t) = \sin(2t) \).

- Signals whose values cannot be predicted with complete accuracy for all time are known as random signals.

- Random signals are generally characterized by mean, mean square, etc.

Eg: Thermal noise generated by a resistor. The intensity of the thermal noise depends on the temperature of resistors and cannot be predicted accurately.

1.4 Systems and Classification:

A system designated T is a mathematically describable operation or transformation that acts on an input signal to produce an output.

![Image of system](image)

Eg: In a communication system, the input signal could be a speech signal, the system is combination of Tx channel and Rx, output signal is an estimate of information contained in the original message.

### 4. Causal and Noncausal Signals:

A signal is causal if \( x(t) = 0 \) for \( t < 0 \) and \( x(t) = a_0 \) for \( t > 0 \).

![Image of causal and non-causal signals](image)

- Multiplying the signal by the unit step function, the resulting signal is causal.

### 5. Deterministic and Random Signals:

- Decisions which of the following signals are periodic, if periodic find the fundamental period?

(a) \( x(t) = \cos(12\pi t) + \sin(12\pi t) \)

(b) \( x(t) = \sin(2\pi t) + \cos(4\pi t) \)

(c) \( x(t) = e^{2t} \)

(d) \( x(t) = \cos(2\pi t) + \sin(2\pi t) \)

(e) \( x(t) = \cos(2\pi t + \pi) \)

(f) \( x(t) = \cos(2\pi t + \pi/2) \)

(g) \( x(t) = e^{2t} \cdot \cos(2\pi t) \)

(h) \( x(t) = e^{-2t} \cdot \sin(2\pi t) \)

(i) \( x(t) = e^{-t} \cdot \cos(2\pi t) \)

(j) \( x(t) = e^{-t} \cdot \sin(2\pi t) \)

(k) \( x(n) = u(n) - u(-n) \)

(l) \( x(n) = -x(-n) \)

(m) \( x(n) = \sum_{k=-\infty}^{\infty} \delta(n - k) \cdot \delta(n - k - 1) \)

- The behavior of a system is governed not only by the input but also by the state of the system at the instant at which the input is applied.
1. Linear And Non Linear System:

For the system to be linear, it should satisfy 2 properties:

(a) \( x_1(t) \rightarrow y_1(t) \) and \( x_2(t) \rightarrow y_2(t) \) then \( x_1(t) + x_2(t) \rightarrow y_1(t) + y_2(t) \) - Additivity

(b) \( cx_1(t) \rightarrow cy_1(t) \) or scaling (or) homogeneity

\( c \) is a constant.


A system is T.I. if the input and output characteristic don’t change with time. T.I. implies that the shape of the response \( y(t) \) depends only on the shape of the input \( x(t) \) and not on the time when it is applied.

For T.I. system if \( x(0) \rightarrow y(0) \) then \( x(t-\tau) \rightarrow y(t-\tau) \)

3. Causal & Non Causal System:

A causal system’s response at any instant does not depend on future values of the input. A system is causal if its output at \( t = t_0 \) depends on the values of the input in the past \( \tau \leq t_0 \) and doesn’t require future values of input \( t > t_0 \).

4.4 Check whether the following systems are causal or non-causal:

(a) \( y(t) = (t+1) \cdot x(t) \)

(b) \( y(t) = x^2(t) \)

(c) \( y(t) = sin(x(t)) \)

(d) \( y(t) = cos(x(t)) \)

(e) \( y(t) = 1/x(t) \)

(f) \( y(t) = e^{-t} x(t) \)

(g) \( y(t) = \sum_{i=0}^{n} x(t-a_i) \) if \( a_i \) is finite

(h) \( y(t) = \sum_{i=0}^{\infty} x(0) \) if \( a_i \) is infinite

(i) \( y(t) = \sum_{i=0}^{\infty} x(t) \)

(j) \( y(t) = \sum_{i=0}^{\infty} x(t) \)

(k) \( y(t) = x(t) \)

(l) \( y(t) = x(t) + x(t-\tau) \)

(m) \( y(t) = x(t) + x(t-\tau) \)

4.4.1 Consider a system with input \( x(t) \) and output \( y(t) \) related as \( y(t) = x(t) + g(t-1) \)

Check for time-invariance if \( g(t) = \), \( g(t) = 0 \), \( g(t) = 1 \), \( g(t) = \frac{x(t)}{2} \), \( g(t) = 0 \).

4.4.3 Consider an LTI system whose response to the input signal \( x(t) \) is \( y(t) \) as shown in figure. Find the response of the system due to the input \( x(t) \) and \( x(t+1) \)?
4. Static (Or) Memoryless And Dynamic (With Memory) System:

A system is said to be memoryless if its output at $t = 0$ depends only on the value of the input at $t = 0$ and no other value of the input signal.

P1.4.7: Check whether the following systems are static or dynamic?

(a) $y(t) = x(t)$
(b) $y(t) = x^2(t)$
(c) $y(t) = e^{2t}$
(d) $y(t) = \int x(t) \, dt$

Q) All memoryless systems are shift-invariant (TRUE/FALSE)

5. Stable And Unstable System:

A system is said to be BIBO stable if and only if every bounded input results in a bounded output. If $|x(t)| \leq M_x < \infty$ then $|y(t)| \leq M_y < \infty$ then $y(t)$ is bounded.

P1.4.8: Check whether the following systems are stable or not?

(a) $y(t) = x^2(t)$
(b) $y(t) = x(t) \cos \lambda t$
(c) $y(t) = x(t - 3)$
(d) $y(t) = \int x(t) \, dt$

Q) $\frac{y(t)}{x(t)} = 2$, if $x(t)$ is bounded, then $y(t)$ is bounded.

6. Invertible And Inverse System:

A system is said to be invertible if the input of the system can be recovered from the output.

$$x(0) \rightarrow \mathbf{T} \rightarrow y(0) \rightarrow \mathbf{T}^{-1} \rightarrow x(0)$$

In any event, a system is not invertible unless distinct inputs applied to the system produces distinct outputs.

1.5 Previous Questions:

(1) Which one of the following Polns is NOT correctly matched (Input $x(t)$ and output $y(t)$)

(a) Unstable system $dy(t)/dt + 0.5y(t) = 0$
(b) N.L. system $dy/dt + 2y(t) = x(t)$
(c) Non causal system $y(t) = x(t + 2)$
(d) Non linear system $y(t) = 3x^2(t)$

(2) The discrete-time equation $y(n + 1) = 0.5y(n) + 0.5x(n + 1)$ is not attributable to

A) memory less system  
B) $T$  
C) linear  
D) causal system

(3) Find whether the following systems are linear, T.I., and dynamic.

(a) $y(t) = x(t)$
(b) $y(t) = 2x(t)$
(c) $y(t) = 3x(t)$

(4) $\frac{dy(t)}{dt} = x(t)$
(b) $y(t) = x(t)$
(c) $y(t) = x(t)$

4. Match the following
List I (System) List II (System category) IES
(A) \( y(n + 2) + y(n + 1) + y(n) = 2x(n + 1) + x(n) \) (1) Linear, T.V., dynamic
(B) \( y(n) = x^2(n) \) (2) Linear, T.I., dynamic
(C) \( y(n + 1) + ny(n) = 4cos(n) \) (3) N.L., T.V., dynamic
(D) \( y(n + 1) = 4x(n) \) (4) N.L., T.I., dynamic
(E) N.L., T.V., memory less

5. The output \( y(t) \) (und output \( y(t) \) of a C.T.S related with the following equations. Which system is causal?
(A) \( y(t) = (t - 2) x(t - 2) \) (B) \( y(t) = (t - 2) x(t + 1) \)
(C) \( y(t) = (t + 2) x(t - 2) \) (D) \( y(t) = (t + 2) x(t + 5) \)

6. Match the correct pairs
List I (System) List II (Properties) GATE
R1: \( y(t) = \int x(t) dt \) (P1) Linear but not T.I.
R2: \( y(t) = 2x(t) \) (P2) T.I. for N.L.
R3: \( y(t) = x(t) \) (P3) T.I.
R4: \( y(t) = x(t) \) (P4) N.L. and not T.I.

7. The power in the signal is \( x(t) = 3 \cos(2t - \pi/2) + \sin(\pi t) \) is \( E_1 = \) (A) 20 \( \) (B) 40 \( \) (C) 42 \( \) (D) 82

8. The system represented by the input-output relationship \( y(t) = \int_{-\infty}^{t} x(\tau) d\tau \) is (a) linear, T.V. causal
(b) linear, T.V. non-causal
(c) linear, T.I. causal
(d) linear, T.I. non-causal

9. Which one of the following gives correct description of the waveform shown in the figure? IES
(a) \( v(t) = u(t) - 1 \) (b) \( v(t) = (t - 1) u(t) - 1 \)
(c) \( v(t) = (t - 1) u(t) - 2 \) (d) \( v(t) = (t - 2) u(t) - 2 \)

10. What is the period of \( \sin(\pi x^2) \)? IES
(a) 10 \( \) (b) 5 \( \) (c) 1 \( \) (d) 0

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1.6 PRACTICE PROBLEM SET:
P1.6.1. Evaluate the following integrals:
(a) \( \int_{-\infty}^{\infty} \sin(\pi x) x(2x + 4) dx \) (b) \( \int_{-\infty}^{\infty} \delta(t - b) \sin(\pi t - 4) dt \)
(c) \( \int_{-\infty}^{\infty} e^{(t - 1)} \delta(t - 5) dt \) (d) \( \int_{-\infty}^{\infty} e^{(n - 1)} \delta(t - 5) dt \)
(e) \( \int_{-\infty}^{\infty} e^{(n - 1)} \cos(2x) (x - 5) \delta(x - 3) dx \) (f) \( \int_{-\infty}^{\infty} e^{(n - 1)} \delta(t - 10) dt \)

P1.6.2. Let \( x(t) = \) (A) \( 1 + 4t - 1 \) \( 1 \leq t \leq 5 \)
(B) \( 0 \leq t \leq 2 \) \( 2 \leq t \leq 3 \)
(C) \( 0 \)
Sketch the signals \( x(t) \) where \( x(t) = 2t - 2 \), \( x(t) = 3t - 2 \).

P1.6.3. Given \( x(t) = 3u(t) - u(t) + 3u(t - 3) - 5u(t - 6) \). Sketch the signal \( x(t) \) and \( \Delta t \).

P1.6.4. Consider the D.T. signal \( x(n) = \) (a) \( u(n + 5) - u(n - 5) \)
Sketch the following signals
(b) \( x(n) = u(n - 10) \) \( 0 \leq n \leq 3 \)
(c) \( x(n) = (n + 1) \)
(d) \( x(n) = 4n - 3 \)

P1.6.5. Determine whether the following signals are energy (x) power signals (x), neither.
(a) \( x(t) = e^{at} \) \( 0 \leq t < \infty \)
(b) \( x(t) = e^{at} \) \( 0 < t < \infty \)
(c) \( x(t) = u(t) \) \( 0 \leq t < \infty \)
(d) \( x(n) = u(n) \) \( 0 \leq n < \infty \)

P1.6.6. (a) Find the energy in \( x(t) \)
(b) Find the energy in \( y(t) = 4x(0) \)
P1.6.7. Find whether the following signals are periodic (n) or, if periodic, find the period?
(a) \( x(t) = 2 \cos(\pi t) \)
(b) \( x(t) = \cos^2(\pi t) \)
(c) \( x(t) = \sum_{n=1}^{\infty} \delta(t - 2n) \)
(d) \( y(t) = \cos(2\pi t) + \sin(3\pi t) + \cos(5\pi t - 3\pi/4) \)
(e) \( x(n) = \cos\left(\frac{\pi n}{2}\right) \)
(f) \( y(n) = \cos\left(\frac{3\pi n}{2}\right) \)

P1.6.8. Sketch the even and odd parts of the signal shown in figure.

P1.6.9. Given in figure are the parts of a signal \( x(n) \) and its even part \( x_e(n) \) only for \( n \geq 0 \) and \( x(n) = -x(-n) \). Compute the plots of \( x(n) \) and \( x_e(n) \) and give a plot of the odd part, \( x_o(n) \) of the signal.

P1.6.10. Sketch the waveforms of the following signals?
(a) \( x(t) = 1 - u(t - 2) \)
(b) \( x(t) = u(t + 1) - u(t - 2) \)
(c) \( x(t) = 2(t + 1) + (t + 2) \)
(d) \( y(t) = 3u(\tau - 3) - r(\tau - 2) + 2r(\tau - 3) - 2u(\tau - 4) \)

P1.6.11. The signal \( x(n) = \begin{cases} 1; & n = 1 \\ 1; & n = 0 \\ 0; & n \neq 0 \text{ or } |n| > 1 \end{cases} \). Find the signal \( y(n) = x(n) * x(-n) \)

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P1.6.12. The following input – output pairs have been observed during the operation of a linear system:

\[ x_1(n) = [0, 1, 2, 3] \]
\[ y_1(n) = [1, 2, 3, 4] \]
\[ x_2(n) = [0, 1, 2, 3, -1, 0, 1] \]
\[ y_2(n) = [0, 1, 2, 3, -1, 0, 1] \]

Can you draw any conclusions about the linearity and causality of this system?
20. Show that the system is linear, T.I., stable, causal, dynamic, invertible?

\[
\begin{align*}
x(n) & \quad 0.25 \quad + \quad y(n) \quad 1/4 \quad + \quad 1/2 \quad y(n) \\
\end{align*}
\]
An LTI System is always considered as an impulse response denoted as $h(t)$ or $h[n]$. If the input is impulse, then the output is impulse response. The SPOW property states that any signal can be produced as a combination of impulses. Convolution may be regarded as a method of finding the zero-state state response of a related LTI system. Any DT signal is the sum of scaled and shifted unit impulses.

$$x[n] = \sum x[n]\delta[n-k]$$

$\tau = -\infty$

Convolution may be treated as the shift + multiply + time = area method.

### CONTINUOUS CONVOLUTION

$$\int_{-\infty}^{\infty} x(t)h(t)dt$$

$$y(t) = \int_{-\infty}^{\infty} x(t-h)h(t)dt$$

### DISCRETE CONVOLUTION

$$x[n]*h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[n-k]h[k]$$

**Steps:**

1. $x(t) \rightarrow x(t), h(t) \rightarrow h(t)$
2. Folding $x[-t]$
3. Shifting $x[-t] \rightarrow x[-t-k]$ $h[t] \rightarrow h[t-k]$ $x[-t-k] \rightarrow x[-t-k]$
4. Multiplication $x[-t-k]h[t-k]$
5. Integration $\rightarrow x[-t-k]h[t-k]$
In the convolution integral, the "t" determines the relative location of h(t - q) w.r.t. x(t). The convolution will yield a non-zero result only if the values of "t" over which h(t - q) & x(t) overlap.

P2.1.1 Find the convolution of the signals x(t) = $e^{-|t|}$ & h(t) = u(t - 1).

P2.1.2 Find the convolution of the signals shown in figure 1.

P2.1.3 An L.T.I. system is having impulse response h(t) = u(t - 1) for which the input signal applied is shown in figure. Find the output at t = 4 & t = 0.5.

P2.1.4. Suppose $x(t) = \int_{-\infty}^{t} [x(t + \tau) h(\tau) + \delta(t - n t)] d\tau$. Express $x(t)$ in terms of $x(t) = x(t) h(t)$.

REVIEW:

P2.1.5 Convolution Property of Continuous Impulse:

\[ x(t) \ast \delta(t - b) = x(t - b) \]

P2.1.6 Find the following limits:

(a) \[ x(t) = x(t) \ast x(t) \]

(b) \[ x(t) \ast \delta(t - 1) \]

(c) \[ x(t) \ast \delta(t - 1) \]

P2.1.7 Write \[ x(t) = x(t - 5) - x(t - 5) \] & h(t) = $e^{-|t|}$. Find \[ \frac{dx(t)}{dt} + h(t)? \]

P2.1.8. An input signal x(t) shown in figure is applied to the system with impulse response

\[ h(t) = \sum_{n=-\infty}^{\infty} x(t - 3n). \] Find the output.

P2.1.9. Suppose the input signal is \[ x(t) = u(t + 0.5) - u(t - 0.5) \] & the impulse response is \[ h(t) = e^{-t^2}. \] If y(t) = x(t) * h(t), find a value of \( a \) which makes that \( y(t) = 0 \).
2.2 Properties of LTI Systems

Stability:

\[ h(t) \leq 0; t \geq 0 \]

\[ x(t) = 0; n < 0 \]

\[ \sum h(n) / n = x(t) \]

Memoryless:

\[ h(t) = 0 \text{ for } t < 0 \]

\[ h(t) = 0 \text{ for } t = 0 \]

\[ h(t) = 0 \text{ for } t > 0 \]

\[ h(t) = h(n) = 0(t) \]

\[ h(t) = h(n) = 0(t) \]

P2.2.1 Find whether the following systems are causal & stable?

(a) h(t) = e^{-n^2} \text{ for } n \geq 0

(b) h(t) = e^{-|n|} \text{ for } n \leq 0

(c) h(t) = 0 \text{ for } n \neq 0

P2.2.2. Consider a LTI system \( S_1 \), with I.F. \( h(n) = (1/3)n^n \).

(a) Find \( x(t) \) such that \( h(t) = -x(t) \).

(b) Using result from part (a), determine the I.F. of an LTI system \( S_2 \) which is inverse of \( S_1 \).

P2.2.3. Consider the system in Fig. 1:

(a) Find I.F. of overall system.

(b) Is this system causal? Under what condition the system is stable?

P2.2.4. The range of \( a \) and \( b \) for the impulse response \( h(n) = 0.5^n \).

P2.2.5. Given \( h(n) = a^n \), \( a \neq 0 \) for what values of \( a \) is the system stable?

\[ a < 0, \quad \beta < 0 \]

\[ a > 0, \quad \beta > 0 \]

\[ a > 0, \quad \beta < 0 \]

\[ a < 0, \quad \beta > 0 \]
2.5 **Step Response of An L.T.I. System:**

- Step response is the response when the input is unit step function.

\[ x(t) = u(t) \]

\[ y(t) = h(t) \times u(t) \]

\[ y(n) = h(n) \times u(n) \]

\[ s(n) = \sum_{k=0}^{n} h(k) \]

\[ h(n) = \delta(n) - s(n-1) \]

P2.3.1 Find the step response of the system if the impulse response is \( h(n) = [0.5^n u(n) \]

P2.3.2 Find the impulse response of the system if the step response is \( s(n) = \cos(n) \)

P2.3.3 An LTI system with input \( u(t) \) produces the output as \( y(t) \), then find the output due to the input \( u(t) \)?

P2.4.1 Find the overall impulse response for the parallel-connected system shown in figure.

\[ h(n) = \begin{cases} 1 & n = 0 \\ \frac{1}{4} & n = 1 \\ \frac{1}{4} & n = 2 \\ 0 & \text{otherwise} \end{cases} \]

\[ h(n) = u(n) + 2 \]

\[ h(n) = 6(n-2) \]

P2.4.2 Consider the interconnection of LTI systems shown in figure. Find \( h(n) \) when

\[ h(n) = \begin{cases} 1 & n = 0 \\ \frac{1}{4} & n = 1 \\ \frac{1}{4} & n = 2 \\ 0 & \text{otherwise} \end{cases} \]

\[ h(n) = u(n) + 2 \]

\[ h(n) = 6(n-2) \]

P2.4.3 The I.R. of a system is \( h(t) = \delta(t) - 0.5 \). If 2 such systems are cascaded, the I.R. of the overall system will be

\[ h(t) = \delta(t) - 0.5 \]

\[ h(t) = \delta(t) - 0.5 \]

P2.4.4 The I.R. of a system consists of 2 delta functions as shown in figure. The input to the system is a unit amplitude square pulse of one unit time duration. Find the output.

\[ h(t) = \delta(t) + 1 \]

\[ x(t) = \begin{cases} 1 & 0 < t < 1 \\ 0 & \text{otherwise} \end{cases} \]

P2.4.5 Two rectangular waveforms of duration \( T \) and \( T \) are convolved. What is the shape of the resulting waveform?

- Triangular
- Parabolic
- Trapezoidal
- Sinusoidal - circle

P2.3.6 For the interconnected system shown in Fig. find the overall impulse response.

\[ h(t) = y(n) - 0.5 \delta(n-1) \]

\[ h(n) = [0.5^n u(n) \]

P2.3.7 Determine whether each of the following statements are TRUE or FALSE. Justify your answers.

1. The cascade of n non-causal LTI systems is necessarily non-causal.
2. If an LTI system is causal, it is stable.
3. If \( h(t) \) is the I.R. of an LTI system which is periodic & non-zero, the system is unstable.
Prob: Find the convolution of the signals shown in figure?

2.5 CONVOLUTION & DIFFERENTIATION:

\[ x(t) * y(t) = \int_{-\infty}^{\infty} x(\tau) y(t-\tau) \, d\tau \]

\[ x(0) * x(0) = \int_{-\infty}^{\infty} x(\tau) x(0-\tau) \, d\tau \]

\[ x(0) * \delta(\tau) = x(0) \delta(\tau) \]

\[ \delta(t) * x(0) = x(0) \]

\[ \frac{d}{dt} x(t) = \frac{d}{dt} \int_{-\infty}^{t} x(\tau) \, d\tau \]

This last formula provides an easier and quicker method for solving convolution problems.
2.6 Practice Problem Set:

2.6.1. Let $x(t) = x(t-1) + 2x(t-2) + u(t) - 3u(t-2)$ and $h(t) = d(t) + 3x(t-1) + 2x(t-2)$

calculate $y(0) = x(0)$

(1) $y(0) = x(0)(x + 2)$

(2) $y(0) = x(0)(x + 2)$

2.6.2. Let $y(t) = e^{-a(t-2)} \cdot u(t-2)$

- Show that $y(t) = 0$ for $0 < t < 2$, and determine the value of $A$

2.6.3. Find whether the following LTI systems are causal & stable?

(a) $h(t) = e^{-t}$ and $x(0)$

(b) $h(t) = e^{-t}$ and $x(0)$

(c) $h(t) = e^{-t}$ and $x(0)$

2.6.4. Find the convolution of the following signals?

(a) $x(t) = e^{-t}$ and $h(t) = u(t)$

(b) $x(t) = e^{-t}$ and $h(t) = u(t)$

(c) $x(t) = e^{-t}$ and $h(t) = u(t)$

2.6.5. An LTI system has the impulse response shown in figure. Find the output if the input

2.6.6. For the interconnected system shown in fig. if the impulse response are

$h(t) = h(t-1)$

$h(t) = h(t-1)$

$h(t) = h(t-1)$

$h(t) = h(t-1)$

Find the overall impulse response?
2.6.7. Consider an LTI system with impulse input related by $y(t) = \int x(t) \, dt$.
   (a) Find $L(R)$. Is this system causal? Why?
   (b) Determine the system output for the input $x(t) = \delta(t) + 1$.

