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GATE - SYLLABUS

Signals and Systems: Definitions and properties of Laplace transform, continuous-time and discrete-time Fourier series, continuous-time and discrete-time Fourier Transform, DFT and FFT, z-transform. Sampling theorem. Linear Time-Invariant (LTI) Systems: definitions and properties; causality, stability, impulse response, convolution, poles and zeros, parallel and cascade structure, frequency response, group delay, phase delay. Signal transmission through LTI systems.

Plan of the book:

The book begins with chapter 1 by introducing the basic concepts of signal & system models and system classification. This material, which is basic to the remainder of the book, considers the mathematical representation of signals.

Chapter 2 is devoted to the time – domain characterization of C.T. L.T.I. system & the convolution.To this point the focus is on the time –domain description of signals & systems. Starting with chapter 3, we consider frequency – domain descriptions. We begin the chapter with a consideration of the orthogonal representations of arbitrary signals. The F.S & its properties are presented.

Chapter 4 begins with the development of the F.T. conditions under which the F.T. exists are presented and its properties discussed. Applications of F.T, such as sampling theorem, Distortionless transmission are considered.

Chapter 5 deals with both unilateral & bilateral L.T. The concept of transfer function is introduced and other applications of the L.T. such as for the solution of differential equations are discussed.

Chapter 6 considers the Fourier analysis of discrete – time signals. The relation between the continuous – time & discrete – time Fourier transforms of sampled analog signals is derived.

Chapter 7 discusses the Z.T. of D.T. signals. The development follows closely that of chapter 5 for the L.T, Properties of the Z.T. are derived & their application in the analysis of discrete –time systems developed. Finally mapping of the s- plane into the z – plane & realization structures is discussed.

Chapter 8 introduces the DFT for analyzing finite length sequences. The properties of the DFT are derived. Two popular fast Fourier transform (FFT) algorithms for the efficient computation of the DFT are presented.Chapter 9 introduces additional questions of all the above topics discussed.

We wish to thank all the people who have contributed to the development of this book. Iam pleased to acknowledge my indebtedness to Mr. Brahmaiah, Mr.Senthil, Mr venu, Mr.Kamalakar who prepared diagrams and alignment of the data Readers comments & criticisms are gratefully appreciated.

For any queries (or) suggestions for further improvement of the book, the readers can write to me at **narasimhamml@gmail.com**

M.L.NARASIMHAM

SIGNALS & SYSTEMS

INDEX

Managing Director

Y.V. GOPALA KRISHNA MURTHY

Chapter 1. INTRODUCTION

- Anything that bears information can be considered a signal.

  E.g.: Speech, Music, speed of an automobile

- Mathematically, signals are modeled as functions of one or more independent variables. Examples of independent variables used to represent signals are time, frequency, or spatial coordinates.

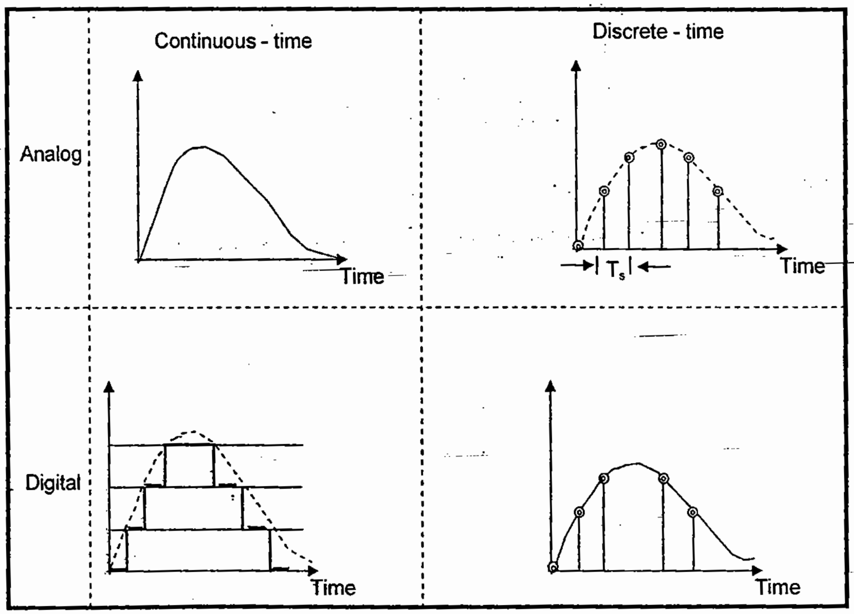
(A) **Continuous – time (CT) or Analog signals:** Continuous in time and continuous in amplitude. Continuous in amplitude means that the amplitude can assume any value in the continuous range form $-\infty$ to $+\infty$.