2.6.8. Let $x[n] = [2, 4, 0, 3]$.
   (a) Find $y[n] = h[n] * x[n]$
   (b) Find $y_1[n] = x[2n] * x[2n]$
   (c) Find $y_2[n] = x[2n]^2 * x[2n]$

2.6.9. Find the periodic convolution of $x[n] = [1, 2, 0, 1]$ & $h[n] = [2, 3, 0]$

2.6.10. Consider 2 systems described by $h_1[n] = 0.5^n \cdot u[n]$ & $h_2[n] = 0.5^n \cdot u[n]$
     (a) Find the response to the input $x[n] = (0.5^n \cdot u[n])$
     (b) Two systems are connected in parallel with $a = 0.5$. - 0.5

2.6.11. Given 3 signals $x_1[n], x_2[n], x_3[n]$, express $y[n]$ in terms of $x[n]$
     If $y[n] = \sum_{k=-\infty}^{\infty} x[n+k] * a[k]$ then $y[n] = \sum_{k=-\infty}^{\infty} x[n-k] * a[k]$

2.6.12. If $h(n) = \delta(n) + \frac{1}{2} \delta(n)$ is the unit sample response of a LTI system, and $x[n]$ is the step response, find $\mathcal{L}(x[n])$.

2.6.13. The LTI system is given by $y[n] = [0.5^n \cdot u[n]]$. Find the output when $x[n] = 1 \& n = 4$.

2.6.14. An FIR system is characterized by $y[n] = 0.5^n x[n] - 0.5 x[n-2] + 0.4 x[n-3]$
     Given the input sequence $[1, 1, 0, -1]$ applied to this system, find the output.

2.6.15. Given the IL filter has impulse response $h[n] = [0.5, 1, 0.5]$
     Find the step & ramp responses?

\[ y[n] = \sum_{k=0}^{n} x[k] \]

\[ y[n] = \sum_{k=-\infty}^{n} x[k] \]
CHAPTER 3. FOURIER SERIES

1. Representing CT signals as superposition of complex exponentials leads to frequency-domain characterization. 
2. A human ear is sensitive to audio signals within the frequency range 20 Hz to 20 kHz. Typically, musical notes occupy a much wider frequency range than the audible range and reject other frequency components.
3. Therefore, the human ear perceives only the frequency components within the audible range and rejects other frequency components. In such applications, frequency-domain analysis provides a convenient means of solving for the response of LTI systems to arbitrary inputs.

4. To use an LTIC system to arbitrary inputs, the sinusoidal signals are used in describing motion of planets & periodic behavior of earth's climate. AC power sources generate sinusoidal voltages & currents.

3.1 ANALOGY BETWEEN VECTORS & SIGNALS

1. Signals are not just like vectors.
2. A vector can be represented as a sum of its components, depending on the choice of coordinate system. A signal can also be represented as a sum of its components.

3. We know that an arbitrary M-dimensional vector can be represented in terms of M orthogonal coordinate. A vector is specified by its magnitude & direction.

\[ \begin{align*} \mathbf{X} &= \mathbf{a} + \mathbf{e}_1 + \mathbf{e}_2 \end{align*} \]

Consider 2 vectors \( \mathbf{f} \) & \( \mathbf{X} \) as shown in figure. Let the component of \( \mathbf{f} \) along \( \mathbf{X} \) be \( \mathbf{C} \).

From Fig. (a) \( \mathbf{f} = \mathbf{C} \mathbf{X} + \mathbf{e} \)

From Fig. (b) \( \mathbf{e} = \mathbf{C} \mathbf{X} - \mathbf{e}_1 = \mathbf{C} \mathbf{X} + \mathbf{e}_2 \)

If we approximate the vector \( \mathbf{f} \approx \mathbf{C} \mathbf{X} \)

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Error in the approximation \( \approx \mathbf{C} \mathbf{X} \)

Maximize the error vector, such that \( \mathbf{X} \) and \( \mathbf{X} \) are approximated

length of the component \( \mathbf{f} \) along \( \mathbf{X} \) = \( \mathbf{C} \mathbf{X} = || \mathbf{X} || \mathbf{C} \mathbf{X} \)

\[ \mathbf{C} = \frac{\mathbf{X} \cdot \mathbf{X}}{|| \mathbf{X} ||^2} \]

2 vectors \( \mathbf{f} \) & \( \mathbf{X} \) are orthogonal if inner (or) scalar product \( \mathbf{f} \cdot \mathbf{X} = 0 \)

If we consider 2 basis vectors \( \mathbf{f} \) & \( \mathbf{X} \)

orthogonality \( \mathbf{f} \cdot \mathbf{X} = 0 \)

unit magnitude \( || \mathbf{f} || = \sqrt{f_x^2 + f_y^2} = 1 \)

Orthogonal Property

Computation of A Signal

\[ \begin{align*} \mathbf{g}(t) &= \mathbf{C} \mathbf{X} \cdot \mathbf{C} = \mathbf{X} \cdot \mathbf{C} \\ \mathbf{g}(0) &= \mathbf{C} \mathbf{X} (0) \cdot \mathbf{C} = \mathbf{X} (0) \cdot \mathbf{C} \\ \mathbf{g}(t) &= \mathbf{C} \mathbf{X} (t) \cdot \mathbf{C} = \mathbf{X} (t) \cdot \mathbf{C} \\ \mathbf{C} &= \mathbf{X} \cdot \mathbf{X} \end{align*} \]

Note:

2 signals \( g(t) \) & \( x(t) \) are said to be orthogonal over the interval \( (a, b) \) if \[ g(t)x(t)dt = 0 \] for all \( a, b \). They are also said to be orthonormal if they satisfy \[ g(t)x(t)dt = ||g(t)||^2 ||x(t)||^2 = 1 \] (Unit magnitude)
For the three continuous functions shown in figure 3.1

\[ \phi_0(t) \quad \phi_1(t) \quad \phi_2(t) \]

\[ \begin{array}{c|c|c|c}
\text{fig. 3.1} & 0 & T & -T \\
\hline
\phi_0(t) & 1 & 0 & -1 \\
\phi_1(t) & 0 & 1 & 0 \\
\phi_2(t) & -1 & 0 & 1 \\
\end{array} \]

(a) Show that the functions form an orthogonal set
(b) Find the sum of T that makes f functions orthogonal
(c) Express the signal s(t) = \int_0^T f(t) dt in terms of orthogonal functions determined in (a)

Sub.

a) For the three signals

\[ \int_0^T f(t)^2 dt = \int_0^T \phi_0(t)^2 dt + \int_0^T \phi_1(t)^2 dt + \int_0^T \phi_2(t)^2 dt = 2T \]

Orthogonality

\[ \int_0^T \phi_0(t)\phi_1(t) dt = \int_0^T \phi_0(t)\phi_2(t) dt = \int_0^T \phi_1(t)\phi_2(t) dt = 0 \]

b) For orthornormality, 2T = 1 => T = \frac{1}{2}

c) s(t) = C_0 \phi_0(t) + C_1 \phi_1(t) + C_2 \phi_2(t)

\[ C_0 = \frac{1}{2T} \int_0^T s(t) \phi_0(t) dt \]

\[ C_1 = \frac{1}{2T} \int_0^T s(t) \phi_1(t) dt \]

\[ C_2 = \frac{1}{2T} \int_0^T s(t) \phi_2(t) dt \]

Simplifying

\[ s(t) = A/2 \{ \phi_0(t) - \phi_1(t) \} \]

3.2.1 Trigonometric F.S. (T.F.S)

Any periodic function of frequency \( \omega_0 \) can be expressed as an infinite sum of sine (or) cosine functions that are integral multiples of \( \omega_0 \)

\[ g(t) = \sum \omega_0 \cos(\omega_0 \omega_0 + \cdots + b_0 \sin(\omega_0 + \cdots) \]

\[ a_0 = \text{Fundamental frequency} \]

\[ a_0, b_0, \cdots \Rightarrow \text{S. coefficients} \]

\[ \omega_0 = 2\pi, \omega_1 = \omega_0 + \omega_0 \]

\[ a_0 \rightarrow \text{Average value} \]

\[ b_n = 2T \int_0^T \sin(n\omega_0) dt \]

Polar form of T.F.S.

\[ g(t) = \sum \omega_0 m(n=0, \omega_0) \]

\[ d_t \rightarrow d_0 \]

\[ a_d = d_0 \]

\[ 1 + d_0^2 + b_0^2 \rightarrow \text{Magnitude spectrum} \]

\[ \omega_0 = 2\pi, d_0 \rightarrow 2\pi \]

\[ b_0 \rightarrow \text{Phase spectrum} \]
3.2.2. EXponential (OE) Complex F.S.:

\[ g(t) = \sum_{n=0}^{\infty} C_n e^{j\omega_n t} \]

where \( C_n = \int_{-T/2}^{T/2} g(t) e^{-j\omega_n t} dt \)

- Exponential F.S. coefficient
- In terms of T.F.S. coefficient: \( C_n = a_n e^{-j\phi_n} \)
- \( C_n = a_n + j\phi_n \)

P3.2.1. A periodic signal is given by \( x(t) = 3 \cos(6t) + 2 \sin(12t) \). Find the amplitude of the second harmonic.

P3.2.2. Which of the following signals is not the representation of F.S.?

- a) \( \cos(3t + \sin(2t)) \)
- b) \( \cos(3t) + \sin(2t) \)
- c) \( \alpha^2 \)
- d) \( \cos(2t) + \sin(3t) \)

P3.2.3. Find the T.F.S representation of the periodic signal \( x(t) \) shown in Fig. 3.2.3.

P3.2.4. Find the symmetry of the signals shown in figure 3.2.4.

P3.2.5. A periodic input signal \( x(t) \) shown below is applied to an LTI system with frequency response \( H(\omega) = \begin{cases} 1 & |\omega| < 4 \\ 0 & 4 < |\omega| \end{cases} \). Find the output?

P3.2.6. Obtain the T.F.S. and E.F.S. representations of the periodic signal \( x(t) \) shown in Fig. 3.2.6.
P3.2.7. For the periodic signal \( x(t) = 2 \cos \left( \frac{2\pi}{3} t \right) + \cos \left( \frac{4\pi}{3} t \right) \), find the E.F.S. coefficients.

P3.2.8. Consider an ideal LPF with frequency response \( H(f) = 1 \) for \( |f| < 100 \) and \( H(f) = 0 \) for \( |f| > 100 \).

When the output to this filter is a signal \( x(t) \) with \( T = \pi/6 \) and E.F.S. Coefficient \( C_n \), it is found that \( x(t) \rightarrow (t-n) \), for what values of \( n \) it is guaranteed that \( C_n \neq 0 \)?

P3.2.9. Consider the two-sided signal spectrum shown in figure for signal \( x(t) \), find \( x(t) \).

Convergence of E.F.S. (Dirichlet Conditions):

1) \( x(t) \) is absolutely integrable i.e., \( \int_{-\infty}^{\infty} |x(t)| dt < \infty \)
2) \( x(t) \) has only a finite number of maxima & minima
3) The number of discontinuities in \( x(t) \) must be finite.

These conditions are sufficient, but not necessary.

---

7.3 Properties of F.S.

1. Linearity: \( x(t) \rightarrow C_n \) Period = \( T \)
   \( y(t) \rightarrow d_n \)
   Then \( a_x(t) + b_y(t) \rightarrow aC_n + bd_n \)

2. Time-shift: \( x(t) \rightarrow C_n \) then \( x(t+\alpha) \rightarrow C_n e^{j\alpha} \)
   when we shift in the time-domain, it changes the phase of each harmonic in proportion to its frequency \( m\alpha \).

P3.3.1: The F.S. coefficients of the signal \( x(t) \) shown in fig(a) are \( C_0 = 10, C_n = -0.55, C_n = \frac{1}{\sqrt{2}} \) for (even) Find F.S. coefficients of \( x(t), f(t) \) and \( g(t) \).
P3.3.2- Let x(t) be a periodic signal with period T and F.S. coefficients $c_n$. Let $y(t) = x(t-a)$. The F.S. coefficient of $y(t)$ is $c_n$. If $a = T/n$, then $c_n$ can be:

<table>
<thead>
<tr>
<th>a</th>
<th>T/n</th>
<th>T2/n</th>
<th>2T/n</th>
</tr>
</thead>
</table>

3) Frequency Shift $x(t-a) \rightarrow c_n$, then $y(t) = \text{exp}(j\omega t) \rightarrow c_n + M$

4) Time-Scale $x(t) \rightarrow c_n$, $x(\lambda t) \rightarrow c_{n\lambda}$

Time-Compressing by a change frequency from $\omega_0$ to $\lambda \omega_0$

---

5) Differentiation in time $x(t) \rightarrow c_n$, $\frac{dx(t)}{dt} \rightarrow jn\omega c_n$

P3.3.3- By using derivative method, find F.S. coefficient of the signal shown in figure:

---

6) Parseval's Power Theorem:

$\int_{-T/2}^{T/2} |x(t)|^2 dt = \frac{1}{T} \sum_{n=-\infty}^{\infty} |c_n|^2$

P3.3.4- Find the power up to the 2 harmonic for the periodic signal shown in figure:

---

P3.3.5- A periodic signal has the F.S. representation $x(t) \rightarrow c_n + \sum_{n=1}^{N} c_n \text{exp}(j\omega nt)$. Without finding $x(t)$, find the F.S. representation $x(t) + c_0 x(\lambda t)$ if:

(a) $x(t) = x(t)$
(b) $x(t) = c_0 x(t)$
(c) $x(t) = x(t) + c_0 x(\lambda t)$
3.4 SYSTEMS WITH PERIODIC INPUTS:

- If we apply an input of the form $x(t)e^{j\omega t}$, then the output is $y(t) = e^{j\omega t}H(0)$, where $H(0)$ is known as the frequency response of the system.

- Knowing $H(0)$, we can determine whether the system amplifies or attenuates a given sinusoidal component of the input & how much of a phase shift adds to that particular component.

- The response $y(t)$ of an LTI system to the periodic input $x(t)$ is $y(t) = \sum C_n \sin(n\omega_0 t + \phi_n)$.

3.5 PREVIOUS QUESTIONS:

1. One period of $x(t)$ consists of 2 periodic waveforms $w_1$ and $w_2$ for $x(t)$ a odd are respectively proportional to $w_1$ and $w_2$.  

   a) $|w_1| + |w_2|$
   b) $|w_1| - |w_2|$
   c) $|w_1| + j|w_1|$
   d) $|w_1| - j|w_1|$

2. Choose the function $f(t) = e^{j\omega t}$ for which F.S. can't be defined
   a) $3\sin(2\pi t)$
   b) $4\cos(2\pi t) + 2\sin(3\pi t)$
   c) $e^{-j\omega t}$
   d) $\sin(2\pi t)$

3. For the signal $x(t)$, the coefficient is $C_0$, one of the coefficients is observed to be $C_1 = 2 + j3$, then $C_0$ is
   a) $-2j$
   b) $2j$
   c) $3 + j2$
   d) $3j2$

4. Consider the signal $x(t) = (10\cos(10\pi t + \pi/3) + 4\sin(10\pi t + \pi/6)$ which lies within the frequency band 10Hz to 20Hz in 

   a) 1W
   b) 8W
   c) 50W
   d) 500W

5. Consider the trigonometric series, which holds true $y(t)$ given by

   $x(t) = \sin(3\pi t) + 1/3\sin(3\pi t) + 1/5\sin(5\pi t)$

   a) 0.5
   b) 0.25
   c) 0.125
   d) 0.0625

6. The $m$th harmonic component of the periodic waveform shown in the figure has an amplitude of

   a) $\frac{1}{m}$
   b) $\frac{1}{2\pi}$
   c) $\frac{1}{2\pi m}$

   a) $\frac{1}{m}$
   b) $\frac{1}{2\pi}$
   c) $\frac{1}{2\pi m}$
7) The TFS for the waveform g(t) shown consists
   (a) only cosine terms with zero d.c. component
   (b) only cosine terms with positive d.c. component
   (c) only sine terms with negative d.c. component
   (d) only sine terms & negative d.c. component

8) A function is given by f(t) = sin(5t) + cos 2t. Which of the following is TRUE? GATE
   (a) F has frequency components at 0 and 1/2π Hz
   (b) F has frequency components at 0 and 1/π Hz
   (c) F has frequency components at 1/2π and 1/π Hz
   (d) F has frequency components at 0, 1/2π & 1/π Hz

9) The fundamental frequency of the composite signal
   \[ x(t) = 2 + 3\cos(0.2t) + 3\cos(0.25\pi t) + 4\cos(0.3\pi t) \]
   (a) 0.05 rad/sec
   (b) 0.1 rad/sec
   (c) 0.2 rad/sec
   (d) 0.25 rad/sec

10) The average value of the periodic signal \( x(t) \) shown in figure is
    (a) 5/6
    (b) 1
    (c) 3/4
    (d) 5/6

11) \( f(t) \) shown in figure is represented by \( f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + b_n \sin nt \). The value of \( a_0 \) is
    (a) 0
    (b) π/2
    (c) π
    (d) 2π

3.6 Practice Problem Set:

P16.1 Obtain the F.S expression for the waveform shown in fig.17

P16.2 Calculate the F.S expression for the function shown in figure 2, (3)?
3. The F.S representation of \( x(t) \) shown in the figure is given by
\[
x(t) = (0.125 + 0.25) \sin(5t) + 0.125 \cos(10t)
\]
Determine the F.S of \( x(t) \).

4. Determine whether the functions given can be represented by a Fourier series?
   a) \( x(t) = \cos(\omega t) + \sin(\omega t) + \cos(2\omega t) + \sin(2\omega t) \)
   b) \( x(t) = 3 \sin(\omega t) + \cos(2\omega t) \)
   c) \( x(t) = (t - 1) \sin(\omega t) \) where \( x(0) = x(2\pi) \) & \( x(t) = \sin(\pi t/2) \)

5. Find the F.S coefficients of
\[
x(t) = \sum_{n=-\infty}^{\infty} 2(1-n) \text{sin}(nt)
\]
   where \( n \) is an integer.

6. The magnitude & phase spectra of a periodic signal \( x(t) \) are shown in the figure.

7. Sketch the output spectrum of the filter if
   a) Ideal filter blocks all frequencies. The input is \( x(t) \).
   b) Ideal filter blocks all frequencies. The input is \( x(2t) \).
   c) Ideal filter passes only frequencies in the range 200-400Hz.

8. The periodic signal \( x(t) = \sin(2\pi t) \) is applied to an ideal filter as shown.

9. Let \( x(t) \rightarrow x_c(t) \) with period \( T \). Find F.S coefficients of the following signals in terms of \( x_c(t) \):
   a) \( x(t) + x_c(t) \)
   b) \( \Re(x(t)) \)
   c) \( \text{Im}(x(t)) \)
   d) \( x(T-t) \)
   e) \( \omega(t) \)

10. The power in the first two harmonics of the signal shown in the figure is \( 1.28 \) joules, what is the power in the rest of the harmonics?

11. Consider an LTI system with impulse response \( h(t) = e^{-2t} \). Find F.S. representation of the output if input is \( x(t) = \sum_{n=-\infty}^{\infty} n^2 \delta(t-n) \).
12) The F.W.R. sinusoidal voltage is \( f(t) \) applied to LPF of \( f(t) \) obtain the output voltage \( v_o(t) \) of the filter?

13) Given the set of functions shown in fig(a) show that this is an orthogonal set & that each member of the set is normalized.

(i) If \( x(t) = \cos(2\pi t) \), \( \phi = \pi \), find \( y(t) \) expressed as \( y(t) = \sum_{k=1}^{\infty} \phi[k] \delta[k] \).

(ii) Repeat part (i) for \( x(t) = \sin(2\pi t) \), \( \phi = \pi / 2 \).

HINT: Use \( \delta(t) = 0 \) or \( \delta(t) = \delta(t) \) for \( \phi \).

14) Consider basis functions of the form \( y(t) = A \cos(2\pi f t) + B \sin(2\pi f t) \), \( f > 0 \). Find \( A \) & \( B \) such that \( y(t) \) and \( y(t) = 0 \) are orthogonal over \( (0, \infty) \).

15) The F.E.S. representation of a signal \( x(t) \) over \( (0, T) \) is \( x(t) = \sum_{n=\pm1}^{\infty} \frac{A_n}{n} \sin(n\pi t/T) \).

a) Find \( T \).
b) One of the components of \( x(t) \) is \( A \cos(3\pi t) \). Find \( A \).
Chapter 4: Fourier Transform (F.T.)

4.1 INTRODUCTION:

- Fourier Transform (F.T.) provides a frequency domain description of time domain signals and is extension of F.S to non-periodic signals.
- F.T. expresses signals as linear combination of complex Sinoids.
- Transformation makes the analysis of signals much easier because certain features which may be obscure in one form may be obvious in other forms.
- Spectrum of F.T. is continuous whereas spectrum of F.S is discrete.

F.T. (or) spectrum of a signal: \( x(t) = \int X(\omega) e^{j\omega t} \, d\omega \) ................................ (1)

Inverse F.T. (or) F.T is: \( x(t) = \frac{1}{2\pi} \int X(\omega) e^{j\omega t} \, d\omega \) ................................ (2)

\[ X(\omega) = \int x(t) e^{-j\omega t} \, dt \]

\[ x(t) = \frac{1}{2\pi} \int X(\omega) e^{j\omega t} \, d\omega \]

P4.1.1 If \( x(t) \) is a voltage waveform, then what are the units of \( X(\omega) \) ?

P4.1.2 For the signal \( x(t) \) shown in figure, find:

a) \( X(\omega) \)

b) \( \int X(\omega) \, d\omega \)

Convergence of F.T.:

1) F.T. is defined for all stable signals i.e., \( |x(t)| \leq M \) for all \( t \).

2) Periodic signals, which are neither absolutely integrable nor square integrable over an infinite interval, can be converted to have F.T. if impulse functions are permitted in the transform.

3) \( x(t) \) has a finite number of discontinuities and finite number of maxima and minima within any finite interval.

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M.I. NARASIMEEAM

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[53] S & S – Fourier Transform

F.T. Of Standard Signals:

1) Decaying exponential

\[ x(t) = e^{-at}, \quad t > 0 \]

2) Increasing exponential

\[ x(t) = e^{at}, \quad t > 0 \]

3) C.T. impulse function \( \delta(t) \)

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ACE ACADEMY
4.2 Properties Of F.T.

1) Linearity: \(x(t) \leftrightarrow X(\omega)\) and \(x_2(t) \leftrightarrow X_2(\omega)\)
then \(a_1 x_1(t) + a_2 x_2(t) \leftrightarrow a_1 X_1(\omega) + a_2 X_2(\omega)\)

P4.2.2 Find the F.T. of the signals
(i) \(x(t) = e^{\text{at}}\) [Two sided exponential]
(ii) \(x(t) = \text{Sign}(t)\) [Signum function]

2) Time - Scaling: \(x(t) \leftrightarrow X(\omega)\) then \(x(at) \leftrightarrow \frac{1}{|a|} X\left(\frac{\omega}{a}\right)\)
Expansion \(\leftrightarrow\) Compression

Linear scaling in time by a factor \(a\) corresponds to linear scaling in frequency by a factor of \(1/a\)

P4.2.3 Find the F.T. of the signals
(i) \(x_1(t) = A \text{rect}(1/2T)\)  
(ii) \(x_2(t) = A \text{sin}(\pi \omega T/2)\)

P4.2.5. Find the F.T. of the signal shown in figure?

\(x(t) \rightarrow X(\omega)\) then \(X(2\pi) = 2 \pi x(t)\),

P4.2.4. Find F.T. of the following signals:
(i) \(x(t) = 1\)  
(ii) \(x(t) = 1 / (\omega + j)\)  
(iii) \(x(t) = 2 / (\omega^2 + 1)\)  
(iv) \(x(t) = 1 / \omega\)

P4.2.5 Find the F.T. of \(x(t) = u(t)\)

4) Time - Shifts: \(x(t) \leftrightarrow X(\omega)\) then \(x(t - t_0) \leftrightarrow e^{j\omega t_0} X(\omega)\)
Time - delay in a signal causes a linear phase shift in its spectrum

P4.2.6 Find the F.T. of the signals
(i) \(x(t) = e^{\text{au}} (\omega - 1)\)  
(ii) \(x(t) = \pi [\omega - 1/2]\)  
(iii) \(y(t) = e^{j\omega t}\)

5) Frequency - Shift (or) Modulation:
If \(x(t) \leftrightarrow X,\) then \(x(t) \leftrightarrow Y(\omega - \omega_0)\)
Modulation spreads the signal spectrum to higher frequencies

P4.2.7 Find the F.T. of the following signals:
(i) \(\text{Cos} \omega t\)  
(ii) \(\text{Sin} \omega t\)  
(iii) \(e^{\omega t} \text{Sin} \omega, t_0(t)\)  
(iv) \(A \text{rect}(1/2T) \text{Cos} \omega t\)

P4.2.8 Find the F.T. of \(Y(t) = \frac{4}{\pi} \text{Cos} 2\omega t\)

6) Time - Reversal: \(x(t) \leftrightarrow X(\omega)\) then \(x(-t) \leftrightarrow X(-\omega)\)
Only phase spectrum changes

P4.2.9 Let \(x(t) = \text{rect}(1 - |t|)\) where \(\text{rect}(1) = 1\) for \(\frac{1}{2} \leq x \leq \frac{1}{2}\) then
if \(\text{Sin} \omega t = \sum x(t) = \sum x(0)\) will be given by
\(\pi \omega t\)
7) Differentiation in time:
\[ x(t) \rightarrow X(j\omega), \; \text{then} \; \frac{d}{dt} x(t) \rightarrow j\omega X(j\omega) \]
Differentiation in time corresponds to multiplying the frequency components of the signal by \( j\omega \).

Differentiation destroys any DC component of \( x(t) \), i.e., \( \text{F.T. of the differentiated signal at } \omega = 0 \) is zero.

P4.2.10: Find the F.T. of the signal \( x(t) \) shown in fig.

P4.2.11: Find the F.T. of the signal \( y(t) = 1(t - 2) + \delta(t - 1 - 2) \)

P4.2.12: For the spectrum \( X(j\omega) \) shown in figure, find \( \frac{d}{dt} X(j\omega) \) at \( \omega = 0 \). 

5) Frequency Differentiation:
\[ x(t) \rightarrow X(j\omega) \; \text{then} \; j\omega x(t) \rightarrow X(j\omega) \frac{d}{dt} \]

P4.2.13: Find the F.T. of \( y(t) = e^{-t} u(t) \)

P4.2.14: Find the F.T. of \( e^{-t} u(t) \), hence find the transform of \( \frac{\sin t}{1 + t^2} \).

P4.2.25: Given \( x(t) \rightarrow X(j\omega) \), express the F.T. of the following signals in terms of \( X(j\omega) \)?
(i) \( x(t) = x(t - 2) + x(t - 4) \)
(ii) \( x(t) = \delta(t - 3) \)
(iii) \( x(t) = t \delta(t) \)

P4.2.16: Given \( x(t) = \begin{cases} 1, & \text{if } |t| < 1 \\ 0, & \text{elsewhere} \end{cases} \)
Find the F.T. of the following signals:
(a) \( y(t) \)
(b) \( z(t) \)

9) Convolution in time:
\[ x(t) \rightarrow X(j\omega), \; h(t) \rightarrow H(j\omega), \; \text{then} \; x(t) * h(t) \rightarrow X(j\omega) \cdot H(j\omega) \]
→ Convolution in one-domain corresponds to multiplication in frequency domain.
→ F.T. of impulse response, \( h(t) \) is known as the frequency response, \( H(j\omega) \).
→ Since \( h(t) \) completely characterizes an LTI system, then so must \( H(j\omega) \).
P4.2.13 Given \( y(t) = x(t) \ast 7(t) \) and \( g(t) = e^{-t} u(t) \) asp. that \( g(0) = A \), find A & 7.

P4.2.18 An LTI system in having I.R. h(t) = \( \sin(4t) \) for which the input applied is

\[
x(t) = \cos(2t) + \sin(6t)
\]

Find the output.

P4.2.19 Consider a causal LTI system with frequency response

\[
H(j\omega) = \frac{e^{j\omega} - 1}{j}\]

Find the impulse response?

b. What is the c/p when the 1/p applied is \( x(t) = e^{j} u(t) - 1 \cdot e^{j} x(t) \)?

P4.2.20 A causal & stable LTI system has the frequency response: \( H(j\omega) = \frac{j \omega}{\omega^2 + 4} \)

a. Find the impulse response?

b. What is the output when the input applied is \( x(t) = e^{-t} u(t) - 1 \cdot e^{-t} x(t) \)?

P4.2.21 Consider the system shown in fig(a). The F.T. of the input signal shown in fig(b).

Find F.T. of \( y(t) \) given \( w(t) = \cos(5t) \) and \( h(t) = \sin(4t) \).

\[
\text{Fig(a)} \quad \text{Fig(b)}
\]

10) Frequency Conversion:

\[
x(t) \longrightarrow X_0(j\omega) \quad \text{and} \quad x(t) \rightarrow \rightarrow X_0(j\omega) \quad \text{then} \quad x(0) \ast 3(0) \rightarrow 1 \cdot \frac{1}{2}\pi \cdot X_0(j\omega) * X_0(j\omega)
\]

P4.2.22 Find the F.T. of \( y(t) = 0 \) (Constant)

11) Integration in Time:

\[
\text{If} \quad x(t) \rightarrow X(j\omega), \text{then} \quad \int x(t) dt \rightarrow X(j\omega) \quad \text{or} \quad X(j\omega) \rightarrow \pi X(0) u(0) \quad \text{if} \quad X(0) > 0
\]

P4.2.23 Find the F.T. of \( \sqrt{3/\pi} \cdot \sin(2\pi t) \)

P4.2.24 Find the energy in the signal \( x(t) = \sin \pi t \).

P4.2.25 Find the energy in the spectrum shown in fig.

\[
\text{Fig(a)} \quad \text{Fig(b)}
\]

P4.2.26 An input signal \( x(t) = e^{-t} u(t) \) is applied to an ideal L.P.F. with frequency response characteristics \( \frac{1}{1 + j\omega} \), where \( 0 \leq \omega \leq \omega_0 \).

Find the output energy in the output is half that of Input energy.

P4.2.27 Consider \( x(t) = 0 \) (Constant), suppose we are given the following facts:

i) \( E \{ (t+\tau) x^2(t) \} = 4x^2 \tau \)

ii) \( E \{ x^2(t) \} = 2 \tau \), find a closed-form expression for \( x(t) \).

P4.2.28 Find the value of the integral

\[
\int_{-\infty}^{\infty} \frac{1}{t^2 + 4} \, dt
\]
### Fourier Transform Properties

<table>
<thead>
<tr>
<th>PROPERTY</th>
<th>X(t) - form</th>
<th>X(ω) - form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linearity</td>
<td>x(t)+bX(t0)</td>
<td>ax(t)+bx(t0)</td>
</tr>
<tr>
<td>Time-Scaling</td>
<td>1/a(X(at))</td>
<td>1/a(X(at))</td>
</tr>
<tr>
<td>Time-reversal x(-t)</td>
<td>X(-t)</td>
<td>X(-t)</td>
</tr>
<tr>
<td>Time-Shift e^(jωt)x(t)</td>
<td>e^(jωt)X(t)</td>
<td>e^(jωt)X(t)</td>
</tr>
<tr>
<td>Frequency Shift e^(jωo)X(t)</td>
<td>X(t-ωo)</td>
<td>X(t-ωo)</td>
</tr>
<tr>
<td>Differentiation in time d/dt</td>
<td>d/dt</td>
<td>d/dt</td>
</tr>
<tr>
<td>Frequency Differentiation</td>
<td>df/dX(t)</td>
<td>df/dX(t)</td>
</tr>
<tr>
<td>Convolution in time x(t)*y(t)</td>
<td>X(t)*Y(t)</td>
<td>X(t)*Y(t)</td>
</tr>
<tr>
<td>Frequency convolution x(t)*X(ω)</td>
<td>X(t)*X(ω)</td>
<td>X(t)*X(ω)</td>
</tr>
</tbody>
</table>

### Fourier Transform Of Useful Signals:

<table>
<thead>
<tr>
<th>Signal, x(t)</th>
<th>X(t) - form</th>
<th>X(ω) - form</th>
</tr>
</thead>
<tbody>
<tr>
<td>e(t)+a(t)</td>
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<td>1</td>
</tr>
<tr>
<td>e^(-t)+a(t)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>δ(t)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Aδ(t)</td>
<td>2πAδ(ω)</td>
<td>2πAδ(ω)</td>
</tr>
<tr>
<td>A rect(T)</td>
<td>AT/sinc(T)</td>
<td>AT/sinc(T/2)</td>
</tr>
<tr>
<td>Sinc(t)</td>
<td>Rect(t)</td>
<td>Rect(t/2)</td>
</tr>
<tr>
<td>e^(-t)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Sgn(t)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>s(t)</td>
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<td>1</td>
</tr>
<tr>
<td>cos(t)</td>
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<td>1</td>
</tr>
<tr>
<td>sin(t)</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

### Integral Properties:

- **Parseval's theorem**
  \[
  \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega
  \]
4.3 DISSIPATIONLESS TRANSMISSION:

In several applications such as signal amplification or microwave signal transmission over communication channels, we require that the output waveform be a replica of the input waveform. Transmission is said to be distortionless if the input and output have identical waveforms within a multiplicative constant (or) a delayed output that retains the input waveform is considered to be distortionless.

For distortionless transmission, the input $x(t)$ and output $y(t)$ satisfies the condition

$$y(t) = k x(t - \tau)$$

$$H(o) = \frac{Y(o)}{X(o)} = Ke^{j\phi}$$

$$|H(o)| = k$$

$$\phi = -\tau o$$

$$\tau$$

For distortionless transmission, magnitude response must be a constant, phase response must be a linear function of $o$ with slope $-\tau o$, where $\tau$ is delay in output with respective to input.

If the gain is not constant over the required frequency range, we have amplitude distortion. If the phase shift is not linear with frequency, we have phase distortion as the signal undergoes different delays for different frequencies.

Ideal filters are necessary, useful, and physically unrealizable.

For a physically realizable system, $h(t)$ must be causal i.e., $h(t) = 0$ for $t < 0$. In the frequency domain, this condition is known as Paley - Wannier Criterion which states that the necessary and sufficient condition for the magnitude response $|H(o)|$ to be realizable is

$$\frac{1}{|H(o)|} = \frac{1}{o} < \infty$$

Phase delay $\tau(o)$ is the delay occurring at a single frequency, as the signal propagates from source to destination, the amount of delay caused is known as phase delay.

$$\tau(o) = \frac{\phi}{o}$$

→ Phase delay is not necessarily the true signal delay. A slowly Sinusoidal signal doesn't carry information. Information must be transmitted only by applying some appropriate change to Sinusoidal wave. Suppose that a slowly varying signal is multiplied by a Sinusoidal wave so that resulting modulated wave consists of a narrow group of frequencies. When this modulated wave is transmitted through the channel, we find that there is a delay between envelope of Input and received signal. This is known as envelope (or) group delay (True signal delay)

$$\tau_g(o) = -\frac{d\phi}{do}$$

P4.3.1 For a linear phase channel, what is $\tau(o)$ & $\tau_g(o)$?

P4.3.2 The system under consideration is an RC LPF with $R = 1 k \Omega$ & $C = 1 \mu F$.

(a) Let $H(o)$ denote the frequency response of RC LPF. Let $f_0$ be the highest frequency component such that $0 < f_1 < f_2$. $|H(f_0)| = 0.95$ then $\tau_g$ (in Hz) is

- $a) 327.8$
- $b) 163.9$
- $c) 52.2$
- $d) 104.4$

(b) Let $\tau_g(o)$ denote the group delay of RC LPF and $f_1 = 100 Hz$, then $\tau_g(2)$ in msec, is

- $a) 0.017$
- $b) 7.17$
- $c) 71.7$
- $d) 4.205$

P4.3.3 Consider a distortionless transmission system $H(o)$ with magnitude and phase response as shown in figure. If an input signal $x(t) = 2 \cos 10 \pi t + \sin 26 \pi t$ is given to the system, the output will be

- $a) e^{Cos 10 \pi t} + \sin 26 \pi t$
- $b) 8 \cos 10 \pi t + \sin 26 \pi t$

(c) $4 \cos (10 \pi t - \pi/4) + \sin (26 \pi t + 13 \pi/30)$
(d) $8 \cos (10 \pi t - \pi/2) + \sin (26 \pi t - \pi/2)$

P4.3.4 An RC LPF with a time constant of 16 msec is excited by a modulated signal $x(t) = \sin (2\pi t) + 2 \cos (2\pi t)$. Find the phase and group delays at 10 Hz?

Ans: $12.54$ msec, $7.95$ msec
P4.3.5 Suppose a transmission system has the frequency response as shown in figure. For what range of frequency there is no distortion?

\[ H(f) \]

Ampl. H(f)

\[ 0 \quad 20 \quad 30 \quad 50 \quad \text{kHz} \]

4.4. **HILBERT TRANSFORM (H.T.)**

The Hilbert transform is an operation that shifts the phase of \( x(t) \) by \( -\pi/2 \), while the amplitude spectrum of the signal remains unaltered. An ideal H.T. is an all pass 90° phase shifter. H.T. is used in number of applications such as representation of base pass signals, phase shift modulators, generation of SSB.

\[ x(t) \quad \frac{1}{\pi t} \quad X(t) = x(t) * 1/\pi t \]

Frequency response of the H.T. = \( \text{sgn}(a) \)

\[ H(\omega) \]

Properties of H.T.:
1) H.T. doesn't change the domain of a signal
2) H.T. doesn't alter the amplitude spectrum of a signal
3) If \( H(\omega) \) is H.T. of \( x(t) \), then H.T. of \( \tilde{x}(t) \) is \( -x(t) \)
4) \( x(t) \) and \( \tilde{x}(t) \) are orthogonal to each other.

P4.4.1 Find H.T. of:
1) \( x(t) = \text{Cos} \omega t \)
2) \( x(t) = \text{Sin} \omega t \)
\[
\begin{align*}
(1) \ x(t) &= \sum_{n=-\infty}^{\infty} a_n \text{Cos} \omega_0 (t + nT) \\
&= \sum_{n=-\infty}^{\infty} b_n \text{Sin} \omega_0 (t + nT) \\
&= 1
\end{align*}
\]
\[
(4) \ x(t) = \text{Cos}(2\pi f t), \ \text{where} \ x(t) \ \text{represents a signal band limited to} \ B, f < B
\]

4.5. **CORRELATION**

- It provides a measure of the similarity between 2 waveforms as the function of search parameter.
- An application of correlation to signal detection in a radar, where a signal pulse is transmitted in order to detect a target. If the target is present, the pulse will be reflected by it. If the target is not present, there will be no reflection pulse, just noise. By detecting the presence or absence of the reflected pulse we confirm the presence of signal or absence of the target.
- In digital communication, the important thing is that is 1s and 0s in the data stream be distinguishable from each other so the receiver can reproduce the bit pattern that was transmitted.
- Auto-correlation function of an energy signal \( x(t) \) is

\[
R_x(t) = \int_{-\infty}^{\infty} x(t) x(t - \tau) d\tau = \int_{-\infty}^{\infty} |x(t)|^2 \delta(t - \tau) d\tau
\]

Properties of ACF:
1) ACF is an even function of \( \tau \) i.e., \( R_x(t) = R_x(-t) \)
2) ACF at origin indicates total energy (or) power in the signal
3) Maximum value of ACF occurs at origin i.e., \( |R_x(0)| = R_x(0) \forall T \)
4) \( R_x(t) = x(t) * x(-t) \)
5) F.T. of ACF is known as ESD (or) PSD

For an LTI system:

\[
Y(\omega) = X(\omega) H(\omega)
\]

Output spectral density = \( |H(\omega)|^2 \)
P4.5.1 Find the auto-correlation and power in the signal
\[ x(t) = 6 \cos (4 \pi t + 2t) \]

P4.5.2 Find the ACF of \( x(t) = e^{-t} u(t) \)

P4.5.3 Consider a filter with \( H(w) = \frac{1}{1 + jw} \) and input \( x(t) = e^{2t} u(t) \).
   a) Find the ESD of the output.
   b) Show that total energy in the input is one-third of the output energy?

P4.6.4 A power signal \( x(t) \) whose PSD is shown in fig. is applied to an ideal differentiator, find the mean square value of the output of the differentiator.

P4.5.5 A power signal \( x(t) \) whose PSD is a constant \( K \) is applied to a RC low-pass filter. Find the MSV of output?

P4.5.6 Find the C.C.F at \( x(t) = e^{t} u(t) \) and \( y(t) = e^{-t} u(t) \).?

F.T. of periodic signals:

\[ x(t) = \sum_{n} x(nT) \delta(t-nT) \]

F.T. of periodic signal consists of a sequence of equally spaced impulses located at harmonic frequencies of the signal.

\[ \sum_{n} x(nT) \]}

P4.6.2 Find the Nyquist rate & Nyquist interval for each of the following signals?

(a) \( x(t) = \sin(200t) + \cos(500t) \)

(b) \( x(t) = \sin(200t) - \cos(500t) \)

(c) \( x(t) = 5 \cos(100t) + 3 \sin(200t) \)

P4.6.2. Let \( x(t) \) be a signal with Nyquist rate \( f_N \). Determine the Nyquist rate for each of the following signals.

(a) \( x(t) + \sin(2t) \)

(b) \( x(t) \cos(200t) \)
4.7 Previous Questions:

1. The F.T. of the signal \( x(t) = e^{-t} \) is of the following form, where \( A \) & \( B \) are constants.
   (a) \( e^{-at} \)  (b) \( A + Bt \)  (c) \( Ae^{-Bt} \)  (d) \( Ae^{-at} \)

2. \( y(t) \) is the input to an LTI system. The required output is \( y(2) \). The transfer function of the system should be \( \frac{Y(s)}{X(s)} = \frac{4}{s} \). The transfer function of the system should be
   (a) \( 4e^{2s} \)  (b) \( 2e^{2s} \)  (c) \( 4e^{s} \)  (d) \( 2e^{s} \)

3. Let \( x(0) \) and \( y(0) \) [with F.T.s X(0) and Y(0) respectively] be related as shown in figure. Then
   \( Y(0) = \) \( 0 \)  (a) \( \frac{1}{2} X(0) e^{-2s} \)  (b) \( X(0) \)  (c) \( X(0) \)  (d) \( X(0) e^{-2s} \)

4. The output \( y(t) \) of an LTI system is related to its input by the following equation:
   \( y(t) = 0.5x(0) \)  (a) \( h(t) \)  (b) \( X(0) e^{-0.5t} \)  (c) \( (1 - 0.5) X(0) e^{-0.5t} \)
   \( Y(0) = 0.5x(0) \)  (d) \( (1 + 0.5) X(0) e^{-0.5t} \)

5. For a signal \( x(t) \), the F.T. is \( X(0) \). Then \( L.F.T. \) of \( X(0) e^{-2t} \) is
   (a) \( 125 \times 0.2e^{-2s} \)  (b) \( 125 \times 0.2 \)  (c) \( 3x(0) \)  (d) \( \alpha e^{-2t} \)

6. Match the following:

   **List I (CT function)**
   A) \( e^{-t} \)  
   B) \( \frac{1}{s^{2}} \)  
   C) \( \frac{1}{s} \)  
   D) \( \frac{1}{1+s} \)

   **List II (CTFT)**
   1) \( e^{at} \)  
   2) \( \sin(at) \)  
   3) \( 0 \)  
   4) \( \sin(\theta) \)  

7. S & S – Fourier Transform

8. A signal \( x(t) = 6 \cos(0.5t) \) is sampled at a rate of 14 Hz to recover the original signal.
   (a) \( f \leq 0 \)  (b) \( 9 \)  (c) \( 10 \)  (d) \( 14 \)

9. S & S – Fourier Transform
7) In the figure shown, \( m(t) = 2\sin(2\pi t) \), \( S(t) = \cos(200t) \) and \( v(t) = 5\sin(299t) \). The output \( y(t) \) will be \[ y(t) = \text{GATE} \]

8) Consider a sampled signal \( y(t) = 5 \times 10^{-5} \sum_{n=-\infty}^{\infty} x(t - nT_s) \). 

where \( x(t) = 10\cos(2\pi t) \) and \( y(t) \) is the output. Print the output of the filter.