(B) **Discrete – time (DT) Signals:** Discretised in time and continuous in amplitude

Most DT signals arise from sampling CT signals.

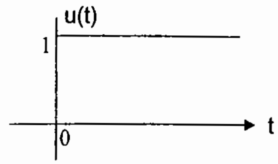
(C) **Digital Signals:** Discretised in time and quantized in amplitude

If the amplitude of a signal can assume only a value from a finite set of numbers, the amplitude is said to be discretised or quantized.



1.1 ELEMENTARY SIGNALS:

(1) Unit step function, u(t):

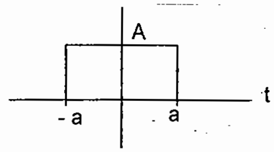


$$u(t) = \begin{cases} 1; t > 0 \\ 0: t < 0 \end{cases}$$

u(t) at t = 0 is discontinuous, u(0) = ½.

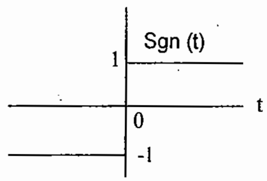
An example of unit step function is the output of 1V dc voltage source in series with a switch that is turned on at t = 0

(2) Rectangular (or Gate) Pulse: A rect (t / 2a)



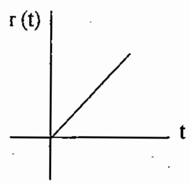
The rectangular function is the result of an ON – OFF switching operation of a constant voltage source in an electrical circuit.

(3) Signum function: Sgn(t)



$$\text{Sgn}(t) = \begin{cases} 1: t > 0 \\ 0: t = 0 \\ -1; t < 0 \end{cases}$$

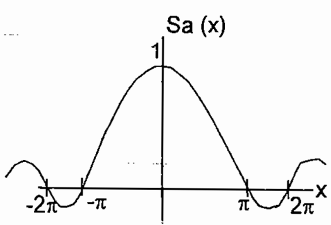
(4) Ramp function: r(t)

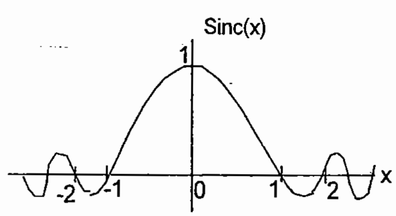


$$r(t) = \begin{cases} t: t > 0 \\ 0: t = 0 \end{cases}$$

An example of a ramp function is the linear – sweep waveform of a cathode – ray tube.

(5) Sampling function: $Sa(x) = \dfrac{\sin x}{x}$ $\quad \text{Sinc}(x) = \dfrac{\sin \pi x}{\pi x}$



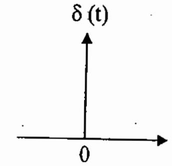


(6) Unit impulse function (or) Dirac delta function: $\delta(t)$

Many physical phenomenon such as point sources, point charges, voltage sources acting for very short time can be modeled as delta functions.

(i) $\delta(0) \to \infty$ (II) $\delta(t) = 0,\ t \neq 0$

(iii) $\int_{-\infty}^{+\infty} \delta(t)\,dt = 1$ (iv) $\delta(-t) = \delta(t)$



(v) Sampling property $\quad \mathbf{x(t)\,\delta(t - t_0) = x(t_0)\,\delta(t - t_0)}$

Eg: $(2t + 1)\,\delta(t - 3) = 7\delta(t - 3)$

(iv) Sifting property

$$\int_{t_1}^{t_2} x(t)\,\delta(t - t_0)\,dt = \begin{cases} x(t_0);\ t_1 < t_0 < t_2 \\ 0:\ \text{elsewhere} \end{cases}$$

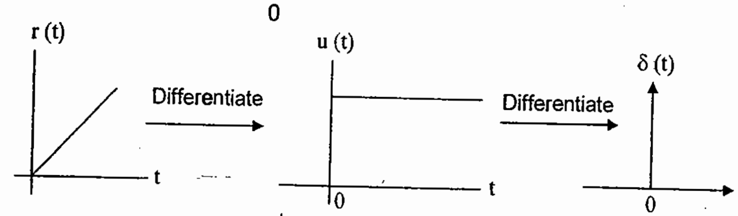
Eg: $\int_{-2}^{4} (t + t^2)\,\delta(t - 3)\,dt = 3 + 3^2 = 12$

P1.1.1: Find the value of the following integrals

(A) $\int_{-1}^{2} (t + t^2)\,\delta(t - 4)\,dt$ (B) $\int_{-2}^{\infty} (t + \cos \pi t)\,\delta(t - 1)\,dt$