9) Let \( x(t) = 2\cos(800t) + \cos(400t) \) and \( x(t) \) is sampled with the rectangular pulse train shown in fig. The only spectral component (in kHz) present in the sampled signal in the frequency range 3.6kHz to 3.8kHz.

   a) 2.7, 3.4
   b) 3.3, 3.6
   c) 2.4, 2.7, 3.3, 3.4, 3.6
   d) 2.7, 3.3

10) Which of the following is/are not a property/properties of PFD?
   a) \( S_1(0) \) is not a function of \( \omega \)
   b) \( S_1(0) \) is an even function of \( \omega \)
   c) \( S_1(0) \) is not \( \omega \)-invariant, \( S_1(\omega) < 0 \)
   d) All the above

11) Match the following

   List I (Application of signals)
   (a) Reconstruction
   (b) Over-sampling
   (c) Interpolation
   (d) Decimation

   List II (Definition)
   (1) A sampling rate is chosen significantly > the Nyquist rate
   (2) A mixture of continuous & discrete signals
   (3) To convert discrete sequence back to continuous signal & then resample
   (4) Assign values between samples & signals

12) F.T. of \( x(t) = 2\sin(\omega) \) then F.T. of \( x(t) \) is ________ IES

13) What is the I.F.T. of \( e^{i\omega} \) ? IES

14) Match the following

   List I
   (a) periodic function
   (b) periodic function
   (c) \( S(\omega) \)
   (d) \( s(t) \)
   (e) \( s(t) \) is discrete

   List II (Fourier spectrum/F.T)
   (1) Continuous spectrum at all frequencies
   (2) \( S(\omega) \)
   (3) Line discrete spectrum
   (4) \( s(t) \) is

15) A signal represented by \( x(t) = 5\cos(800t) \) is sampled at a rate of 300 samples/sec. The resulting samples are passed through an ideal LPF with cut-off frequency of 150 kHz. Which of the following will NOT be contained in the output of LPF ? IES

   (a) 200 Hz  (b) 100 Hz, 150 Hz  (c) 50 Hz, 100 Hz  (d) 20, 100, 150 Hz

16) I.F.T of \( X(\omega) = 2\cos(\omega) + \sin(\omega) + 4\) is in Hz

   (a) 1 - cos(4t)  (b) 1 - cos(4t)  (c) 2(1 - cos(4t))  (d) 2(1 + cos(4t))

17) If a C.T. signal \( x(t) = 2 + \cos(50t) \) is sampled with a periodic function, aliasing occurs when the sampling period is __________ DRDO

   (a) 0.01 sec  (b) 0.015 sec  (c) 0.019 sec  (d) 0.025 sec

18) If a C.T. signal \( x(t) = 2 + \cos(50t) \) is sampled with a periodic function, period 0.025 sec then it is passed through an ideal LPF with a response of 1. Within the range of \( -\pi \leq \omega \leq \pi \) (in radians), the reconstructed signal is ________ DRDO

   (a) 2 + cos(50t)  (b) 2cos(50t)  (c) 2cos(50t)  (d) 2cos(50t)
4.8 Practice Problems:

4.8.1) Let \( X(\omega) = \text{rect} \left( \frac{\omega - 2\pi}{2} \right) \) Find the F.T. of the following signals:

a) \( x(t) \)

b) \( x(t+4) \)

c) \( x(t) - x(t-1) \)

d) \( x(t)e^{-j\omega t} \)

4.8.2) Given \( x(t) \rightarrow H(\omega) \rightarrow x_0(t) \) Obtain F.T. of the following signals:

- \( x(t) \)

4.8.3) The F.T. of 1 signal \( x(t) \) is defined as

\[ X(t) = \text{rect}(t) \]

a) Find closed-form expression for \( X(t) \) and \( y(t) \)

b) Design the system shown in the block diagram in terms of choosing the parameters \( A, f_1, f_2 \) so that the output is \( y(t) \)

4.8.4) Find the I.F.T. of the following:

a) \( X(f) = \frac{e^{-j\omega t}}{1 + 4\pi^2} \)

b) \( X(f) = \frac{\sin(2\pi f)}{1 + j2\pi f} \)

4.8.5) A signal \( x(t) = 8 - 8\cos^2(4\pi t) \) is passed through an ideal LPF. The filter blocks frequencies above 5 Hz. Find the output?

4.8.6) The transfer function of a system is \( H(\omega) = \frac{2+j\omega}{4+6\omega - \omega^2} \)

Find the output if the input is \( x(t) = e^{-j\omega t} \)

4.8.7) The signal \( x(t) = \text{Sinc}(0.5t) + \text{Sinc}(0.25t) \) is applied to a filter whose impulse response is \( h(t) = a\text{Sinc}(at) \). For what value \( a \) of \( A \) is the filter output equal \( x(t) \)?

4.8.8) Find the frequency response & impulse response of a filter whose input - output relation is described by the following equation

\[ y(t) = x(t) - 2\frac{1}{\omega}e^{j\omega t} + (\omega - 2) \]

4.8.9) Find the output \( y(t) \) for each cascaded system. Do the 2 systems yield the same output?

4.8.10) The input to the system shown in figure has the spectrum shown. Let \( P(t) = \text{Cos}(t), 0 < t < \pi \)

Find the spectrum \( Y(\omega) \) of the output if \( h(t) = \text{Simpul} \)

Consider the cases \( \omega_0 < 0 \) and \( \omega_0 > 0 \).
4.8.11 The input signal \( x(t) = 2 \text{cos}(3t) + \text{sin}(6t) \) is applied to the system shown in figure. 

(a) Find \( y(t) \) if \( a_1 = a_2 = 1 \) 
(b) \( a_1 = 2 \), \( a_2 = 1 \) 
(c) \( a_1 = 1 \), \( a_2 = 2 \) 

4.8.12 Find \( x(t) \) if \( X(e^{j\omega}) = \frac{1}{1+e^{j\omega}} \) 

4.8.13 For the F.T. \( X(e^{j\omega}) \) shown in figure, evaluate the following quantities without calculating \( X(e^{j\omega}) \) :

(a) \( \frac{\infty}{-\infty} \int \frac{X(e^{j\omega})e^{j\omega}}{\omega} d\omega \) 
(b) \( \frac{\infty}{-\infty} \int X(e^{j\omega})e^{j\omega} d\omega \) 

4.8.14. Find the F.T. of \( X(e^{j\omega}) = |X(e^{j\omega})|e^{j\omega} \), where 
   \( |X(e^{j\omega})| = \frac{2|\omega|}{\omega} \) and \( |X(0)| = \frac{2\omega}{\omega} \) 

4.8.15. The output \( y(t) \) of a causal L.T.I. system related by the D.E.
\[ \frac{dy}{dt} + 2y(t) = 2x(t) \]

(a) Find the I.R. 
(b) Find the response if \( x(t) = e^{-2t} u(t) \) 

4.8.16. A causal L.T.I. filter has the frequency response \( H(e^{j\omega}) \) shown in figure. For each of the input signals given below, find output:

(a) \( x(t) = 5 \text{sin}(2t + \pi) \) 
(b) \( X(e^{j\omega}) = \frac{1}{1 + e^{j\omega}} \) 

4.8.17 The F.T. of a triangle pulse \( f(t) \) shown in figure is \( 7\text{sin}^2(\pi t) \). Using this find the F.T. of the signal shown in figure:

4.8.18 What percentage of the total energy in the signal \( f(t) = e^{-2t} u(t) \) is contained in the frequency band \( 0 \leq \omega \leq \pi \)? (Hint: Rayleigh's theorem.)

4.8.19. If the input to a system is \( \text{Cos}(10t + 1 + 2 \text{Cos}(20t + 3) \), tell what kinds of distortion, if any, a system would have if its output is:
   (a) \( \text{Cos}(10t + 4) + 5 \text{Cos}(20 t + \pi) \)
   (b) \( \text{Cos}(10 t + 1) + 2 \text{Cos}(20 t + \pi) \)
   (c) \( \text{Cos}(10 t + 1) + 2 \text{Cos}(20 t + \pi) \)
   (d) \( 2 \text{Cos}(10t + 1) + 2 \text{Cos}(20 t + \pi) \)

4.8.20. Consider a fixed, linear system with amplitude and phase responses as shown. Obtain the output for the following inputs:
   (a) \( x(t) = 3 \text{Cos}(3t) + 3 \text{Cos}(20t) \)
   (b) \( x(t) = 2 \text{Cos}(3t) + 2 \text{Cos}(20t) \)

4.8.21. Find the average auto Correlation of \( x(t) = \text{Cos}(2t) + 2 \text{Cos}(4t) \)
4.8.22) A signal \( x(t) = e^{j\omega_0 t} \) is passed through an ideal LPF with cut-off frequency \( \omega_c = 1 \text{ rad/sec} \). Find the ratio of output energy to input energy?

4.8.23) Find the average power at the output of the circuit shown in figure, if \( x(t) \) is a finite power signal with PSD

\[
\begin{align*}
(i) & \quad S_x(n) = k \\
(ii) & \quad S_x(n) = \pi \delta(n + 2\pi) + \delta(n - 2\pi)
\end{align*}
\]

4.8.24) The P.S.D. of a signal is shown figure,

- (a) Find the normalized average power of the signal.
- (b) Find the amount of power contained in the frequency range 5-10 KHz.

4.8.25) The input to RC N.W. shown in figure is \( x(t) = 1 + 2 \cos \omega t + 0.5 \sin \omega t \).

- (a) Find the input & output spectral densities
- (b) Find the normalized average power content of \( y(t) \).

4.8.26) Find the F.T. of the periodic signal shown in figure.

4.8.27) 2 signals \( x_1(t) = 10 \cos 100\pi t \) and \( x_2(t) = 10 \cos 50\pi t \) are both sampled with \( f_s = 75 \text{ Hz} \). Show that the 2 sequences at samples are identical?
Chapter 5. LAPLACE TRANSFORM

1. L.T expresses signals as linear combination of complex exponentials, which are eigen functions of D.E which describe continuous-time L.T.I systems.

2. The primary role of the L.T in engineering is the transient & stability analysis of causal L.T.I systems.

3. L.T provides a broader characterization of systems & their interaction with signals than is possible with F.T.

4. In addition to its simplicity, many design techniques in circuits, filters & control systems have been developed in L.T. domain.

5. Consider applying an input of the form $x(t)e^{st}$ where $X(s)$ is transfer function of the system.

6. L.T. of a general signal $x(t) = X(s)$ $x(t)dt ightarrow X(s)$

7. $e^{st}$ may be decaying (as) growing depending on whether a" is +ve (re > 0).

8. $x(t) ightarrow X(s)$

9. Region of Convergence (R.O.C) of L.T.

The range of values of "s" for which $x(t)$ is satisfied i.e., $x(t)e^{st}$ for $t < 0$ is known as R.O.C of L.T.

10. L.T. calculated on the jω-axis (α = 0) is F.T.

5.1 L.T. of Standard Signals:

1) $x(t)e^{-st} = 0$ $t > 0$ $\frac{1}{s}$ $\rightarrow$ Re(s) > 0

2) $x(t) = e^{at} (t > 0)$ $\frac{1}{s}$ $\rightarrow$ Re(s) < 0

5.2 Properties of L.T.

1) Linearity: $X_{1}(s)$ $X_{2}(s)$ with ROC = $R_{1}$ $R_{2}$

2) Convolution $x_{1}(t)x_{2}(t)$ $x_{1}(s)x_{2}(s)$ with ROC = $R_{1}$ $R_{2}$

5.2.1 Given $X(s) = \frac{2s+5}{s^{2}+1}$, find all the time-domain signals?

5.2.2 Find the L.T. of the following signals with R.O.C?

1) $x_{1}(t) = e^{at}e^{at}$

2) $x_{2}(t) = e^{at}e^{bt}$

3) $x_{3}(t) = e^{at}e^{bt}$

4) $x_{4}(t) = e^{at}$

5) $x_{5}(t) = sm(t)$
2) Time-shifter : \( x(t) \to X(\sigma) \), ROC=R
Then \( x(t-a) \leftrightarrow e^{-\alpha t}X(\sigma) \), with ROC=R

PS1.5 Find the L.T. of the following signals:
( a ) unit ramp starting at \( t=0 \)
( b ) \( x(t) = u(t) \)
( c ) \( y(t) = e^{a t}(a+3) \)

PS1.6 Find the L.T. of \( Y(s) = \frac{e^{-\sigma t}}{(s+1)(s+2)} \) with ROC=R

PS1.7 Consider the signal \( x(t) = e^{-\sigma t}u(t-1) \) with L.T. \( X(s) \)
( a ) Find \( X(s) \) with ROC=R
( b ) Find the value of \( X(0) \) when the L.T. \( X(s) = L\{e^{-\sigma t}u(t-1)\} \) has a rational form as \( X(s) \). What is the ROC corresponding to \( X(s) \)?

3) Shift in s-domain : \( x(s) \to X(s-a) \) with ROC=R
then \( x(t) \leftrightarrow X(s-a) \) with ROC=R

PS1.8 Find the L.T of 1) \( x(t) = u(t-1) \) 2) \( x(t) = u(t) \) 3) \( x(t) = e^{a t} \text{ sign}(a t) \)

PS1.9 Let \( x(t) \) be a signal that has a rational L.T. with exactly 2 poles located at \( s=-1 \) and \( s = -3 \). If \( g(t) = e^{-3 t} \) and \( X(s) \) converges, determine whether \( g \) is causal.
(a) Left-sided  (b) right-sided  (c) one-sided  (d) finite-duration.

4) Time-reversal : \( x(t) \leftrightarrow X(-\sigma) \) ROC=R

PS1.10 Let \( g(t) = x(t) + \alpha \) where \( x(t) \) is causal with ROC=R
( a ) Find \( X(s) \), ROC=R

Determination in time-
\[ x(t) \leftrightarrow X(s) \text{ with ROC=R then } \frac{dX(s)}{dt} = sX(s) \text{ with ROC=R} \]

PS1.11 Consider 2 right-sided signals \( x(t) \) and \( y(t) \) related through the equation
\[ \frac{dx(t)}{dt} = 2y(t-1) \text{ and } \frac{dy(t)}{dt} = 2x(t) \text{ Find } x(t) \text{ and } y(t) \text{ with ROC=R} \]

PS1.12 Find the L.T. of the following signals with ROC=R
( a ) \( x(t) = \frac{e^{-\sigma t}}{(s+1)(s+2)} u(t-2) \)
( b ) \( x(t) = e^{-\sigma t} \text{ sign}(a t) \)

Determination in S-domain-
\[ x(t) \leftrightarrow X(s) \text{ ROC=R then } \frac{dX(s)}{dt} = sX(s) \text{ ROC=R} \]

PS1.13 Find the L.T. of \( x(t) = e^{-\sigma t} u(t-1) \) \( y(t) = \text{ sign}(a t) \)

PS1.14 Find the L.T. of \( X(s) = \frac{\text{sign}(a t)}{s+1} \)

PS1.15 Find the L.T. of \( x(t) = \frac{4}{(s+1)^2} \)
( b ) \( X(s) = e^{-\sigma t} \text{ sign}(a t) \)

Convolution in time-
\[ x(t) \leftrightarrow X(s) \text{ with ROC=R then } \frac{dX(s)}{dt} = sX(s) \text{ ROC=R} \]

1.7 of impulse response is known as system (or) transfer function

PS1.16 Solve the following D.E. \( y(t) + 4y(t) = x(t) \text{ if } x(t) = 0 \text{ for } t < 0 \)

PS1.17 Consider a signal \( y(t) = x(t-2) * x(t+3) \) where \( x(t) = e^{-t}(t) \text{ sign}(0) \text{ for } t < 0 \)
Find \( Y(s) \) with ROC=R

PS1.18 Find the impulse response of a Linear causal system describe by the equation.
\[ \frac{dy(t)}{dt} + 4y(t) + 3 \int y(t) \text{ dt} = x(t) \text{ for } x(t) = 0 \text{ for } t < 0 \]
5.3

5.3.1

5.3.2

5.3.3

5.3.4

L.T. of periodic signals

\[ X(\omega) = \int_{0}^{2\pi} x(t) e^{-j\omega t} dt \]

Find the L.T. of the periodic signal shown in fig. 7.

\[ x(t) \]

\[ 0 \]

\[ 1 \]

\[ 2 \]

\[ 3 \]

\[ 4 \]
5.4 PREVIOUS QUESTIONS:

1) $L\{r(t)\} = \frac{s+2}{s^2+1}, L\{g(t)\} = \frac{s^2+1}{s(s+2)}$ then $L\{f(t)(-t)\}$ is ________ GATE

2) T.F. of a system is $H(s) = \frac{1}{s^2+2}$ then I.R. of the system is ________ GATE

   A) $t^2e^{-2t}$  B) $(t^2+2)u(t)$  C) $te^{-2t}(t)$  D) $e^{-2t}$

3) What does the T.F. of a system describe for the system?

4) What is the I.T. of the waveform shown in Fig. 7?
   A) $F(s) = \frac{1}{s}e^{-t}$  B) $F(s) = \frac{1}{s^2} + \frac{2}{s}$  C) $F(s) = \frac{1}{s} + \frac{2}{s^2}$  D) $F(s) = \frac{1}{s^2} + \frac{2}{s}$

5) For the signal shown:

6) An LTI system is having transfer function $\frac{s^2+1}{s^2+2s+1}$ & input $u(t)=\sin(\pi t+1)$ is in steady state. The output is sampled at $\alpha$, rad/sec to obtain final output $y(k)$ which of the following is true?
   A) $y(k) = \alpha$ for all $\alpha$  B) $y(k) = 0$ for all $\alpha$  C) $y(k) = 0$ for $\alpha > 2$ but nonzero for $\alpha < 2$  D) $y(k) = 0$ for $\alpha > 2$ but nonzero for $\alpha < 2$

7) What is the output as $t \to \infty$ for a system that has T.F. $F(s) = \frac{2}{s^2+1}$ when subjected to a step input?
   A) $-1$  B) $1$  C) $2$  D) unbounded

8) I.T. of $r(t)$ is ________ IES

9) I.T. of $x(t)e^{-u(t)}u(t)$ is ________ IES

10) Let $r$ be the waveform shown in Fig. 10 is $\frac{1}{2} \left[ (\cos \omega t + 1) + (\cos \omega t + 1) \right]$ then $D$ is ________ IES

   A) 0.5  B) 1.5  C) 0.5  D) 2

11) The I.L.T. of is $X(s) = \frac{1}{s(s+1)}$ then initial & final values of $x(t)$ are respectively ________ IES

   A) 0 & 1  B) 0 & 1  C) 0 & 0  D) -1 & 0

12) The I.T. of $x(t)$ is $X(s) = \frac{1}{s(s+1)}$ then waveform of $x(t)$ is ________ IES

   A) 1  B) 1  C) 1  D) 1

13) What is the impulse response $h(t)$ for a system specified by differential equation $y(t) = (t+2)y(t-1) + (t+1)y(t)$?