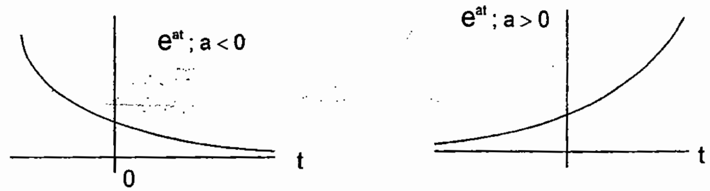
(C) $\int_{0}^{\infty} \text{Cos}\, t\, u(t - 3)\,\delta(t)\,dt$ (D) $\int_{-2}^{\infty} e^{(t-2)}\,\delta(2t - 4)\,dt$

(E) $\int_{0}^{2\pi} t \sin t\,\delta(\pi/2 - t)\,dt$

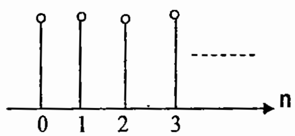


$\delta(t) = \dfrac{d}{dt}\,u(t)$ $\quad u(t) = \int_{-\infty}^{t} \delta(t)\,dt$

(7) Exponential signals:



(8) Unit Step sequence: u(n)

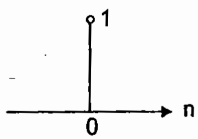


$$u(n) = \begin{cases} 1; & n \geq 0 \\ 0; & n < 0 \end{cases}$$

(9) D.T. impulse function: δ (n)

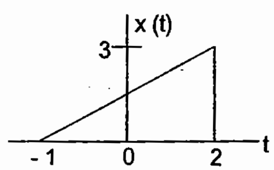
$$u(n) = \sum_{K=-\infty}^{n} \delta(k) = \sum_{K=0}^{\infty} \delta(n-k)$$

$$\delta(n) = u(n) - u(n-1)$$



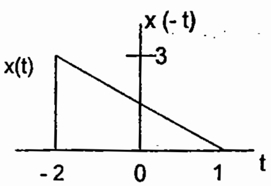
1.2. TRANSFORMATIONS OF THE INDEPENDENT VARIABLE:

Eg.: (1)



$$x(t) = \begin{cases} t+1; & -1 < t < 2 \\ 0; & \text{elsewhere} \end{cases}$$

To get x (- t) reflect x(t) about t = 0

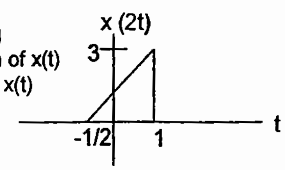


$$x(-t) = \begin{cases} -t+1; \ -1 < -t < 2 \\ \quad 1 > t > -2 \\ \quad \rightarrow -2 < t < 1 \end{cases}$$

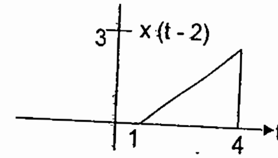
x(at) → Time Scaling

|a| > 1 → Compression of x(t)

|a| < 1 → expansion of x(t)



$$x(2t) = \begin{cases} 2t+1; \ -1 < 2t < 2 \\ \quad -1/2 < t < 1 \end{cases}$$



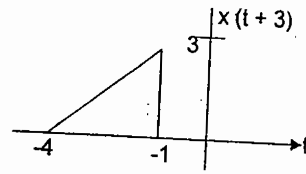
$x(t - t_0)$

$t_0 > 0 \rightarrow$ delay by t_0

$t_0 < 0 \rightarrow$ Advance by t_0

x (t – 2) → shift x (t) right by 2

~~x (t + 3)~~ Shift x(t) ~~left by 3~~



Note: Suppose we want x (-2t + 1), follow the sequence of steps

x (t) → x (t + 1) → x (-t +1) → x (-2t +1)

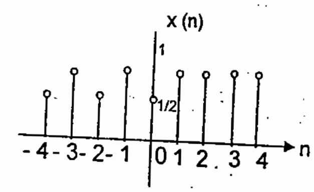
Alternatively x (-2t +1) = x [-2 (t – ½)] follow the following operations

x (t) → x (-t) → x(-2t) → x [-2 (t – ½)]

Eg. (2):

Sketch the waveform

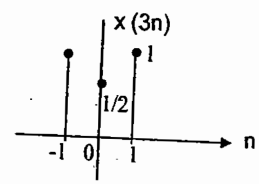
$x\left(-6-3n\right)$

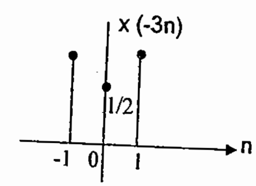


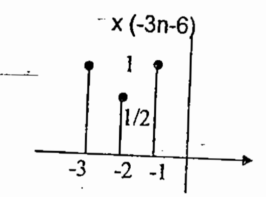
(i) Compress x (n) by a factor of 3 to obtain x (3n)

(ii) Time – reverse x (3n) to obtain x (-3n)

(iii) x[- 3n – 6] = x[- 3 (n+2)] . Shift x (- 3n) left by 2 units.





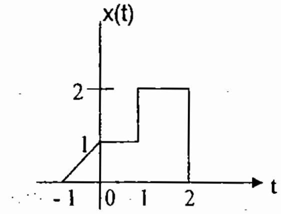


P1.2.1. For the continuous – time signal x (t) shown in figure, sketch the following signals?

(A) $x(t-4)$ (B) $x(2t+1)$

(C) $x(5-t)$ (D) $x(-t-2)$

(E) $[x(t)+x(-t)]u(t)$ (F) $x(t)\,\delta(t-3/2)$

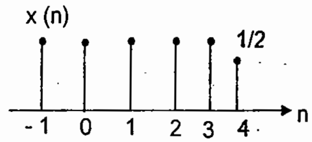


P1.2.2. A discrete – time signal x (n) is shown in figure. Sketch the following signals?

(A) $x[n+3]$ (B) $x[6-n]$

(C) $x[3n+1]$ (D) $x[n/2]$

(E) $x[n]\,u[2-n]$ (F) $x[n-1]\,\delta[n-3]$



P1.2.3. Consider the D.T. signal $x(n) = 1 - \sum_{K=3}^{\infty} \delta[n-1-k]$

Find the values of M and n_0 so that $x(n) = u[Mn - n_0]$

P1.2.4 Sketch the wave forms of the following signals?

(a) $x(t) = u(t+1) - 2u(t) + u(t-1)$

(b) $x(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$ where r(t) is ramp function

1.3. CLASSIFICATION OF SIGNALS:

(1) Energy and Power signals:

→ A signal x (t) (or) x (n) is called an energy signal if total energy has a non – zero finite value $0 < E_x < \infty$

→ A signal is called a power signal if it has non – zero finite power i.e, $0 < P_x < \infty$.

→ A signal can't be both an energy and power signal simultaneously.

→ The term instantaneous power is reserved for the true rate of change of energy in a system. In most cases, when the term power is used it refers to average power i.e, the average rate of energy utilization, a constant quantity independent of time.

$$E_{x(t)} = \underset{T\to\infty}{\mathrm{Lt}} \int_{-T}^{T} |x(t)|^2\, dt$$

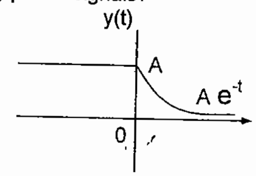
$$P_{avx(t)} = \underset{T\to\infty}{\mathrm{Lt}} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\, dt$$

$$E_{x(n)} = \underset{N\to\infty}{\mathrm{Lt}} \sum_{n=-N}^{+N} |x(n)|^2$$

$$P_{avx(n)} = \underset{N\to\infty}{\mathrm{Lt}} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x(n)|^2$$

P1.3.1 : Determine whether the following signals are energy (or) power signals?

(A) $x(t) = e^{-t}u(t)$ (B) $x(t) = A$ (C)



(D) $x(t) = A\cos(wt+\theta)$ (E) $x(t) = t\,u(t)$

(F) $x(t) = e^{3t}u(t)$

P1.3.2 Consider $x(t) = \delta(t+2) - \delta(t-1)$. Calculate the energy for the signal

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

P1.3.3. If a signal g (t) has energy E, then the signal g(3t) has energy of

(A) E (B) E/3 (C) 3E (D) E/9

P1.3.4. Find whether the following signals are energy (or) power signals?

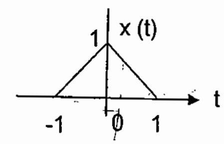
(A) $x[n] = (1/2)^n u[n]$ (B) $x[n] = u[n]$ (C) $x[n] = 2^n u[n]$

2. Even and Odd signals:

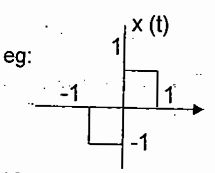
A signal is even if $x(t) = x(-t)$
$x(n) = x(-n)$

A signal is odd if $x(t) = -x(-t)$
$x(n) = -x(-n)$

eg:



eg:



P1.3.5 Find the even and odd parts of the signal $x(t) = u(t)$?

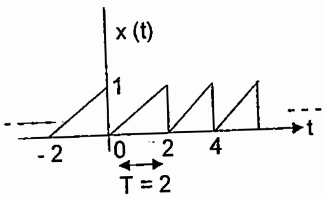
P1.3.6 Find the conjugate anti-symmetric part of $x(n) = \{1 + j2, \underset{\uparrow}{2}, j5\}$

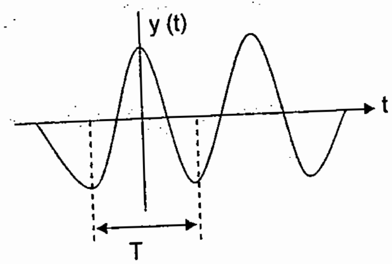
P1.3.7: Suppose even part of a signal x (n) is $x_e(n) = (1/2)^{|n|}$ and total energy in x (n) = 5, find energy in odd part?