14) The response of a system to a unit ramp input is $tu(t) + \frac{1}{2} e^{-t}u(t)$. Then the unit impulse response of the system is ________ IES

15) Let a signal $u(t) = u(t)$ be applied to a stable LTI system. Let the corresponding output be represented as $y(t) = y(t)$. Then which of the following statement is TRUE?
   A) $F$ is not necessarily a "sine" or "Cosine" function but must be periodic & $\omega = \omega_0$
   B) $F$ must be "Sine" or "Cosine" with $\omega = \omega_0$
   C) $F$ must be "Sine", $\omega_0 u(t)$, $a_1 u(t)$
   D) $F$ must be "Sine" or "Cosine" functions with $\omega = \omega_0$

16) Let $r(t) = u(t)$ be applied to a stable LTI system. Let the corresponding output be represented as $y(t) = y(t)$. Then which of the following statement is TRUE?
   A) $F$ is not necessarily a "sine" or "Cosine" function but must be periodic & $\omega = \omega_0$
   B) $F$ must be "Sine" or "Cosine" with $\omega = \omega_0$
   C) $F$ must be "Sine", $\omega_0 u(t)$, $a_1 u(t)$
   D) $F$ must be "Sine" or "Cosine" functions with $\omega = \omega_0$

17) If a signal $\text{Re}(s)$ remains so that the L.T. of the function $e^{(s^2)} x(t)$ exists?
   A) Re(s) $\neq 0$  B) Re(s) $\neq 7$  C) Re(s) $\neq 0$  D) Re(s) $\neq 5$ GATE

18) If $r(t) = u(t)$ be applied to a stable LTI system. Let the corresponding output be represented as $y(t) = y(t)$. Then which of the following statement is TRUE?
   A) $F$ is not necessarily a "sine" or "Cosine" function but must be periodic & $\omega = \omega_0$
   B) $F$ must be "Sine" or "Cosine" with $\omega = \omega_0$
   C) $F$ must be "Sine", $\omega_0 u(t)$, $a_1 u(t)$
   D) $F$ must be "Sine" or "Cosine" functions with $\omega = \omega_0$
5.5 PRACTICE PROBLEM SET:

5.5.1 Find the L.T. & associated R.O.C of the following signals:

- a) \( x(t) = e^{-at} u(t) + e^{-2at} u(t) \)
- b) \( x(t) = e^{-at} u(t) - e^{-2at} u(-t) \)
- c) \( x(t) = e^{-at} u(t) + u(t) \)
- d) \( x(t) = e^{-at} u(t) + e^{-2at} u(-t) \)

5.5.2 Find the corresponding signal for each of the following L.T & their associated R.O.C:

- a) \( \frac{s}{s^2 + 9} \)  
  \( \text{R.O.C} \): \( s > 0 \)
- b) \( \frac{s}{s^2 + 6s + 5} \)  
  \( \text{R.O.C} \): \( s > -1 \)
- c) \( \frac{s+1}{s^2 + 3s + 2} \)  
  \( \text{R.O.C} \): \( s > -2 \)
- d) \( \frac{s+1}{s^2 + 3s + 2} \)  
  \( \text{R.O.C} \): \( s > -2 \)

5.5.3 (Given the transform pair \( \cos(\omega_0 t) \leftrightarrow X(s) \), find the time signals corresponding to the following L.T:

- a) \( \frac{s^2 + s + 1}{s^4 + 2s^3 + 2s^2 + s + 1} \)  
- b) \( \frac{2}{s^2 + 3s + 2} \)  
- c) \( \frac{e^{-at} u(t)}{s} \)

5.5.4 Consider an LTI system for which the transfer function \( H(s) \) has the pole-zero pattern shown in fig.5.5.4.

- a) Indicate all possible R.O.Cs
- b) For each R.O.C of part (a), specify whether the system is stable and/or causal.

5.5.5 Find the I.L.T. of \( X(s) = \frac{s^2 + s + 1}{s^3 + 2s^2 + 5} \)  
\( \text{R.O.C} \): \( s > -1 \)

5.5.6 Solve the differential equation:

\[ \frac{dy(t)}{dt} + 5y(t) + 6y(t) = -4x(t) - 3 \text{delta}(t) \]

Given \( y(0) = 1, \text{delta}(0) = 5 \). Find the response.

5.5.7 A non-causal signal \( x(t) = e^{-at} u(t) + e^{-at} u(-t) \) is the excitation of a filter whose impulse response is \( h(t) = 5 \delta(t) - 5 \). Find the response.

5.5.8 The step response of C.T. LTI system is given by \( (1 - e^{-at})u(t) \). For a certain unknown input \( x(t) \), the output \( y(t) \) is observed to be \( 2(3e^{-at} - 2)u(t) \). Find the input \( x(t) \).

5.5.9 Determine the initial & final values of \( y(t) \) if they exist, given that:

- a) \( F(s) = \frac{s^2 + 2s + 1}{(s - 1)(s + 2)} \)  
- b) \( F(s) = \frac{s^2 + 2s + 1}{(s - 1)(s + 2)(s + 4)} \)

5.5.10 Find the U.L.T. of the periodic signals shown in fig.5.5.10.

5.5.11 Consider a C.T. L.T.I system for which the input \( x(t) \) & output \( y(t) \) are related by the D.E:

\[ \frac{dy(t)}{dt} + 5y(t) + 2y(t) = x(t) \]  
Find impulse response, \( h(t) \) under the following conditions:
- a) Stable
- b) Causal
- c) Neither stable nor Causal

5.5.12 The signal \( y(t) = e^{-at} u(t) \) is the output of a Causal all-pass system for which the system function is \( H(s) = \frac{s-1}{s+1} \). Find at least 2 possible input's \( x(t) \) that could produce \( y(t) \).

5.5.13 The relation between the input \( x(t) \) & output \( y(t) \) of a system is described by D.E:

\[ \frac{d^2y(t)}{dt^2} + 5y(t) + 2y(t) = \frac{d^2x(t)}{dt^2} - 2 \frac{dx(t)}{dt} + x(t) \]

a) Does this system have a stable & Causal inverse?

b) Find D.E. of the inverse system.

5.5.14 A Causal L.T.I system has the B.D. shown in fig.5.5.14. Find the D.E. relating input \( x(t) \) & output \( y(t) \) of the system, is the system stable?

---

Fig. 5.5.10

Fig. 5.5.4

Fig. 5.5.14
Chapter 6. DTFT

- The DTFT describes the spectrum of discrete signals and formalizes that discrete signals have periodic spectra. The frequency range for a discrete signal is unique over \((-\pi, \pi]\) or \([0, 2\pi]\).

\[
X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}
\]

\[
x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega
\]

- X(e^{j\omega}) is decomposition of x(n) into its frequency components.

Convergence of DTFT

A sufficient condition for existence of DTFT is \(\sum_{n=-\infty}^{\infty} |x(n)| < \infty\).

- Some sequences are not absolutely summable, but they are square summable.
- There are signals that are neither absolutely summable nor have finite energy, but still have DTFT.

\[
s^2 u(n), |a| < 1 \quad \longleftrightarrow \quad \frac{1}{1-a e^{-j\omega}}
\]

- \(\delta(n) \quad \longleftrightarrow \quad 1\)

Periodicity property: \(X(e^{j(\omega+2\pi)n}) = X(e^{j\omega n})\)

P6.1.1 Find the F.T. of the signal shown in figure?

P6.1.2 Let \(x(n) = (1/2)^n u(n)\), \(y(n) = x^2(n)\) & \(Y(e^{j\omega})\) be the F.T. of \(y(n)\). Then \(Y(e^{j\omega})\) is ___

M.L. NARASIMHAM
Find the autocorrelation of \( x(t) = (1, 2, 3, 4) \) ?

Continuous time signal \( x(t) \) is to be filtered to remove frequency component in the range 5kHz \( t \leq f \leq 10 \) kHz. The maximum frequency present in \( x(t) \) is 20kHz. Find the minimum sampling frequency & first frequency response of ideal digital filter that will remove the desired frequencies from \( x(t) \).

A signal \( x(t) = \sin(\omega_0t + \Phi) \) is input to a LTI system having frequency response \( H(e^{j\omega}) \). If the output is the system in A \( x(t) = h(t) \), then the most general form of \( |H(e^{j\omega})| \) will be

(a) \( h(t) = n \) for any arbitrary roll \( n \)
(b) \( h(t) = 2n \) arbitrary integer \( n \)
(c) \( h(t) = 2n \) for any arbitrary value \( n \)
(d) \( h(t) = 2n \) arbitrary

A 3 point sequence \( x[n] \) is given by

\[ x(-3) = 1, x(-2) = 1, x(-1) = 0, x(0) = 5, x(1) = 1 \]

Let \( x[n] \Leftrightarrow X(e^{j\omega}) \), then the value of \( \int_{-\pi}^{\pi} X(e^{j\omega}) \, d\omega \) is ____________

An LTI system is having impulse Response \( h(t) = (1/3)u(t) \). Find the response when the input applied is \( x(t) = \cos(\pi t + \pi/3) \).

Compute the Fourier transform of the following signals?

(a) \( x(t) = u(t-2) - u(t-6) \)
(b) \( x(t) = (1/2) \quad u(t-1) \)
(c) \( x(t) = \cos(7\pi t/2) \)
(d) \( x(t) = \{-2, 1, 0, 2, 1\} \)
(e) \( x(t) = 2^{|n|} \)

Determine the signal corresponding to the following transforms?

(a) \( X(e^{j\omega}) = \text{rect}(\omega) \)
(b) \( X(e^{j\omega}) = \text{sinc}(\omega/2) \)
(c) \( X(e^{j\omega}) = \text{sinc}(2\pi \omega) \)
(d) \( X(e^{j\omega}) = \cos(\pi \omega) \)

P6.2.3 Find the magnitude and phase responses of the following systems:
(a) \( y(n) = 0.5x(n+1) - x(n-1) \)
(b) \( y(n) = 2x(n-3) - x(n-2) \)
(c) \( y(n) = x(n) - x(n-3) \)

P6.2.4 A signal \( x(n) \) has the DFT \( X(e^{j\omega}) = \frac{1}{1-e^{-j\omega}} \).
Find the DFT of the following signals:
(a) \( x(2n+1) \)
(b) \( x(n) + x(2n) \)
(c) \( \cos(0.3n) \)
(d) \( x(n)^* x(n-1) \)

P6.2.5 Consider the filter \( y(n) = 0.9y(n-1) + b(n) \).
(a) Find \( h(n) \) so that \( |H(e^{j\omega})| = 1 \)
(b) Find the frequency at which \( |H(e^{j\omega})| = 1/2 \)

P6.2.6 For the system shown in figure, let \( h(n) \) be the unit sample response of an ideal LPF with cut-off frequency \( \omega_c = \pi/4 \). Find the overall frequency response.

P6.2.7 Find the group delay of the system that has a frequency response

P6.2.8 What type of filter has a unit sample response

P6.2.9 Consider a D.T ideal HPF whose frequency response is specified as

If \( h(n) \) is the impulse response of this filter, determine a function \( p(n) \) such that

P6.2.10 Find the DFT of \( X(e^{j\omega}) = |X(e^{j\omega})| e^{j\pi \omega^2} \)
Where \( |X(e^{j\omega})| \) is

P6.2.11 An LTI system with impulse response \( h(n) = (1/3)^n u(n) \) is connected in parallel with another causal LTI system with I.R. \( h_2(n) \). The resulting parallel

interconnection has the frequency response
\[ X(e^{j\omega}) = \frac{12 - 7j \cdot e^{j\omega} + e^{2j\omega}}{12 + 7j \cdot e^{j\omega} + e^{2j\omega}} \]
Find \( h_2(n) \)?

P6.2.12 Find the I.R corresponding to the frequency response

P6.2.13 Let \( x(n) = 28(n+1) - 5(n-1) + 38(n) - 5(n-1) + 28(n-2) \) evaluate the following quantities without solving DFT
(a) \( X(e^{j\omega}) \)
(b) \( \int X(e^{j\omega}) \) \( e^{j\omega} \) \( d\omega \)
(c) \( \int X(e^{j\omega}) \) \( d\omega \)

P6.2.14 For the DFT spectrum shown in figure, find the value of centroid of the signal \( x(n) \)

P6.2.15 Calculate the output of a system for which input is \( x(n) = (1)^n u(n) \) & the impulse response is \( h(n) = (\frac{1}{2})^n u(n) \)

P6.2.16 Find the auto-correlation of the sequences
(i) \( x(n) = (1,2,1,1) \) & \( y(n) = (1,2,1,1) \)
Is there exist any relation between auto correlations?

P6.2.17 Find the cross correlation of the sequences
\( x(n) = (1,1,2,1) \) & \( y(n) = (1,2,1,3) \)

P6.2.18 Find the IDFT of spectrum shown in fig 6.2.17

FIG 6.2.18
P6.2.19 The DFT of a sequence is \( X(e^{j\omega}) = \frac{1}{(1 - 0.8 e^{j\omega})^2} \). Evaluate \( \sum_{n=-\infty}^{\infty} x(n) \).

P6.2.20 Evaluate the integral \( \int_{-\infty}^{\infty} \frac{e^{j\omega}}{1 - 0.3 e^{j\omega}} \, d\omega \).

P6.2.21 Given that the frequency response of a system is given by \( H(e^{j\omega}) = \frac{1}{1 + 3 e^{j\omega}} \), find and sketch:

(i) \( \text{Re} \{H(e^{j\omega})\} \)
(ii) \( H(e^{j\omega}) \)
(iii) \( \text{Im} \{H(e^{j\omega})\} \)
(iv) \( |H(e^{j\omega})| \)

P6.2.22 Determine whether or not the DT systems with these frequency response are causal?

(a) \( H(n) = \frac{\sin(\pi/2)}{\sin(n\pi/2)} \)
(b) \( H(n) = \frac{\sin(3\pi/2)}{\sin(n\pi/2)} e^{j\omega n} \)
(c) \( H(n) = e^{-j\pi n} + e^{-j2\pi} \)
Chapter 7. Z - TRANSFORM

7.1 Introduction to Z-Transform

- Discrete-time counterpart of L.T.I. is Z.T.
- For a D.T.L.T.I system with impulse response \( h(n) \), the response \( y(n) \) of the system to a complex exponential input of the form \( Z^n \) is \( Y(Z) = \mathcal{Z}\{Y(n)\} \) where \( H(Z) \) is known as transfer function of the system.

Z.T. of a general D.T. signal \( x(n) \) is \( \mathcal{Z}\{X(n)\} = X(Z) = F(x(n)Z^{-n}) \)

- Z.T. calculated on the unit circle is D.T.F.T.
- The range of values of \( Z \) for which \( \sum_{n=-\infty}^{\infty} x(n)Z^{-n} \) is R.O.C. of Z.T.
- D.T.F.T. is defined only for stable signals whereas Z.T. is defined for unstable signals also.
- The primary role of Z.T. in engineering are the study of system characteristics & the derivation of computational structures for implementing discrete systems on computers.

Z.T. of standard signals:

* Linearly:
  - \( x(n) \) \( \longleftrightarrow \) \( X(Z) \), ROC\(=\mathbb{R}_+ \)
  - \( x(n) \) \( \longleftrightarrow \) \( X(Z) \), ROC\(=\mathbb{R}_- \)
  - \( x(n) \) \( \rightarrow \) \( X(Z) \), ROC\(=\mathbb{R} \)

- Time delay:
  - \( x(n) \) \( \longleftrightarrow \) \( X(Z) \), ROC\(=\mathbb{R}_+ \)
  - \( x(n) \) \( \longleftrightarrow \) \( X(Z) \), ROC\(=\mathbb{R}_- \)
  - \( x(n) \) \( \rightarrow \) \( X(Z) \), ROC\(=\mathbb{R} \)

- Time shrinking:
  - \( x(n) \) \( \longleftrightarrow \) \( X(Z) \), ROC\(=\mathbb{R}_+ \)
  - \( x(n) \) \( \longleftrightarrow \) \( X(Z) \), ROC\(=\mathbb{R}_- \)
  - \( x(n) \) \( \rightarrow \) \( X(Z) \), ROC\(=\mathbb{R} \)

7.2 Properties of the Z.T.:

1) Linearity:
   - \( x_1(n) \) \( \longleftrightarrow \) \( X_1(Z) \), ROC\(=\mathbb{R}_+ \)
   - \( x_2(n) \) \( \longleftrightarrow \) \( X_2(Z) \), ROC\(=\mathbb{R}_- \)
   - \( x_3(n) \) \( \rightarrow \) \( X_3(Z) \), ROC\(=\mathbb{R} \)
   - Then \( a_1 x_1(n) + a_2 x_2(n) \) \( \longleftrightarrow \) \( a_1 X_1(Z) + a_2 X_2(Z) \), ROC\(=\mathbb{R} \)

2) Time delay:
   - \( x(n) \) \( \longleftrightarrow \) \( X(Z) \), ROC\(=\mathbb{R}_+ \)
   - \( x(n) \) \( \rightarrow \) \( X(Z) \), ROC\(=\mathbb{R}_- \)
   - \( x(n) \) \( \rightarrow \) \( X(Z) \), ROC\(=\mathbb{R} \)

P7.2.1 Find the Z.T. of \( x(n) = a^n, |a| < 1 \)

P7.2.2 The ZT \( X(Z) \) of a real & right-sided sequence \( x(n) \) has exactly 2 poles and one of them is at \( Z = e^{j\theta} \) and there are 2 zeros at the origin. If \( X(1) = 1 \), which one of the following is TRUE?

- (a) \( X(2e^{-j\theta}) = \frac{1}{(2e^{j\theta})} \) \( |e^{j\theta}| < 1 \)
- (b) \( X(2e^{j\theta}) = \frac{1}{2} \) \( |e^{j\theta}| > 1 \)
- (c) \( X(2e^{-j\theta}) = \frac{1}{2} \) \( |e^{j\theta}| > 1 \)
- (d) \( X(2e^{j\theta}) = \frac{1}{2} \) \( |e^{j\theta}| > 1 \)
2) Time-shift: $x(n) \leftrightarrow X(Z)$, ROC = R

Then $x(n-a) \leftrightarrow Z^{-a}X(Z)$, ROC = R except for the possibility of detection of 0 (or a)

P7.2.3 Find the Z.T. of $y(n) = x(n+a)Z^{-a}$

P7.2.4 Consider the signal $x(n) = \begin{cases} 1, & 0 \leq n \leq 5 \\ 0, & \text{otherwise} \end{cases}$

Let $y(n) = n(n-1)$. Find $OZ$ with ROC.

P7.2.5 Given $X(Z) = \frac{Z^2-2Z}{Z-2}$, & $x(n)$ is left-sided, find $x(n)$?

3) Exponential multiplication (or) Scaling in Z-domain:

$x(n) \leftrightarrow X(Z)$ with ROC = R then $x^n(n) \leftrightarrow X^n(Z)$ with ROC = $|z| < R$

P7.2.6 Find the Z.T. of $y(n) = \cos(\omega n(t))$

P7.2.7 The pole-zero plot shown in fig. for a signal $x(n)$.

Find the Z.T. of $y(n) = (1/2)X^2(n)$?

4) Time-reversal: $x(n) \leftrightarrow X(Z)$, then $(-a) \leftrightarrow X(-z^a)$, ROC = Z

P7.2.8 Find the Z.T. of $y(n) = x^2(n-a)$

5) Differentiation in Z-domain: $nx(n) \leftrightarrow -Z\frac{d}{dz}X(Z)$, ROC = R

P7.2.9 Find the Z.T. of $x^n(Z) = \log(1+z^a)$, $|z'| = 1$

P7.2.10 Find the Z.T. of $y(n) = u^2(n-a)$

6) Convolutions in time $x(n) \leftrightarrow X(Z)$ with ROC = R2 and $h(n) \leftrightarrow H(Z)$ with ROC = R3 then $x(n)h(n) \leftrightarrow X(Z)H(Z)$ with ROC = R, R1

Z.T. of impulse response is known as system (or) transfer function

P7.2.11 A sequence $x(n) \leftrightarrow X(Z) = Z^2 + 2Z + 1$ is applied as an input to a LTI system with impulse response $h(n) = \delta(n-3)$. The output at $n = -5$ is

P7.2.12 Consider a signal $y(n) = x(n-3) + x(n-1)$. Find $Y(Z)$ with ROC.

P7.2.13 Find the response of the system $y(n) = 5x(n+1) + 2y(n-2) - x(n)$ to the input signal, $x(n) = 5\delta(n) - 3\delta(n-1)$

P7.2.14 Find the impulse response & step response of the system described by the D.E. $y(n) + 1/4 y(n-1) - 1/8 y(n-2) = x(n) - x(n-1)$

P7.2.15 $G(Z) = az^{-1}$, $Z^{-1}$ represents a digital LTI filter with linear phase if and only if

$A) a > 1$  $B) a < 1$

P7.2.16 The following is known about a D.T.LTI system with input $x(n)$ & output $y(n)$

1. If $x(n) = (2^n)\delta(n)$, then $y(n) = (4^n)\delta(n)$

2. If $y(n) = (2^n)\delta(n)$, then $y(n) = (4^n)\delta(n)$

Where 'a' is a constant.

a) Find the value of 'a'

b) Find the response $y(n)$ if the input is $x(n+1) \delta(n)$

P7.2.17 A system with T.F. $H(Z)$ has B.Z. defined as $h(n) = 1, h(3) = -1$ and $h(k) = 0$ otherwise, consider the following statements

1. $H(Z)$ is a LTI

2. $H(Z)$ is a FIR filter

a) only S2 is true
b) both are false
c) both are true and S1 is the reason for S1

d) both are true but S1 is not the reason for S1

7. Accumulation: $x(n) \leftrightarrow X(Z)$, ROC = R

Then $x(n) \leftrightarrow \sum_{k=0}^{l} x(k)$

UNILATERAL Z.T.

P7.2.18 Find the U.Z.T. of the following signals

- $x(n) = 2^n\delta(n)
- x(n) = \delta(n) + (2/3)^n\delta(n)
- x(n) = (1/2)2^n\delta(n)
- x(n) = (1/2)^n\delta(n)$

1. Left-Shift: $x(n+1) \leftrightarrow X(z)/(z-1)$

9. Right-Shift: $x(n) \leftrightarrow Z^nX(Z)\delta(n)$

10. Initial Value Theorem: $x(0) = \delta(0)$

11. Final Value Theorem: $x^{(n)}(\infty) = \lim_{z \to 1} (1-z)X(z)$ or $X_z(Z = 1) X(Z)$
P.7.2.20. Given \( X(Z) = \frac{0.5}{Z - 2} \). Is it given that the ROC of \( X(Z) \) includes the unit circle. Then \( x[n] \) is

P.7.2.21. Apply F.V.T. for \( x[n] = \begin{cases} 1, & n \text{ even} \\ 0, & n \text{ odd} \end{cases} \). Assume the signal is right-sided?

P.7.2.22. Consider a system whose input \( x[n] \) & output \( y[n] \) are related by \( y[n] = 1 + y[n-1] \). Find the output of the system when \( x[n] = u[n] \) & \( y[0] = 2 \).

7.3 Causality & Stability:

A. A D.T. LTI system with rational system function \( H(Z) \) is causal if and only if (a) ROC is exterior of a circle outside the outermost pole (b) \( H(Z) \) expressed as a ratio of polynomials is in \( \mathbb{Z} \) order of \( Nc \). Can't be greater than order of \( Dc \).

B. An LTI system is stable if and only if the ROC of \( H(Z) \) includes the unit circle.

P.7.3.1 Find the corresponding impulse response for the given \( Z \)-\( T \), \( H(Z) = \frac{3}{1 - 0.3Z^{-1} + 0.2Z^{-2}} \).