(3) Periodic And Non Pereiodic (Or Aperiodic) Signals:

A periodic function is one which has been repeating an exact pattern for an infinite and will continue to repeat that exact pattern for an infinite time.

A signal is periodic if g (t) = g (t + nT) for any integer "n" T → period of a function.





HINT: The sum of harmonic signals $y(t) = x_1(t) + x_2(t) + x_3(t) +$ ______

is periodic with overall period $T = LCM(T_1, T_2, T_3, \ldots)$

→ A discrete signal x (n) is periodic if x [n] = x[n + N] ; Where N → periodic of x[n]

HINT: For finding fundamental period of discrete sinusoid (or) Complex exponentials always use the equation $\omega_0 / 2\pi = m / N$

P1.3.8. Determine which of the following signals are periodic, if periodic find the fundamental period?

(A) $x(t) = \cos(18\pi t) + \sin(12\pi t)$ (B) $x(t) = \sin(2\pi t/3)\cos(4\pi t/5)$

(C) $x(t) = \cos 3t + \sin 5\pi t$ (D) $x(t) = je^{j10t}$

(E) $x(t) = \cos 5t u(t)$ (F) $x(t) = Ev[\cos(2\pi t)\, u(t)]$

(G) $x(n) = \sin\left(\frac{5\pi n}{3}\right)$

(H) $x(n) = 2\cos\left(\frac{\pi n}{4}\right) + \sin\left(\frac{\pi n}{8}\right) - 2\cos\left(\frac{\pi n}{2} + \frac{\pi}{6}\right)$

(I) $x(n) = e^{j7\pi n}$ (J) $x(n) = \cos(n/4)\sin\frac{\pi n}{8}$

(K) $x(n) = u(n) + u(-n)$ (L) $x(n) = (-1)^n$

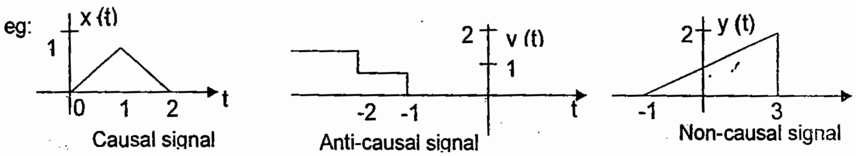
(M) $x(n) = \sum_{K=-\infty}^{+\infty} \delta(n-4k) - \delta(n-1-4k)$

P1.3.9: A signal $x(t) = 2\cos(150\pi t + 30^0)$ is sampled at 200Hz. Find the fundamental period of discrete signal?

(4) Causal & Noncausal Signals:

A signal is causal if x (t) = 0 for t < 0
x (n) = 0 for n < 0

eg:



→ Multiplying the signal by the unit step ensures that the resulting signal is causal.

5. Deterministic and Random signals:

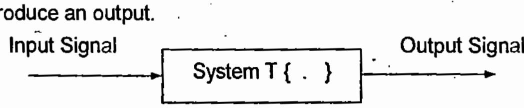
→ If the value of a signal can be predicted for all time (t or n) in advance without any error, is a deterministic signal. Eg: $x(t) = e^{-t}u(t)$, x (t) = rect (t)

→ Signals whose values can't be predicted with complete accuracy for all time are known as random signal.

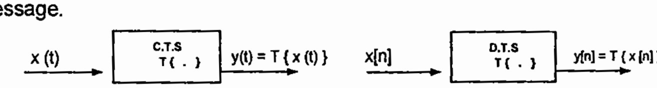
→ Random signals are generally characterized by mean, mean square value.
Eg: Thermal noise generated by a resistor. The intensity of the thermal noise depends on the movement of electrons and can't be predicted accurately

1.4 Systems And Classification:

A System, designated T, is a mathematically describable operation or transformation that acts on an input signal to produce an output.



Eg: In a communication system, the input signal could be a speech signal, the system is combination of Tx, channel and Rx, output signal is an estimate of information contained in the original message.



→ All physical systems are referred to a zero – energy state (or) ground state (or) relaxed state at t = - ∞

→ The behavior of a system is governed not only by the input but also by the state of the system at the instant at which the input is applied

1. Linear And Non Linear System:

For the system to be linear, it should satisfy 2 properties

(A) $x_1(t) \rightarrow y_1(t)$ and $x_2(t) \rightarrow y_2(t)$

Then $x_1(t) + x_2(t) \rightarrow y_1(t) + y_2(t) \rightarrow$ Additivity

(B) $cx(t) \rightarrow cy(t) \rightarrow$ Scaling (or) Homogenity

(or)

$ax_1(t) + bx_2(t) \rightarrow ay_1(t) + by_2(t) \rightarrow$ Super position

P1.4.1 Determine which of the following systems is linear?