(a) Causal
(b) Stable

P.7.3.2. The step response of an \( LTI \) system is \( x[n] = (1/4)^{n}(2n-2) \).

(a) Find the system function & impulse response?

(b) Check if the system is causal & stable?

P.7.3.3 Consider an \( LTI \) system whose pole-zero pattern is shown in figure

(a) Find the ROC of system function if it is known to be stable?

(b) Is it possible for the given pole-zero plot to be causal & stable system?

(c) How many possible ROC's are there?

P.7.3.4 The impulse response \( h[n] \) of a \( LTI \) system is real. The transfer function \( H(z) \) of the system has only pole at zero and it is at \( z = 0.5 \). The zeros of \( H(z) \) are non-real and located at \( 2 \angle 30 \). The system is

(a) stable & causal
(b) unstable & causal
(c) unstable & non-causal

(d) unstable & non-causal

P.7.3.5 Find the first 2 samples of \( X(Z) = \frac{1}{1 - 1.5Z^{-1} + 0.2Z^{-2}} \).

(a) \( |Z| > 1 \)

(b) \( |Z| = 1 \) using partial fraction expansion method?

P.7.3.6 Given \( h[n] = x[n] \). If the ZT converges on the unit circle find impulse response?

7.4 Relation between \( S \)-Plane & \( Z \)-Plane:

The \( j\Omega \) axis in the \( S \)-plane should map into the unit circle in the \( Z \)-plane & the left-half plane of the \( S \)-plane should map into the inside of the unit circle in the \( Z \)-plane. Thus a stable analog filter will be converted into a stable digital filter.

\[ Z = e^{j\Omega} \Rightarrow \omega = \Omega T \]

Table: Mapping at frequencies from the \( S \)-plane to \( Z \)-plane

<table>
<thead>
<tr>
<th>( S )-Plane</th>
<th>( Z )-Plane</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Omega _P )</td>
<td>( \pi / T )</td>
</tr>
<tr>
<td>( \Omega _G )</td>
<td>( \pi )</td>
</tr>
<tr>
<td>( \Omega _D )</td>
<td>( \omega )</td>
</tr>
<tr>
<td>( \Omega _G )</td>
<td>( \omega )</td>
</tr>
<tr>
<td>( \Omega _D )</td>
<td>( \omega )</td>
</tr>
</tbody>
</table>

Example: The pole-zero plot of the transfer function \( H(s) \) of an \( LTI \) system in the \( S \)-plane is shown in fig. The corresponding impulse response \( h(t) \) is sampled at 2 Hz to get the discrete impulse response \( h[n] \). Find the equivalent pole-zero plot in the \( Z \)-plane.

\[ x[n] = \begin{cases} 1, & n = \pm 1 \\ 0, & \text{otherwise} \end{cases} \]
7.5 Realization of Digital Systems

Realization involves converting a given transfer function $H(z)$ into a suitable filter structure. Block diagrams are used to depict filter structures and they show the computational procedures for implementing the digital filter. The basic elements of realization structures are:

1. Multiplier
2. Adder/Accumulator
3. Delay

General form of IIR filter is

$$y(n) = \sum_{k=0}^{M} b_k x(n-k) - \sum_{k=0}^{N} a_k y(n-k), \quad M \geq N$$

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=0}^{N} a_k z^{-k}}$$

Direct form realization of IIR filter

Cascade II order section:

$$w(n) = x[n] - \sum_{k=0}^{M} b_k x(n-k)$$

$$y[n] = \sum_{k=0}^{N} a_k y(n-k)$$

Cascade section is the most popular because it has a good round off noise property & requires minimum number of storage elements but it is susceptible to internal overflow.

Cascade & parallel realization structures:

To implement higher order filters we are using cascade & parallel structures. In cascade realization the transfer function is factored into $N$ II order factors.

$$H(z) = \prod_{i=1}^{N} \left( \frac{b_k z^{-k}}{1 + a_k z^{-k}} \right)$$

N is assumed to be even

$$H(z) = \sum_{i=0}^{N/2} b_i z^{-i}$$

Three difficulties arise with cascade realizations:

1. How to pair the numerator factors with denominator factors
2. Order in which individual sections should be connected
3. Need to scale the signal levels at various points within the filter to avoid the levels becoming too large or too small.
In parallel realization an Nth order transfer $H(z)$ is expanded using partial fractions as

$$H(z) = C + \sum_{i=1}^{N} \frac{B_i}{1 + a_i z^{-1}}$$

- $C = b_0/a_0$
- $B_i = b_i a_0 + b_0 a_i z^{-1}$

In parallel realization, N numerator coefficients for $z^{-1}$ is zero and order in which the section is connected is not important. Scaling is easier and can be carried out for each block independently zeros of parallel structures are more sensitive to coefficient quantization errors. Most available software packages produce coefficients for cascade realization but not for parallel structure.

- From a practical viewpoint, parallel & cascade forms using low order filters minimize the effects of finite word length.
- In the canonical direct form structure with large N, a small change in a filter coefficient due to parameter quantization results in a large change in location of poles & zeros of the system.
- In a parallel realization, a change in coefficient will effect only a localized segment.

- In cascade realization, higher – order filters are designed using string of II order filters because these lower order filters are easier to design, are less susceptible to coefficient quantization errors & stability problems & their implementations allow easier data word scaling to reduce potential overflow effects of data word size.

Example: - Obtain DF I, DF II, cascaded & parallel structures for the system

$$y[n] = -0.1 y[n-1] + 0.2 y[n-2] + 3x[n] + 3.6x[n-1] + 0.6x[n-2]$$

Solution:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{Z(1+1.2z^{-1}+0.2z^{-2})}{1+0.1z^{-1}-0.2z^{-2}}$$

DF I -

$$y[n] = -0.1 y[n-1] + 0.2 y[n-2] + 1.2 x[n]$$

DF II -

$$y[n] = -0.1 y[n-1] + 1.2 y[n-2] + 0.2 x[n]$$

Cascaded -

$$H(z) = \frac{Z[(1+0.2z^{-1})(1+z^{-1})]}{(1+0.5z^{-1})(1-0.4z^{-1})}$$
7.6 PREVIOUS QUESTIONS:

(1) Z.T. of a signal is \( Z(z) = \frac{z^2(1-z^{-2})}{(1-z^{-1})^2} \). Its final value is ________ GATE
   (a) \( \frac{1}{14} \)  (b) \( 0 \)  (c) \( 1 \)  (d) \( 5 \)

(2) If I.R. of a D.T. system is \( h(n) = -5^n a(n-1) \), then the system function \( H(z) \) is equal to ________ GATE
   (a) \( \frac{-z}{z-5} \) & is stable  (b) \( \frac{z}{z-5} \) & is stable
   (c) \( \frac{-z}{z-3} \) & is unstable  (d) \( \frac{z}{z-3} \) & is unstable

(3) A causal L.T.I. system is described by the D.E \( y(n) = 2y(n-1) - 2x(n) + x(n-1) \)
   The system is stable only if ________ GATE
   (a) \(|x| < 2, |y| < 2, \) (b) \(|y| > 2, \) (c) \(|x| > 2, \) (d) \(|y| < 2, \) (e) \(|x| < 2, \) (f) \(|y| < 2, \) (g) \(a < 2, \) any \(a\)

(4) The R.O.C. of Z.T. of the sequence \( \left[ \frac{5}{6} \right] a(n) - \left[ \frac{5}{6} \right] a(n-1) \) must be ________ GATE
   (a) \(|a| < \frac{5}{6} \)  (b) \(|a| > \frac{5}{6} \)  (c) \(5/6 < |a| < 6/5 \)  (d) \(6/5 < |a| < \infty \)

(5) If \( X(z) = \frac{z^2 + z^{-3}}{z + z^{-1}} \), then \( x(n) \) series has ________ IES
   (a) alternate 0's  (b) alternate 1's  (c) alternate 2's  (d) alternate -1's

6. For the system shown, \( x(n) \rightarrow X(z) \), and \( y(n) \) is related to \( x(n) \) as \( y(n) = \frac{1}{2} x(n-1) = x(n) \).
   What is \( y(0) \)?
   (a) \( y(0) \) \( \rightarrow \) \( X(z) \) \( \rightarrow \) \( Y(z) \) \( \rightarrow \) \( IAS \)

(7) If \( x(n) \rightarrow X(z) \) then Z.T. of \( x(n-2) \) \& \( x[n/2] \) will be ________ DRDO
   (a) \( Z^2 X(z), 2X(2z) \)  (b) \( Z^2 X(z), X(2z) \)
   (c) \( 2X(z), X(2z) \)  (d) \( Z^2 X(z), X(2z) \)

8. Given \( X(z) = 2z^2 + 2z^4 + 2 - 6z^3 + 2z^2 \), it is applied to a system, with a transfer function \( H(z) = 2z^2 + 1 \).
   Let the output be \( y(n) \). Which of the following is TRUE?
   (a) \( y(n) \) is non-Causal with finite support
   (b) \( y(n) \) is Causal with infinite support
   (c) \( y(n) = 6 \)  (d) \( |z| > 1 \)
   (e) \( y(n) \) is Causal with finite support
   (f) \( |z| < 1 \)
   (g) \( y(n) \) is Causal with infinite support
   (h) \( |z| = 1 \)

9. The R.O.C. of \( Z.T. \) of \( x(n) = (1/3)^n \) \( u(n) - (1/3)^n u(n-1) \) is ________ GATE
   (a) \( \frac{1}{3} \leq |z| \leq \frac{1}{2} \)  (b) \(|z| > \frac{1}{2} \)  (c) \(|z| < \frac{1}{2} \)
   (d) \(|z| < \frac{1}{3} \)  (e) \(|z| < \frac{1}{3} \)  (f) \(|z| < \frac{1}{3} \)  (g) \(|z| < \frac{1}{3} \)  (h) \(|z| < \frac{1}{3} \)

10. The unit impulse response of the system described by \( y(n) = y(n-1) - x(n) - x(n-1) \) is ________ IAS
    (a) \( x(n) \)  (b) \( x(n) \)  (c) \( x(n) \)  (d) \( 0 \)

11. Pole-zero plot of a DF is shown below. What is the type of the filter? ________ ISRO
    (A) LPF  (B) HPF  (C) BPF  (D) APF

12. H(z) is a discrete rational TF. To ensure that both H(z) and its inverse are stable its ________ GATE
    (A) Poles must be inside the unit circle and zeros must be outside the unit circle.
    (B) Poles must be outside the unit circle and zeros must be inside the unit circle.
    (C) Poles and zeros must be inside the unit circle.
    (D) Poles and zeros must be outside the unit circle.

13. A DF having I.F. H(z) = \( \frac{P(z)}{1+D(z)} \) using DFII and DFII realizations of IIR, the ________ GATE
    (A) 6 & 6  (B) 6 & 6  (C) 6 & 6  (D) 6 & 6

14. Consider \( X(z) = \frac{5n^2 + 3n + 4}{z^2 + 3z + 4} \), \( 0 < |z| < \infty \). The I.T.Z. \( x(n) \) is ________ GATE
    (A) \( 5(n+2) + 3(n+1) \)  (B) \( 5(n-2) + 3(n+1) \)
    (C) \( 5(n+2) + 3(n+1) \)  (D) \( 5(n+2) + 3(n+1) \)
7.7 PRACTICE PROBLEM SET:

(1) Find the Z.T., R.O.C, and the location of poles & zeros of the following signals:

(a) \( x(n) = \frac{1}{2^n} u(n) + \frac{1}{3^n} u(n) \)
(b) \( x(n) = \frac{1}{2^n} u(n) - \frac{1}{3^n} u(n-1) \)
(c) \( x(n) = \frac{1}{2^n} u(n) - \frac{1}{3^n} u(n-2) \)

(2) Using properties of Z.T., find the Z.T. of the following signals:

(a) \( x(n) = \frac{1}{2^n} u(n) + \frac{1}{3^n} u(n-1) \)
(b) \( x(n) = \frac{1}{2^n} u(n) - \frac{1}{3^n} u(n-2) \)
(c) \( x(n) = \frac{1}{2^n} u(n) - \frac{1}{3^n} u(n-3) \)

(3) Find the time –domain signals corresponding to following Z.T.:

(a) \( X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} = \frac{1}{2} \)
(b) \( X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \)
(c) \( X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{2} \)

(4) Find the impulse response of the system with transfer function:

(a) \( H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \)
(b) \( H(z) = A_0, A_1, A_2, \ldots, y(n) = \frac{1}{2} y(n-1) \)

(5) The system function of a causal LTI system is \( H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \). If the input to the system is \( x(n) = \frac{1}{2^n} u(n) - \frac{1}{3^n} u(n-1) \), find \( y(n) \).

(6) A causal LTI system has the system function:

\( H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \)

(a) Find the impulse response.
(b) Find the output if the input is \( x[n] \).

(7) If the input of an LTI system is \( x[n] = u[n-1] \), and the system function is:

(a) \( H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \)
(b) \( H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \)

(8) When the input to an LTI system is \( x[n] = \frac{1}{2^n} u(n) + \frac{1}{3^n} u(n-2) \), find the output if the system is:

(a) \( H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \)
(b) \( H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \)

(9) Compute the response of the system \( y[n] = 0.7y[n-1] + 0.12y[n-2] + x[n] \) for \( x[n] = n \).

(10) Determine the impulse response of the following causal systems & determine the response of the system for the input \( x[n] = n u[n] \):

(a) \( y[n] = \frac{1}{2} y[n-1] + \frac{1}{2} y[n-2] + x[n] \)
(b) \( y[n] = 0.5y[n-1] + 0.5y[n-2] + x[n] \)

(11) The step response of an LTI system is:

(a) \( y[n] = \frac{1}{2} u[n+1] + \frac{1}{2} u[n] \)
(b) \( y[n] = 0.5u[n-1] + 0.5u[n-2] + x[n] \)

(12) The input to a causal LTI system is:

(a) \( x[n] = u[n-1] + \frac{1}{2} x[n] \)
(b) \( x[n] = u[n] + \frac{1}{2} x[n] \)

The output of the system is:

\( y[n] = \frac{1}{2} u[n+1] + \frac{1}{2} u[n] \)

(a) Find the system function with R.O.C.
(b) What is the R.O.C. of \( y[n] \) & find \( y[0] \).
(13) Find the signal x[n] with Z.T. X(z) = \frac{3}{1 - \frac{3}{2}z^{-1} + z^{-2}} if X(z) converges on the unit circle.

(14) A causal LTI system is described by the D.E. y[n] = y[n-1] + y[n-2] + x[n-1]
(a) Find the T.F. with R.O.C.
(b) Find the unit sample response.
(c) You should have found the system to be unstable. Find a stable (noncausal) unit sample response that satisfies the difference equation.

(15) Consider the Z.T. X(z) whose pole - zero plot is shown in fig. 15
(a) Determine the R.O.C of X(z) if it is known that the T.F. exists.
(b) How many possible two - sided sequences have the pole - zero plot shown in figure.
(c) Is it possible for the pole - zero plot to be associated with a sequence that is both stable & causal?

(16) Explain the condition of BIBO stability in z - domain. A LTI system is characterized by the system function H(z) = \frac{3 - 4z^{-1}}{1 - 3.5z^{-1} + 1.5z^{-2}} specify the ROC and determine h(n), when
(i) system is stable
(ii) system is causal

(17) Consider the digital filter structure shown in fig.17
(a) Find transfer function for this causal filter
(b) For what values of 'K' the system is stable?

(18) Show that the following systems are equivalent:
(a) y[n] = 0.2x[n-1]+0.3x[n-1] + 0.02x[n-2] (b) y[n] = x[n] - 0.1x[n-1]

(19) Determine the L.Z.T. of the following
(a) X(z) = \frac{1}{1 - \frac{2}{3}z^{-1}} x[n] is right - sided \Rightarrow long Division
(b) X(z) = \frac{3}{1 - \frac{3}{4}z^{-1}} x[n] is stable \Rightarrow p.s. expansion
(c) X(z) = \ln(1 - 4z^{-1})|z| < \frac{1}{4} \Rightarrow Power series
(d) X(z) = \frac{1}{1 - \frac{3}{4}z^{-1}} |z| > \frac{3}{4}

(20) Find y[n], n ≥ 0 for the following difference equation:
(a) y[n] = \frac{1}{2}x[n-1] + \frac{1}{2}x[n], x[n] = \frac{1}{2} y[n-1] + \frac{1}{2} x[n-1]
(b) y[n] = \frac{1}{2}x[n-1] + \frac{1}{2} x[n-2] y[n] = 0, x[n] = y[n-1] - y[n-2]

(21) Consider a causal L.T.I. system whose input x(n) & output y(n) are related through the block diagram shown in figure:
(a) Determine a difference equation relating y(n) & x(n)
(b) Is this system stable?

(22) Determine the system function & the impulse response of the system shown in figure:

(23) Consider a sequence x[n] for which X(z) = \frac{1}{1 - \frac{3}{2}z^{-1}} and for which the R.O.C includes the unit circle. Find x[0] using initial value theorem.

(24) Determine a sequence x[n] whose Z.T. is X(z) = e^z + e^{2z}, z ≠ 0

(25) Find the R.O.C of Z.T. of output y[n] given X(z) & H(z):
(a) X(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} H(z) = \frac{1}{1 - \frac{1}{3}z^{-1}} (b) X(z) = \frac{1}{z^3} \Rightarrow H(z) = \frac{1}{1 - \frac{1}{3}z^{-1}} |z| > \frac{1}{3}

(26) Find the L.Z.T. of X(z) = \frac{1}{1 + 0.2z^{-1} + 0.2z^{-2}}

(27) Given x[n] = \frac{4n}{(n+0.5)^2}, |z| > 0.5. Find Z.T. of the following signals & specify ROC:
(a) y[n] = x[n-2] (b) y[n] = 2^ny[n] (c) y[n] = nx[n]
(d) y[n] = n^2 x[n] (e) y[n] = x[n] (f) y[n] = x[n]
\(0\) The transfer function of 2 cascaded systems \(H_1(z) \& H_2(z)\) is known to be \(H(z) = \frac{z^2 + 0.25}{z^2 - 0.25}\)

It is also known that the unit step response of first system is \([3, 0.5]^T x(n)\). Find \(H_1(z) \& H_2(z)\)

\[29\) Apply I.V.T \& F.V.T for \((i)\) \(X_\alpha(z) = \begin{cases} 1 & \text{if } z = 1 \\ 0 & \text{if } z = j \end{cases}\)

\((ii)\) \(X_\mu(z) = \frac{1}{z^2 - 1}\)

\[X_\beta(z) = \frac{1}{z - 1}(z - 0.5)\]

**REVIEW NOTES**

**Chapter 8, DFT & FFT**

### 8.1 Introduction To DFT

The Fourier series describes periodic signals by discrete spectra, whereas the DFT describes discrete signals by periodic spectra. As a result, signals that are both discrete and periodic in one domain are also periodic and discrete in the other. This is the basis for the formulation of the DFT.

→ Sampled version of D.T.F.T. spectrum is D.F.T.

\[\text{The } N \text{ point DFT of a signal } x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)e^{-j2\pi kn/N}, \quad x = 0, 1, \ldots, (N-1)\]

**IDFT of** \[X(k) = x(n) = \frac{1}{N} \sum_{N=0}^{N-1} X(k)e^{-j2\pi nk/N}, \quad x = 0, 1, \ldots, (N-1)\]

The DFT & its IDFT are also periodic with period \(N\), and it is sufficient to compute the results for only one period (0 to \(N-1\)).

\[X(k) = \sum_{n=0}^{N-1} x(n)W_N^{kn}, \quad x(n) = \sum_{k=0}^{N-1} X(k)W_N^{-kn}\]

where \(W_N = e^{-j2\pi/N}\) is the phase factor.

Periodicity: \(w_i^{-kN} = W_N^{-k}\) Symmetry: \(w_i^{kN} = -W_N^{-k}\)

**PR.1.1** Analog data to be spectrum analyzed are sampled at 10 kHz & DFT of 1024 samples are computed. Find the frequency spacing between successive samples?

**PR.1.2** Find the 4 point DFT of \(x(n) = (0, 1, 2, 3)\)

**Note:** For direct calculation of \(N\) point, we require \(N^2\) complex multiplications and \(N(N-1)\) additions.

**PR.1.3** Let \(X(\omega) = \sum x(n)e^{-j2\pi \omega n}\), then prove the following statements.

**Proof:**

(i) \(X(\omega + \frac{N}{2}) = X(-\omega)\)

(ii) \(X(\omega + \pi) = -X(\omega)\)

### 8.1.4 Fig. 8.1.4a Shows a finite length sequence \(x(n)\). Sketch the signals

- \(x(n)\)
- \(x(n-2)\)
- \(x(n+1)\)
- \(x(n-1)\)

**PR.1.5** Find the first five points of an 8 point DFT of a real valued sequence

\((0.25, 0.125, -0.30, 0, 0.25 - 0.30, 0, 0)\). Find other 3 points?

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8.2 PROPERTIES OF DFT:

(1) Circular shift: \( x[n-n_0] \rightarrow e^{-j2\pi n_0/k} X[k] \)

(2) Modulation: \( x[n] e^{j\omega_0 n} \rightarrow X[k-\omega_0] \)

(3) Circular Convolution: \( X[n] = X_1[n] * X_2[n] \)

(4) Central ordinate:

\[
X[0] = \sum_{n=0}^{N-1} x[n] \\
X[0] = \frac{1}{N} \sum_{n=0}^{N-1} X[k] \\
X[0] = \frac{1}{N} \sum_{n=0}^{N-1} (e^{j2\pi n/N})^k X[k]
\]

(5) Parseval's relation:

\[
\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2
\]

PR.2.1 Given \( x[n] = (1, -2, 3, -4, 5, -6) \), without calculating DFT, find the following quantities:

(a) \( X(0) \) 
(b) \( \frac{1}{N} \sum_{n=0}^{N-1} X[n] \) 
(c) \( X(1) \) 
(d) \( \frac{1}{N} \sum_{n=0}^{N-1} (e^{j2\pi n/N})^k X[k] \) 
(e) \( \frac{1}{N} \sum_{n=0}^{N-1} (e^{j2\pi n/N})^k X[k] \)

PR.2.2 The two 8-point sequences \( x[n] \) & \( x_0[n] \) shown in fig.8.2.2 have DFTs \( X(k) \) & \( X_0(k) \) respectively.

Find the relation between \( X(k) \) & \( X_0(k) \)?

PR.2.3 Consider the sequence \( x[n] \) shown in fig.8.2.3. Find \( y[n] \) whose six-point DFT is \( Y(k) = W_6^n X(k) \), where \( X(k) \) is the six-point DFT of \( x[n] \).

8.3 LINEAR CONVOLUTION USING DFT

The circular convolution of 2 sequences, of lengths \( N_1 \) & \( N_2 \) respectively, can be made equal to the linear convolution of the 2 sequences by zero padding both sequences so that they both consist of \( (N_1 + N_2 - 1) \) samples.

Let us consider the computational efficiency of calculating a convolution using the DFT rather than the direct method. In calculating the convolution of two \( N \) point sequences using DFT method, we required \( 3N \log_2 N + 2N \) Complex multiplications whereas a direct convolution of two \( N \) sequences requires \( N^2 \) complex multiplications. \( \therefore \) DFT method is more efficient for \( N \geq 32 \).

PR.3.1 Find the linear convolution of \( x[n] = (1, 2) \) & \( h[n] = (1, 2, 3) \) using DFT method?

8.4 CONVOLUTION OF LONG SEQUENCES:

Some times we have to process a long stream of incoming data by a filter whose impulse response is much shorter than that of incoming data. The convolution of a short sequence \( h[n] \) of length \( N \) with a very long sequence \( x(n) \) of length \( L \gg N \) can involve large amount of computation & memory.
(1) Overlap — Add method:

Suppose h(n) is of length N, and the length of x(n) is L = nN (if not, we can always zero pad it to this length). We partition x(n) into n segments x(0), x(1)…x(n-1), each of length N. We find the regular convolution of each section with h(n) to give partial results y(0), y(1)…y(n-1). Since each regular convolution contains (2N - 1) samples, we zero pad h(n) and each section x(n) with [N - 1] zeros before finding y(n) using the FFT. Splitting x(n) into equal-length segments in n is a strict requirement.

Example: Let x(n) = {1, 2, 3, 4, 5} and h(n) = {1, 1, 1} using Overlap - add method find y(n)?

Solution: L=6 & N=3.

\[y(0) = y(0)^{1,1,1} \times (1,1,1) = (1,1,1)\]
\[y(1) = y(1)^{1,1,1} \times (1,1,1) = (1,1,1)\]
\[y(2) = y(2)^{1,1,1} \times (1,1,1) = (1,1,1)\]

Shifting & superposition results in the required convolution

\[y(n) = y(n) + y(n)\]
\[y(3) \rightarrow 7 4 8 12 9 5\]
\[y(n) = (1, 3, 6, 8, 10, 12, 9, 5)\]

(2) Overlap — Save method:

If L > N, and we zero pad the second sequence to length L, their periodic convolution has (2L - 1) samples. Its first (N-1) samples are contaminated by wraparound, and the rest correspond to the regular convolution. Eq. Let L=6 & N=3. If we pad X by 9 zeros, their regular convolution has 31 (or 2L-1) samples with 9 trailing zeros (L-N=0). For periodic convolution 15 samples (L-N=15) are wasted around. Since the last nine [or L-N=0] zeros, only the first 6 samples of the periodic convolution are contaminated by wraparound, which is the basis idea of this method.

First, we add N zeroes to the longer sequence x(n) & section it into k overlapping (by N - 1) segments of length M. Typically, we choose M=2N.

Next, we zero-pad h(n) (with trailing zeros) to length M, and find the periodic convolution of h(n) with each section of x(n). Finally, we discard the first (N-1) contaminated samples from each convolution & chop (concatenate) the results to give the required convolution.

Example: Find convolution of x(n) = {1, 2, 3, 4, 5} & h(n) = {1, 1, 1} using Overlap Save method.

Solution: First add (N - 1) = 2 zeros to x(n) = {0, 0, 1, 2, 3, 4, 5}.

Take M=2N-1 = 5, section it into k overlapping segments of length M/5

\[x(n) = (0, 0, 1, 2, 3)\]
\[h(n) = (1, 1, 1, 0, 0)\]
\[x(n) = (2, 3, 4, 5)\]

S & S - DFT & FFT

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\[y(n) = x(n) \times h(n)\]

We discard the first 2 samples from each convolution & give the results in the form

\[y(n) = (1, 1, 1, 0, 0)\]

\[x(n) = (2, 3, 4, 5)\]

**5.5 DFTs:** Fast algorithms solve the problem of calculating an N-point DFT to that of calculating many smaller-size DFTs. The computation is carried out separately on even- and odd-indexed samples to reduce the computational effort. All algorithms allocate for complex results. The less the memory required, the more efficient is the algorithm. Many FFT algorithms reduce storage requirements by performing computations in place by storing results in the same memory locations that previously held the data.

### 3 stages in an 8-point DFT-FFT:

- **Stage 1**
  - **2-point DFTs**
  - **Combine 4 point DFTs**

- **Stage 2**
  - **2-point DFTs**
  - **Combine 4 point DFTs**

- **Stage 3**
  - **X(0) X(1)**

**Typical butterfly for 8-point FFT algorithm:**

- **A**
  - **B**
  - **W**

**DIT algorithms for N = 2, 4, 8:**

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>[A = A + BW]</td>
<td>[A = A + BW]</td>
<td></td>
</tr>
</tbody>
</table>
**DIF algorithm for N = 2, 4, 8:**

<table>
<thead>
<tr>
<th>Feature</th>
<th>N-point DFT</th>
<th>N-point FFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algorithm</td>
<td>Solution of N equations in N unknowns</td>
<td>N log2(N/2) butterflies/stage m stages</td>
</tr>
<tr>
<td>Multiplication</td>
<td>N per equation</td>
<td>N per butterfly</td>
</tr>
<tr>
<td>Addition</td>
<td>N/2 per equation</td>
<td>N per butterfly</td>
</tr>
<tr>
<td>Total multiplication</td>
<td>N^2</td>
<td>N^2 log2(N)</td>
</tr>
<tr>
<td>Total additions</td>
<td>N(N-1)</td>
<td>N log2(N)</td>
</tr>
</tbody>
</table>

**Sort - reversal using Bruenee’s algorithm:**

1. Start with (0, 1) multiply by 2 to get (0, 2).
2. Add 1 to the list of numbers obtained above, now it is (1, 3).
3. Append the list in step 2 to that in step 1 to get (0, 2, 1, 3).
4. The list obtained in step 3 now becomes the starting list in step 1. The steps are repeated until the desired length of list is obtained.

```
0 1
 Down x 2
0 2 3
 Down x 2
0 4 5 6
 Down x 2
0 8 4 12 2 10 6 14 ... 1 9 5 13 3 11 7 15
```

**PR.5.1** Let x[n] be a real 8 point sequence & let X(k) be its 8 point DFT

(A) Evaluate \( \sum_{k=0}^{7} X(k)e^{-j2\pi kn/8} \) for \( n = 9 \) in terms of x(n)

(B) Let w(n) be a 4 point sequence for \( 0 \leq n \leq 3 \) & W(k) be its 4 point DFT. If \( W(k) = X(k) + X(k+4) \), express w(n) in terms of x(n)

(C) Let y(n) be an 8 point sequence for \( 0 \leq n \leq 7 \) and Y(k) be its 8 point DFT.
If \( Y(k) = \begin{cases} 2X(k) & \text{if } k = 0, 2, 4, 6 \\ 0 & \text{for } k = 1, 3, 5, 7 \end{cases} \), express y(n) in terms of x(n).

**PR.5.2** Consider the butterfly shown in fig. which was extracted from a SFG implementing an FFT algorithm. Choose the most accurate statement from the following

(A) The butterfly was extracted from DFT-FFT

(B) The butterfly was extracted from DIT-FFT

(C) It is not possible to say from the fig. which kind of FFT algorithm the butterfly came from.

**PR.5.3** The butterfly was taken from DIT-FFT with N=16, where the input sequence was arranged in normal order. A 16 point FFT will have 4 stages (m=1,2,3,4). Which of the stages have butterflies of this form? Justify your answer.

**PR.5.4** The butterfly was taken from DIT-FFT with N=16. Assume that 4 stages of the SFG are indexed by m=1,2,3,4. Which of the 4 stages have butterflies of this form?
PREVIOUS QUESTIONS

01. How many address are required to realize a 256 point radix-2 FFT using DIF?
   (a) 256
   (b) 1024
   (c) 4096
   (d) 3848

02. 4 point DFT of real 3T signal x(n) of length 4 is X(k), k = 0, 1, 2, 3. If X(0) = 5, X(1) = 1 + j, X(2) = 0.5. Then X(3) and x(0) respectively are
   (a) -1 - j, 0.1
   (b) -1 + j, 1.5
   (c) 1 + j, 0.875
   (d) 0.1 + 0.1, 1.5

03. For an N point FFT algorithm with N = 2^n, which one of the following statements is true?
   (a) It is possible to construct a GFG with both bit and digit in normal order
   (b) The no. of butterflies in the mth stage is N/m
   (c) In place computation requires storage of only 2N node data
   (d) Computation of a butterfly requires only one complex multiplication

04. The 4 point DFT of a DT sequence {1,2,1,1} is
   (a) [0, 2+2j, 2-2j]
   (b) [2, 2j, 2-2j]
   (c) [6, 1+3j, 2, 1+3j]
   (d) [6, 1+3j, 2, 1+3j]

05. x[n] is a real valued periodic sequence with period N and its DFT is X[k]. The DFT Y[k] of the sequence y[n] = \sum_{m=0}^{N-1} X[k] \cdot e^{j2\pi km/N}
   (a) |Y[k]|^2
   (b) \sum_{m=0}^{N-1} X[k] \cdot e^{j2\pi km/N}
   (c) \sum_{m=0}^{N-1} X[k] \cdot e^{-j2\pi km/N}
   (d) 0

06. x[n] is a real valued sequence given by x[0], x[1], x[2], x[3], x[4], x[5], x[6]. If first 4 DFT coefficients of x[n] are X[0], X[1], X[2], X[3] then the coefficient X[4] is equal to
   (a) X[2]
   (b) X[1]
   (c) X[0]
   (d) X[3]

07. Suppose N = 32 FFT algorithm has a twiddle factor of W_{32}^{16} for one of the butterflies in its fifth (last) stage. If the FFT is a DIT (or DIF) algorithm

8.6 PRACTICE PROBLEM SET:

(1) Calculate the DFT of the following sequences:
   (a) x(n) = 3
   (b) x(n) = [a, b, a, b, a, b, a, b], 0 \leq a \leq N-1
   (c) y(n) = 3
   (d) y(n) = \sum_{m=0}^{N-1} x[k] \cdot e^{-j2\pi km/N}

(2) Let X[k] denote the DFT of x(n). Let y(n) denote a finite duration sequence of length 10 i.e., y(n) = 0 for n < 0 or n > 9. The 10 point DFT of y(n) is Y(k).
   Which is Y(0) = X[0] + X[9] if I find y(n)?

(3) Match 6 point circular convolution of the following sequences?
   (a) x(n) = [a, b, c, d, e, f]
   (b) x(n) = [a, b, c, d, e, f]
   (c) x(n) = [a, b, c, d, e, f]
   (d) x(n) = [a, b, c, d, e, f]

(4) Find the circular convolution of x(n) = [1, 2, 3, 1] and y(n) = [2, 3, 2, 1]

(5) Consider the sequence x(n) = [2, 1, 1, 2]. The five point DFT of x(n) is X(k). Find the sequence y(n) whose DFT is Y(k) = W_5^{k} \cdot X(k)

(6) Let x(n) be an N - point real sequence with N = point DFT X(k) (N even) and x(n)
   satisfies the symmetry x[n] = x[N-1-n] for n = 0, 1, ..., N-1 property i.e., upper half of the sequence is the negative of the lower half and show that X(k) = 0 for even k.

(7) Figure shows a sequence x(n) for which the value of N) is an unknown constant C.
   Let X[k] be the DFT where X(k) is the 5 point DFT of x(n). Find the value of C?

(8) Given x(n) = [2, 3, 2, 1]. Find the 12-point signal described by y(n) = [x(n), x(n), x(n)] and
   12-point zero interpolated signal is [x(n)] + [x(n)]

HINT: Replication in one domain corresponds to zero interpolation in other. If a signal is replicated by M, its DFT is zero interpolated & scaled by M.

Y(k) = [x(n), x(n), x(n), x(n), x(n), x(n)]

M-fold replication

Y(k) = [X(k), X(k), X(k), X(k), X(k), X(k)]

M-fold replication

Ans: Y(k) = [2, 4, 0, 0, -6, 0, 0, 0, 0, 6, 0, 0]
(9) Given two four-point sequences \( x[n] = [1, 0.75, 0.5, 0.25] \) and \( y[n] = [0.75, 0.5, 0.25, 1] \), express DFT \( X(k) \) in terms of DFT \( Y(k) \).

(10) For each DFT pair shown, compute values of the boxed quantities, using properties.
\[
\begin{align*}
&([x_0], [0.5, 0.25, 1, 0.75]) \\
&([y_0], [0.75, 0.5, 0.25, 1])
\end{align*}
\]

(11) Let \( x[n] = [1, 2, 1] \) & \( y[n] = [1, 2, 1] \) & \( z[n] = [1, 2, 3, 2, 3, 0, 1, 0, 2, 2] \) find their convolution using (a) Overlap-add method (b) overlap-save method

**REVIEW NOTES**

**CHAPTER 9. ADDITIONAL QUESTIONS**

01. Which of the following could not be the Fourier series expansion of a periodic signal?
(a) \( 2 \cos 3 \cos 2 \theta + 3 \cos 3 \theta \)
(b) \( 2 \cos 0.5 \theta + 3 \cos 3 \theta \)
(c) \( 2 \cos 0.25 \theta + 3 \cos 0.005 \theta \)
(d) \( 2 \cos 0.5 \theta + 3 \cos 2 \theta \)

02. The value of \( \sum_{n=0}^{\infty} \sin n \theta \) is
(a) \( 0 \)
(b) \( \infty \)
(c) \( \sin \theta \)
(d) none

03. The value of the integral \( \int (1 - x^2) \, dx \) is
(a) \( 0 \)
(b) \( 3 \)
(c) \( 5 \)
(d) \( 9 \)

04. Given the signal \( x(t) = 2 \cos 3 \cos \theta + 4 \cos \theta \). Then power in \( x(t) \) is \( \ldots \) watts.
(a) 10
(b) 14.5
(c) \( \sqrt{29} \)
(d) none

05. The signal \( x(t) = e^{3t} \) is
(a) Energy Signal (b) Power Signal (c) neither energy nor power (d) none

06. The integral value \( \int (e^{t^2} - e^{-t^2}) \, dx \) is
(a) \( e^{t^3}/(3t) \) (b) \( \sqrt{t} \) (c) \( \sqrt{t} \) (d) \( \sqrt{t} \)

07. Consider an energy signal \( x(t) \), over the range \(-3 \leq t \leq 3\), with energy \( E = 12.1 \). Then the signal energy in \( 2t(t) \) is
(a) \( 24 \) J
(b) \( 12 \) J
(c) \( 48 \) J
(d) \( 6 \) J

08. The fundamental period of \( x(t) \) is \( 2\pi \). Then \( x(t) \) is
(a) \( 10 \)
(b) \( 20 \)
(c) non-periodic (d) none

09. The signal \( x(t) = (j)^n \) is
(a) Periodic with fundamental period \( N = 8 \) (b) Periodic with fundamental period \( N = 4 \) (c) Non-periodic (d) None

10. The differential equation \( y'' + 4y = 0 \) is
(a) \( y(t) \) - invariant (b) \( y(t) \) - variant (c) \( y(t) \) - invariant (d) \( y(t) \) - invariant

11. Consider the sinusoidal signal \( x(t) = 10 \cos (2\pi t) \). Then fundamental period of \( x(t) \) is
(a) \( 3\pi \) (b) \( \pi \) (c) \( 4\pi /3 \) (d) none

12. The system \( y(t) = 2x(t) \) is
(a) Time - invariant & causal (b) Time - variant & causal (c) Time - invariant & causal (d) Time - invariant & non-causal

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13. The value of $Q(0) * 2b(-1) * 3d(-2)$ is
   (a) 2b(-2)  
   (b) 6b(-2)  
   (c) 4b(-2)  
   (d) 65b(-3)

14. Convolution of $e^{at}(t-2)$ with $h(t) = e^{at}$ is
   (a) $e^{at}h(t)$  
   (b) $e^{at}h(t-2)$  
   (c) $e^{at}h(t+2)$  
   (d) $e^{at-2}h(t)$

15. An LTI system with impulse response $h(t) = e^{at}$ is
   (a) Causal & stable  
   (b) Non-causal & stable  
   (c) Causal & unstable  
   (d) Non-causal & unstable

16. The impulse response of a system is $h(t) = e^{at}$. The condition for the system to be
   BIBO stable is
   (a) $'a'$ is real & positive  
   (b) $'a'$ is real & negative  
   (c) $'a' > 0$  
   (d) $'a' < 0$

17. When impulse response of an LTI system is integrated over a short time domain, it gives
   (a) Steady - stable response of system  
   (b) DC response of system  
   (c) Transient response of system  
   (d) Steady- state error of system

18. A periodic signal which can be expanded in Fourier series
   (a) is power signal  
   (b) is a.c. signal  
   (c) is neither power nor an energy signal  
   (d) can be either power or an energy signal depending on the nature

19. The convolution of $u(t)$ with $u(t-4)$ at $t = 5$ is
   (a) 5  
   (b) 2  
   (c) 1  
   (d) 0

20. Given $x(t) = x(t-2) + x(t+2)$, where $x(t) = x(t+18)$ and $x(t+154)$, then $x(t)$ is
   (a) periodic with periodic of 24  
   (b) periodic with periodic of 27  
   (c) periodic with periodic of 18  
   (d) non-periodic with periodic of 18

21. The ideal LTFT is not realizable because it is
   (a) ac-serial  
   (b) non-linear  
   (c) unstable  
   (d) time-variant

22. The inverse LTFT of $x(t)$ is
   (a) $e^{-a}t$  
   (b) $e^{-z}$  
   (c) $e^{-t}$  
   (d) $e^{-z}$ $t$  
   (e) $e^{-at}$

23. An analog RGC signal contains frequency up to 100 Hz. This signal is sampled at a rate
   3250 samples/sec. The highest frequency that can be represented uniquely at this sampling
   rate is
   (a) 125 Hz  
   (b) 625 Hz  
   (c) 100 Hz  
   (d) 250 Hz

24. If the result of convolution of $2$ infinite duration signals is causal signal, then
   (a) both $h(t)$ signals are causal  
   (b) neither signal is causal  
   (c) both signals are & other is non-causal  
   (d) both the signals are causal & other is non-causal

25. Assume $x(t) = e^{at} + e^{at-2} + e^{at}$. Given $y(t) = x(t) * h(t)$ then $y(t)$ in terms of $y(t)$ is
   (a) $y(t) = 1$  
   (b) $y(t) = 0$  
   (c) $y(t) = x(t)$  
   (d) $y(t) = t$
37. The D.E. describing the Network shown in Fig. is
   (a) 4y''(t) + y'(t) + 2y(0) = x(t)
   (b) y''(t) + 4y'(t) + 2y(t) = x(t)
   (c) y''(t) + 2y(t) = x(t)
   (d) None

38. Given a periodic signal x(t) = 3 + sint - 2cos(t) + sint - cos(2t). The amplitude of 11 harmonic of positive exponent term is
   (a) 1.57
   (b) 1.12
   (c) 0.5
   (d) None

39. Consider the transfer function H(s) = \frac{s-2}{s^2 + s + 100.25} Then zero frequency response of the system is
   (a) 0.000625
   (b) 0.0166666
   (c) 0.05
   (d) None

40. Given the signal x(t) = 16cos(20t) + 12sin(30t) + 6cos(30t + \frac{\pi}{6}) + 4cos(40t + \pi/3) the power contained in the frequency interval 12 Hz to 22Hz is
   (a) 22W
   (b) 16W
   (c) 10W
   (d) 20W

41. Given x(t) = 2\sin^2(2\pi\times 10^5 \times t), then the average power is
   (a) 0.75 watts
   (b) 0.5 watts
   (c) 1 watt
   (d) 2 watts

42. If the input to a system is cos(10t) + 2cos(20t), then the type of distortion (if any) introduced by the system is x(t) = cos(10t-n) + 2cos(20t-n) is
   (a) Amplitude
   (b) Phase
   (c) Both (a) and (b)
   (d) None

43. The initial value of X(s) = \frac{s^2 + 5s + 7}{s^4 + 3s + 2}
   (a) 1
   (b) 2
   (c) 3.5
   (d) 0

44. Consider an LTI system with input & output related through the equation
   y(t) = \int_0^\infty x(t-\tau) d\tau. The impulse response of the system is
   (a) e^{10t}u(t)
   (b) e^{50}u(t-2)
   (c) e^{100t}u(t-2)
   (d) e^{-10t}u(t)

45. Define the area under a continuous time signal x(t) is
   A = \int_{-\infty}^{\infty} x(t) dt
   (a) A_x \times A_y
   (b) A_x A_y
   (c) A_x - A_y
   (d) (A_x + A_y)^2

46. Given input of system is x(t) = \delta(t) - 2\delta(t-1) + \delta(t-2) and impulse response h(t) is shown in fig.(1). Then output of the system is
   (a) 2
   (b) 0
   (c) 4
   (d) None

47. The impulse response h(t) = \delta(t-1) is
   (a) Casual & Stable
   (b) Non Casual & Stable
   (c) Casual & Unstable
   (d) Non Casual & Stable