(A) $y(t) = x(t)\, x(t-2)$

(B) $y(t) = \sin\{x(t)\}$

(C) $y(t) = d/dt\, x(t)$

(D) $y(t) = 2x(t) + 3$

(E) $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

(F) $y(t) = x^2(t)$

(G) $y(t) = x(t) \cos \omega_0 t$

(H) $y(n) = \log\{x[n]\}$

(I) $y(n) = |x[n]|$

(J) $y(n) = x^*(n)$

(K) $y(n) = \text{median}\{x[n]\}$

(L) $y(n) = \dfrac{x(n)}{x(n-1)}$

(M) $\dfrac{d^2y(t)}{dt^2} + \dfrac{5\, dy(t)}{dt} + 2 = x(t)$

(N) $y(n) = e^{x(n)}$

P1.4.2 If 2 systems S_1 and S_2 represents nonlinear systems, will the cascade be nonlinear?

2. Time – Invariant (Shift – Invariant) & Time – Variant (Shift – Dependent) Systems:

A system is T.I. if the input output characteristic don't change with time. T.I. implies that the shape of the response y (t) depends only on the shape of the input x (t) and not on the time when it is applied.

For T.I system if $x(t) \rightarrow y(t)$ Then $x(t - t_0) \rightarrow y(t - t_0)$

P1.4.3 Test the following systems for time – invariance?

(a) $y(t) = tx(t) + 3$

(b) $y(t) = e^{x(t)}$

(c) $y(t) = x(t) \cos 3t$

(d) $y(t) = \sin\{x(t)\}$

(e) $y(t) = d/dt\, x(t)$

(f) $y(t) = x^2(t)$

(g) $y(t) = x(2t)$

(h) $y(n) = 2^{x(n)}\, x(n)$

(i) $y(n) = x(n+2) - x(7-n)$

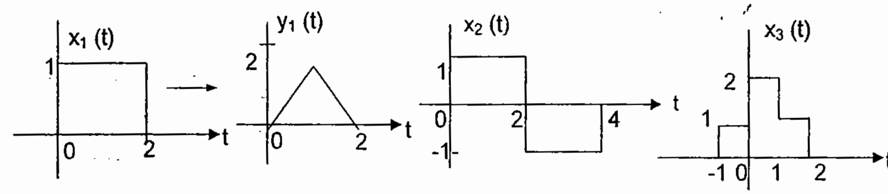
(j) $\dfrac{d^2 y(t)}{dt^2} + y(t)\, y(3t) = x(t)$

(k) $\dfrac{d^2 y(t)}{dt^2} + 3y(t) = 2x(t)$

Q) If 2 shift-variant systems are connected in series, Is the overall system will be shift-varying?

P1.4.4 Consider a system with input x (n) and output y (n) related as $y(n) = x(n)\,[g(n) + g(n-1)]$. Check for time – invariance if (i) $g(n) = 1\ \forall n$ (ii) $g(n) = n\ \forall n$ (iii) $g(n) = 1 + (-1)^n\ \forall n$

P1.4.5: Consider an LTI system whose response to the input signal $x_1(t)$ is $y_1(t)$ as shown in figure. Find the response of the system due to the Input $x_2(t)$ and $x_3(t)$?



3. Causal & Non Causal System:

A causal (or) non – anticipating system is one whose present response does not depend on future values of the input (or) A system is causal if its output at $t = t_0$ depends on the values of the input in the past $t \le t_0$ and doesn't require future value of input ($t > t_0$)

P1.4.6. Check whether the following systems are causal (or) non causal?

(a) $y(t) = (2t + 3)\, x(t)$

(b) $y(t) = x^2(t)$

(c) $y(t) = x(t) \sin 5t$

(d) $y(t) = x\{(\sin t)\}$

(e) $y(t) = |x(t)|$

(f) $y(n) = e^{x(n)}$

(g) $y(n) = \sum_{K=n_0}^{n} x[k]$, "n_0" is finite

(h) $y(n) = \sum_{K=n-n_0}^{n+n_0} x[k]$ "n_0" is finite

(i) $y(n) = \sum_{K=-\infty}^{n} x(k)$

(j) $y(n) = \sum_{K=0}^{n} x(k)$

(k) $y(n) = \dfrac{1}{2m+1} \sum_{K=-m}^{+m} x[n-k]$

(l) $y(t) = x(\alpha t)$

(m) $y^1(t+4) + 2y(t) = x(t+2)$

(n) $y^1(t) + 2y(t) = x(t+3)$

4. Static (Or) Memoryless And Dynamic (With Memory) System:

A system is static if its output at $t = t_0$ depends only on the value of the input at $t = t_0$ and no other value of the input signal.