48. For the system shown in figure System stability is guaranteed when \alpha is
   (a) \alpha > 1
   (b) \alpha < 0
   (c) 0 < \alpha < 1
   (d) None

49. Given y(t) = x(t) \cdot \text{rect}(t) where x(t) = \text{cost}. Then amplitude of d.c. component of y(t) is
   (a) 0
   (b) 2
   (c) 1/2
   (d) None

50. Consider an LTI system with impulse response h(t) = e^{\delta(t)}. Then output y(t) for input x(t) = \sum_{t=-\infty}^{\infty} \delta(t)
   (a) 1/(4-j\pi)
   (b) 1/(4-j\pi)
   (c) 1/(4+j\pi/2)
   (d) 1/(7+j\pi)

51. The Fourier transform of x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) u(t) is
   (a) \frac{1}{T} \text{sinc} \left( \frac{\omega}{2\pi} \right)
   (b) e^{-j\omega T} \text{sinc} \left( \frac{\omega}{2\pi} \right)
   (c) \text{rect}(\omega/2\pi)
   (d) 1/T \text{sinc}(\omega T)

52. The Fourier transform of x(t) = u(t) + 2e^{-3t} is
   (a) 1 + e^{-3j\omega}
   (b) \omega + 2e^{j\omega}
   (c) -j\omega + e^{j\omega}
   (d) j\omega + 2e^{j\omega}
53. The inverse Fourier transform of \( X(w) = \frac{\sin(\omega - 2 \pi)}{(\omega - 2 \pi)^2} \) is

(a) \( e^{j\omega} \), \( |\omega| < 3 \) 
(b) \( 1/2 \), \( |\omega| < 3 \) 
(c) \( e^{j\omega} \cdot \frac{1}{2} \), \( |\omega| < 3 \) 
(d) Data is not sufficient

54. A real, continuous time function \( x(t) \) has a F.T. \( X(w) \). Whose magnitude obeys the relation \( |X(w)| = 1 - |w| \). Then the signal \( x(t) \) is an even function of time if

(a) \( |X(w)| = 1 \) 
(b) \( |X(w)| = 1 \) 
(c) \( X(w) = 1/2 \) 
(d) Data is not sufficient

55. Let \( X(w) \) denote F.T. of the signal \( x(t) \) shown in fig.

The phase of \( X(w) \) is

(a) \( -\pi \) 
(b) \( 0 \) 
(c) \( \pi \) 
(d) \( 2\pi \)

56. In the above problem (55), the value of \( \int X(w) \cdot dw \) is

(a) \( 2\pi \) 
(b) \( 1/2\pi \) 
(c) \( 4\pi \) 
(d) \( 1/4\pi \)

57. The output \( y(t) \) of a causal LTI system is related to the input \( x(t) \) by D.E. \( dy/dt + 2y(t) = x(t) \). If \( x(t) = e^{j\omega} \) then output of the system is

(a) \( e^{j\omega} \) 
(b) \( e^{j\omega} \cdot -1/2 \) 
(c) \( e^{j\omega} \cdot -1/2 \) 
(d) None of these

58. The Hilbert transform of \( x(t) = \cos 3t \) is

(a) \( -\cos 3t \) 
(b) \( \sin 3t \) 
(c) \( -\sin 3t \) 
(d) None

59. Consider the Differential Equation of an LTI system is \( d^2 y(t) / dt^2 + 6dy(t) + 9y(t) = d^2 x(t) / dt^2 \). The impulse response is

(a) \( X(\omega) = 3e^{j\omega} + 3e^{-j\omega} \) 
(b) \( X(\omega) = 3e^{j\omega} + 3e^{-j\omega} \) 
(c) \( X(\omega) = 3e^{j\omega} + 3e^{-j\omega} \) 
(d) None of these

60. A signal \( x(t) \) of finite energy is applied to a square-law device whose output is \( y(t) \). The spectrum of \( x(t) \) is limited in frequency interval \(-\omega \leq f \leq \omega \). Then impulse response is

(a) \( 2\pi \) 
(b) \( -2\pi \) 
(c) \( 0 \) 
(d) \( -2\pi \)

61. The Fourier Transform of a signal \( g(t) \cdot \delta(t) \) is

(a) \( \frac{1}{2\pi} \) 
(b) \( \frac{1}{2\pi} \) 
(c) \( \frac{1}{2\pi} \) 
(d) None of these

62. The frequency response of the linear system shown in Fig(a) is

(a) \( 1/2 \) 
(b) \( 1 \) 
(c) \( 1/2 \) 
(d) \( 1/4 \)

63. The average value of the signal shown is Fig. is

(a) \( -1/4 \) 
(b) \( 0 \) 
(c) \( 1/4 \) 
(d) None

64. A harmonic signal \( x(t) = 3\sin(4\pi t + 2\pi) \) \( \Rightarrow 4 \cos(12\pi t + 4\pi) \). Then the amplitude of the harmonic is

(a) \( 0 \) 
(b) \( 3 \) 
(c) \( -4 \) 
(d) \( -1 \)

65. The exponential Fourier series coefficient for the signal shown in Fig. is

(a) \( -1 \) 
(b) \( -2 \) 
(c) \( -1 \) 
(d) None of these

66. If energy of first 4 harmonics of the signal shown in Fig. is equal to 1.78 Joules. What is the energy contained in the rest of harmonics?

(a) \( 0 \) 
(b) \( 0 \) 
(c) \( 1.78 \) 
(d) \( 1.78 \)

67. Consider a filter with frequency response \( H(\omega) = 1/2 \). If input is \( x(t) = e^{j\omega} \). Then Energy in output is

(a) \( 0 \) 
(b) \( 1 \) 
(c) \( 0 \) 
(d) Data is not sufficient
68. For the system shown in Fig., the relation between input & output is described by D.E.

\[
\begin{align*}
\frac{d^2y(t)}{dt^2} + \frac{6}{5} \frac{dy(t)}{dt} + 2y(t) &= x(t) \\
\frac{d^2x(t)}{dt^2} + \frac{6}{5} \frac{dx(t)}{dt} + 2x(t) &= 0
\end{align*}
\]

69. A discrete-time system is described by the 2nd order differential equation

\[y[n] = -1.5y[n-1] + 0.5x[n-2] + 2x[n]. \text{ Given } y[-1] = 1, y[-2] = 2. \text{ If a step sequence is applied as input then } y[0] = \_ \text{ to be found.} \]

(a) -0.125  (b) 0.125  (c) -1.75  (d) 1.25

70. For an LTI system the input signal \(x(t)\) & impulse response \(h(t)\) are shown in Fig. the output observed at \(t = 3\) sec is

(a) 0  (b) 1  (c) -5  (d) -32

71. The impulse response of the system shown in Fig. is

(a) \(4t^2\)  (b) \(3t\)  (c) \(2t^2\)  (d) \(3t^2\)

72. The laplace transform of \(e^{-nt} u(t)\) is

(a) \(\frac{1}{s+n}\)  (b) \(\frac{1}{s-n}\)  (c) \(\frac{1}{s+n^2}\)  (d) \(\frac{1}{s-n^2}\)

73. If \(X(s) = \frac{1}{s^2+2s+2}\), the initial value is

(a) 0  (b) 2  (c) 2\(\pi\)  (d) 1

74. Given \(X(s) = \frac{1}{(s+1)^2}\), then inverse Laplace transform is

(a) \(e^{-t}\)  (b) \(t\)  (c) \(2t\)  (d) \(e^{-t}\)

75. A CONTINUOUS system has the impulse response \(2\cos t + 4\sin 2t\). Then the response to the input \(\frac{5s^2+10s+9}{s^2+2s+2}\) is

(a) \(-\cos 2t\)  (b) \(-\cos 2t\)  (c) \(\sin 2t\)  (d) \(-\sin 2t\)

76. A CONTINUOUS system has the transfer function \(H(s) = \frac{(s+1)(s+2)}{(s^2+4s+4)}\). If \(Y(s) = \frac{1}{s}\), then \(y(t)\) is

(a) \((-2t-2e^{-t})u(t)\)  (b) \((2t+2e^{-t})u(t)\)  (c) \((2t-2)u(t)\)  (d) \((2t+2)u(t)\)

77. A discrete-time signal \(x[n] = z^{-n}\) has \(Z\)-transform \(X(z) = \frac{1}{z-1} \text{ for } z = e^{j\theta}\). The value of \(X(10,0000)\) is

(a) \(10^{-4}\)  (b) \(-10^{-4}\)  (c) \(-10^{-4}\)  (d) None of these

78. If \(x[n] = u[n]\) & \(h[n] = \sin n\pi\), then \(y[n] = x[n] * h[n]\) will be

(a) \(2\sin n\pi\)  (b) \((n+1)u(n)\)  (c) \(\sin (n+1)\pi\)  (d) \(\sin (2n\pi)\)

79. The signal \(x[n] = \sin (\pi n / 12)\) is

(a) Periodic with period = 1  (b) Periodic with period = 12
(c) A periodic  (d) None of these

80. The Fourier transform of \(4t^2\) is \(X(s)\) is

(a) \(4\pi^2\)  (b) \(6\pi^2\)  (c) \(8\pi^2\)  (d) \(4\pi^2\)

81. The Fourier transform of \(e^{at}\) is \(X(s)\) is

(a) \(\frac{1}{s-a}\)  (b) \(\frac{1}{s+a}\)  (c) \(\frac{1}{s-a}\)  (d) \(e^{as}\)

82. If frequency response of the system is \(-3e^{j\pi}\), then the response if \(x(t) = 3\delta(t-3)\) is

(a) \(3\delta(t-2) + 3\delta(t-5)\)  (b) \(-3\delta(t-2) - 3\delta(t-5)\)
(c) \(3\delta(t-2) + 3\delta(t-5)\)  (d) \(-3\delta(t-2) + 3\delta(t-5)\)

83. The output of the system shown in Fig.

If the input is \(e^{-nt}\) u(t) is

(a) \(e^{nt}\) u(t)  (b) \(e^{2t}\) u(t)  (c) \(2e^{nt}\) u(t)  (d) \(e^{nt}\) u(t)

84. The \(Z\)-transform of \(e^{at}\) is

(a) \(\frac{1}{Z-e^{at}}\)  (b) \(-1/(Z-e^{at})\)  (c) \(Z/(Z-e^{at})\)  (d) \(1/(Z-e^{at})\)

85. The \(Z\)-Transform of the system shown in Fig.

If the input is \(e^{nt}\) u(t) is

(a) \(e^{-nt}\) u(t)  (b) \(e^{-2nt}\) u(t)  (c) \(e^{-nt}\) u(t)  (d) \(e^{-2nt}\) u(t)

86. A CONTINUOUS system has transfer function \(H(s) = 1/(s+1)\). The output when \(x(t) = e^{2t}\) is applied is \(y(t) = \text{cost} + e^{-t}\). The output when \(x(t) = e^{2t}\) is applied is \(y(t) = \text{cost} + e^{-t}\).
87. A LTI continuous-time system has frequency response \(H(\omega)\), it is known that the input \(x(t)\) = 1 + cos(2\pi t) + 1/4 sin(2\pi t) gives the output \(y(t)\) = 2 - sin(2\pi t). Then \(H(\omega)\) at \(\omega = \pi\) is
(a) 0
(b) 1
(c) \(1/2\) e\(^{j\pi}\)
(d) None of these

88. A continuous-time signal \(x(t)\) has F.T. \(X(\omega) = \frac{1}{2} [\delta(\omega) + \delta(\omega - \pi)]\) where "\(\pi\)" is Constant. Then F.T. of \(\delta(t) - x(t)\) is
(a) \(2\pi \delta(\omega)\)
(b) \(2\pi \delta(\omega)\)
(c) \(-2\pi \delta(\omega)\)
(d) None of these

89. The Nyquist rate for the signal \(x(t) = \delta(t) + 2\cos(2\pi t)\) is
(a) 6 rad/sec
(b) 8 rad/sec
(c) 2 rad/sec
(d) 24 rad/sec

90. Given \(h(t) = (1, 2, 1, 0, 2)\) Then system is
(a) Casual & Stable
(b) Non-Casual & Stable
(c) Casual & Unstable
(d) Non-Casual & Unstable

91. A signal \(x(t)\) with band with \(B\) is put on a carrier \(\cos(\omega_c t)\) with \(\omega_c > 2\). The modulated signal \(x(t)\),\( \cos(\omega_c t)\) is then applied to the system shown in figure. The frequency response of the filter is given by
\[H(\omega) = \begin{cases} 0 & \text{if } |\omega| > B \\ \frac{1}{2} & \text{if } |\omega| = B \\ 1 & \text{if } |\omega| < B \end{cases}\]

92. A recursive filter is described by \(y(n) = 0.3y(n-1) + 0.7y(n-2) - 0.3x(n-2) - 0.7x(n-1)\). The static gain of the filter is
(a) 10
(b) 1
(c) 0
(d) None of these

93. The difference equation is described by \(y(n) = 0.3y(n-1) + 0.7y(n-2) - 0.3x(n-2) - 0.7x(n-1)\) where input \(u(n)\) and output \(y(n)\) is
(a) Casual & stable
(b) Casual & unstable
(c) Non-Casual & Stable
(d) Non-Casual & Unstable

94. The power contained in the first 2 harmonics of periodic signal shown in Fig. is given by
(a) 0.3043 watts
(b) 0.33 watts
(c) 0.67 watts
(d) 0.0308 watts

95. For the system shown in Fig. If input is \(x(t)\), then Fourier transform of output is
(a) \(1 - e^{-j\omega} (1/\omega^2)\)
(b) \(1 + e^{-j\omega} (1/\omega^2)\)
(c) \(1/\omega\)u(\(\omega\))
(d) \(1 + e^{-j\omega} (1/\omega^2)\)

100. Consider a continuous-time system \(\phi(t) = s(t), \phi(t) \geq 0\). Let the sequence \(x(n)\) be obtained by uniform sampling of \(x(t)\) such that \(x(n) = x(\pi nT_0)\). The Z.T. of \(x(n)\) is
(a) \(1/(1-e^{-\omega T_0})\)
(b) \(1/(1-e^{-\omega T_0})\)
(c) \(e^{\omega T_0}/(1-e^{-\omega T_0})\)
(d) None of these

ASSERTION AND REASON

In questions, two statements A(Assertion) & R(Reason) are given mark the answer as
(a) If A & R both are true & R is the correct explanation of A.
(b) If A is true & R is false.
(c) If A is false & R is true.
(d) If A & R both are false.

1. Assertion (A): L.T of \(e^{-at}\) is \(1/(a+s)^2\).
   Reason (R): L.T of \(e^{-at}\) is \(1/(a+s)^2\).

2. Assertion (A): L.T of \(\sin(at)\) is \(a/(a^2+1)^{1/2}\).
   Reason (R): L.T of \(\cos(at)\) is \(a/(a^2+1)^{1/2}\).

3. Assertion (A): LTI system is non-causal if its transfer function is \(H(s) = 1/(s+1)\).
   Reason (R): A system is unstable if its transfer function is \(H(s) = 1/(s+1)\).

4. Assertion (A): LTI system is unstable if its transfer function is \(H(s) = 1/(s+1)\).
   Reason (R): A system is unstable if its transfer function is \(H(s) = 1/(s+1)\).

5. Assertion (A): LTI system is unstable if its transfer function is \(H(s) = 1/(s+1)\).
   Reason (R): A system is unstable if its transfer function is \(H(s) = 1/(s+1)\).
6. Assertion (A): The differential equation $y(t) + 2y(t) + 2x(t) + x(t)$ is nonlinear but time invariant.
   Reason (R): An LTI system is causal if $h(t) = 0; t < 0$.

7. Assertion (A): The periodic signal $x(t) = 2\sin(\pi t)$ is having exponential F.S. coefficients $(2, 2, -2)$.
   Reason (R): An odd periodic signal contains only sine terms.

   Reason (R): If a periodic signal to have F.S. it should satisfy Dirichlet condition.

9. Assertion (A): In the exponential Fourier representation of a real-valued periodic function, $f(t)$ of frequency $f_0$, the coefficients of the terms $e^{j2nft}$ and $e^{-j2nft}$ are negatives of each other.
   Reason (R): The discrete magnitude spectrum of $f(t)$ is even & the phase spectrum is odd.

10. Assertion (A): The stability of the system is assured if the ROC includes the unit circle in the z-plane.
    Reason (R): For a causal stable system all the poles should be outside the unit circle in z-plane.

11. Assertion (A): The Laplace transform of the signal $e^{2t}u(t)$ and $\frac{-e^{-2t}u(t)}{s+2}$ is $\frac{1}{s^2+2}$.
    Reason (R): The ROC for $e^{2t}u(t)$ ROC is $s > 2$, and for $-e^{-2t}u(t)$ ROC is $s < 2$.

12. Assertion (A): For a periodic signal $x(t) = 3\sin(\pi t / 2)e^{-3t} \cos(\pi t / 2)$, the amplitude of fundamental harmonic is 3.
    Reason (R): For a signal $x(t)$, fundamental period $p = 2\pi$.

13. Assertion (A): F.T. of unit step function is $1/J\omega^2$.0.
   Reason (R): L.T. evaluated along the imaginary axis becomes F.T.

14. Assertion (A): For an LTI system to be stable $\|h(t)\| < \infty$.
    Reason (R): For an LTI system with system function $H(s)$, ROC must include the imaginary axis.

15. Assertion (A): F.T. of $\sin(\omega t)$ is $\pi [-\delta(t - \omega) + \delta(t + \omega)]$.
    Reason (R): If $x(t) = \delta(t)$ then $X(f) = 2\pi \delta(f)$.

16. Assertion (A): Ideal filters are not causal.
    Reason (R): Ideal filters are not linear function of frequency.

17. Assertion (A): Linear phase characteristics is must condition for FIR filter.
    Reason (R): IIR filters are generally stable.

Match the following:

<table>
<thead>
<tr>
<th>List I (f(t))</th>
<th>List II (F(f))</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: $\frac{10}{s^2+109}$</td>
<td>1: $100(1)$</td>
</tr>
<tr>
<td>B: $\frac{10}{s^2+100}$</td>
<td>2: $2e^{-10t} \cos(10t)(u(t))$</td>
</tr>
<tr>
<td>C: $\frac{s+10}{(s+10)^2+100}$</td>
<td>3: $3\sin(10t)(u(t))$</td>
</tr>
<tr>
<td>D: $4(1-e^{-10t})(u(t))$</td>
<td></td>
</tr>
</tbody>
</table>

A B C D
a) 3 4 1 2
b) 4 3 1 2
c) 4 3 2 1
D: 4 3 2 1

2) List 1 (Input - output relation)
   A) $y(t) = x(t)$
   B) $y(t) = x(t)^2$
   C) $y(t) = x(t)e^{2t}$
   D) $y(t) = x^2(t)$

A B C D
a) 1 4 3 2
b) 3 2 1 4
c) 1 2 3 4
d) 3 4 1 2

3) List 1 (Function in time - domain)
   A) Delta function
   B) Gaus function
   C) Normalized Quantised function
   D) Sinoidal function

A B C D
a) 1 2 4 3
b) 3 4 2 1
c) 1 2 3 2
d) 3 4 4 1
4) The following table gives some time functions and I.T.
<table>
<thead>
<tr>
<th>F(t)</th>
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<tbody>
<tr>
<td>t^n</td>
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<tr>
<td>1/n</td>
</tr>
<tr>
<td>2/n</td>
</tr>
<tr>
<td>4/n</td>
</tr>
</tbody>
</table>

   Of the correctly matched pair is [a] (a) 2 & 4  (b) 1 & 4  (c) 3 & 4  (d) 1 & 2

5) List I (s(v(t)) List II (Z.T. X(s))
   a) s^n v(t)  
   b) s^n (v(t)  
   c) s^n v(t)  

   List II (Z.T. X(s))
   1) \( \frac{s^n}{1 - s} \)
   2) \( \frac{1}{Z - a} \)
   3) \( \frac{z - a}{1 - z} \)
   4) \( \frac{z}{1 - z} \)
   5) \( \frac{z}{1 - z} \)
   6) \( \frac{z}{1 - z} \)
   7) \( \frac{z^{2}}{1 - z} \)

   A B C D
   a) 3 2 4 1
   b) 2 3 4 1
   c) 3 4 2 1
   d) 4 2 3 1

6) List I (properties)
   A) (t^n + n^n) = 0
   B) (t^2 + t^3) = 0
   C) (t^n + n^n) = 0
   D) (t^n + n^n) = 0

   List II (characteristic of trigonometric form)
   1. Even harmonics can exist
   2. Odd harmonics can exist
   3. Even terms can exist
   4. Odd terms can exist
   5. Even terms of even harmonics can exist

   A B C D
   a) 4 5 3 1
   b) 3 4 1 2
   c) 5 4 2 3
   d) 4 3 2 1

7) List I (signal transform)
   A) (t^n + n^n) = 0
   B) (t^2 + t^3) = 0
   C) (t^n + n^n) = 0
   D) (t^n + n^n) = 0

   List II ( ROC)
   A) \( e^{-at} \)
   B) \( e^{at} \)
   C) \( e^{-at} \)
   D) \( e^{at} \)

   A B C D
   a) 1 4 3 2
   b) 3 1 4 2
   c) 3 1 2 4
   d) 1 3 4 2

   A B C D
   a) 1 4 3 2
   b) 3 1 4 2
   c) 3 1 2 4
   d) 1 3 4 2

   A B C D
   a) \( e^{-at} \)
   b) \( e^{at} \)
   c) \( e^{-at} \)
   d) \( e^{at} \)

   A B C D
   a) 1 4 3 2
   b) 3 1 4 2
   c) 3 1 2 4
   d) 1 3 4 2

   A B C D
   a) \( e^{-at} \)
   b) \( e^{at} \)
   c) \( e^{-at} \)
   d) \( e^{at} \)

   A B C D
   a) 1 4 3 2
   b) 3 1 4 2
   c) 3 1 2 4
   d) 1 3 4 2

   A B C D
   a) \( e^{-at} \)
   b) \( e^{at} \)
   c) \( e^{-at} \)
   d) \( e^{at} \)
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