P1.4.7. Check whether the following systems are static or dynamic?

(a) $y(t) = 3x(t) + 5$ (b) $y(t) = x^2(t)$ (c) $y(n) = e^{x(n)}$

(d) $y(n) = \cos\{x(n)\}$ (e) $y(n) = x[3n]$ f) $y(t) = d/dt\ x(t)$

(g) $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$

Q) All memory less systems are shift-invariant (TRUE/FALSE)

5. Stable And Unstable System:

A system is said to be BIBO stable if and only if every bounded input results in a bounded output If $|x(t)| \le M_x < \infty$ then $|y(t)| \le M_y < \infty$

→ A bounded signal has an amplitude remains finite.

P1.4.8: Check whether the following systems are stable (or) not?

(a) $y(t) = x^2(t)$ (b) $y(t) = x(t)\cos 3t$ (c) $y(t) = x(t-3)$

(d) $y(t) = d/dt\ x(t)$ (e) $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$ (f) $y(t) = \sin\{x(t)\}$

(g) $y(t) = t\,x(t)$ (h) $y(n) = e^{x(n)}$ (i) $y(n) = x[3n]$

(j) $y(n) = \sum_{K=n_0}^{n} x[k]$, "n_0" is finite

6. Invertible And Inverse System :

A system is said to be invertible if the input of the system can be recovered from the output

$$x(t) \longrightarrow \boxed{T\{\text{-}\}} \xrightarrow{y(t)} \boxed{T^{-1}\{\text{-}\}} \longrightarrow x(t)$$

$$T.T^{-1} = I$$

In any event, a system is not invertible unless distinct inputs applied to the system produces distinct outputs

P1.4.9: Determined which of the following systems are invertible?

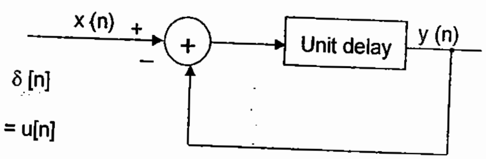
(a) $y(t) = x^2(t)$ (b) $y(t) = |x(t)|$

(c) $y(t) = x(t-3)$ (d) $y(t) = d/dt\ x(t)$

(e) $y(n) = x[n]x[n-1]$ (f) $y(n) = nx(n)$

(g) $y(n) = x[n] - x[n-1]$ (h) $y(n) = \sum_{K=-\infty}^{n} x[k]$

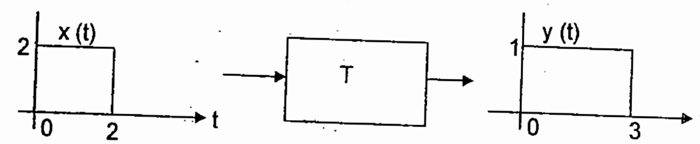
P1.4.10 : Consider the feedback system shown in figure, assume that $y[n] = 0$ for $n < 0$



Find y(n) when the input is (i) $x(n) = \delta[n]$

(ii) $x(n) = u[n]$

P1.4.11. A system has input – output pairs given. Determine whether the system could be memory less, causal, linear and T.I?



1.5 Previous Questions:

(1) Which one of the following Paris is NOT correctly matched (Input x (t) and output y (t) **IES**

(a) Unstable system $dy(t)/dt - 0.1\,y(t) = x(t)$ (b) N.L. system $dy(t)/dt + 2t^2 y(t) = x(t)$

(c) Non causal system $y(t) = x(t+2)$ (d) Non dynamic system $y(t) = 3x^2(t)$

(2) The discrete – time equation $y(n+1) + 0.5n\,y(n) = 0.5\,x(n+1)$ is not attributable to a ______ **IES**

(A) memory less system (B) T.I. (C) linear (D) Causal system

(3) Find whether the following systems are linear, T.I. and dynamic **IES**

(A) $\frac{2t\,dy(t)}{dt} + 4y(t) = 2t\,x(t)$ (B) $\frac{d^2y(t)}{dt^2} + 4y(t) = 2x(t)$

(C) $4\frac{d^2y(t)}{dt^2} + \frac{2\,dy(t)}{dt} + y(t) = \frac{3d}{dt}x(t)$ (D $\left(\frac{dy(t)}{dt}\right)^2 + 2t\,y(t) = \frac{4dx(t)}{dt}$

4. Match the following

List I (System)	List II (system category) IES
(A) $y(n+2) + y(n+1) + y(n) = 2x(n+1) + x(n)$	(1) Linear, T.V., dynamic
(B) $n^2 y^2(n) + y(n) = x^2(n)$	(2) Linear, T.I., dynamic
(C) $y(n+1) + ny(n) = 4nx(n)$	(3) N.L., T.V., dynamic
(D) $y(n+1)\, y(n) = 4\, x(n)$	(4) N.L., T.I., dynamic
	(5) N.L., T.V., memory less.

5. The input x (t) and output y(t) of a C.T.S related with the following equations. Which system is causal? GATE

(A) $y(t) = x(t-2) + x(t+4)$ (B) $y(t) = (t-4)\, x(t+1)$
(C) $y(t) = (t+4)\, x(t-1)$ (D) $y(t) = (t+5)\, x(t+5)$

6. Match the correct pair GATE

List I (System)	Lists II (Properties)
R1: $y(t) = t^2 x(t)$	P1: linear but not T.I
R2: $y(t) = t\,\lvert x(t)\rvert$	P2: T.I. but N.L.
R3: $y(t) = \lvert x(t)\rvert$	P3: LTI
R4: $y(t) = x(t-5)$	P4: N.L. and but not T.I

7. The power in the signal $x(t) = 8\cos(20\pi t - \pi/2) + 4\sin(15\pi t)$ is GATE

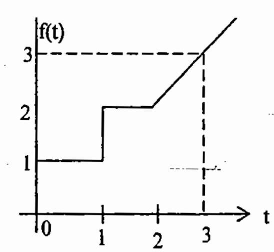
(A) 40 (B) 41 (C) 42 (D) 82

8. The system represented by I/p-o/p relationship $y(t) = \int_{-\infty}^{5t} x(\tau)\,d\tau,\ t > 0$ is ______ GATE

(a) LTI, Causal (b) linear, T.V, Causal
(c) linear, T.V, non causal (d) linear, T.I, non causal

9. Which one of the following gives correct description of the waveform shown in the figure?IES

(a) $u(t) + u(t-1)$
(b) $u(t) + (t-1)\,u(t-1)$
(c) $u(t) + u(t-1) + (t-2)\,u(t-2)$
(d) $u(t) + (t-2)\,u(t-2)$



10. What is the period of $x[n] = 5\cos[0.2\pi n]$ IES

(a) 10 (b) 5 (c) 1 (d) 0

1.6 PRACTICE PROBLEM SET:

P1.6.1. Evaluate the following integrals?

(a) $\int_{-\infty}^{+\infty} \text{Sin}[3(t-1)]\ \delta(2t+4)\,dt$ (b) $\int_{-\infty}^{+\infty} \delta(at-b)\ \sin^2(t-4)\,dt$ for $a > 0$

(c) $\int_{6}^{\infty} (t-1)\ \delta(t-5)\,dt$ (d) $\int_{-\infty}^{+\infty} \left(\frac{2t}{3} - \frac{3}{2}\right) \delta(t-1)\,dt$

(e) $\int_{-\infty}^{+\infty} e^{-(5t+1)}\,\dot{\delta}(t-5)\,dt$ (f) $\int_{-\infty}^{+\infty} (t^3+4)\ \delta(1-t)\,dt$

(g) $\int_{-\infty}^{+\infty} e^{-(x-1)}\,\text{Cos}[\pi/2\,(x-5)]\ \delta(x-3)\,dx$ (h) $\int_{-\infty}^{+\infty} e^{-\alpha t^2}\,\dot{\delta}(t-10)\,dt$

P1.6.2. Let $x(t) = \begin{cases} -t+1; & -1 \le t \le 0 \\ t & ;0 \le t < 2 \\ 2 & ;2 \le t < 3 \\ 0 & ;\text{elsewhere} \end{cases}$

Sketch the signals $x(t)$, $x(t-2)$, $x(t+3)$, $x(-3t-2)$, $x\left(\frac{2t}{3} + \frac{1}{2}\right)$

P1.6.3. Given $x(t) = 3\,u(t+3) - u(t) + 3\,u(t-3) - 5\,u(t-6)$. Sketch the signal $x(-3t-6)$

P1.6.4. Consider the D.T. signal $x(n) = n\left[u(n+5) - u(n-5)\right]$

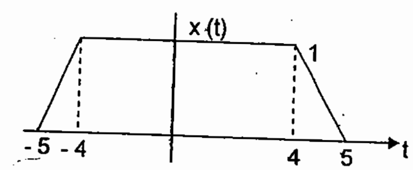
Sketch the following signals

(a) $x(2n)$ (b) $x(3-n)$ (c) $x\left(\frac{3-n}{3}\right)$ (d) $x(n)\,u(-n)$ (e) $x(4-3n)$

P1.6.5. Determine whether the following signals are energy (or) power signals (or) neither?

(a) $x(t) = e^{-3t}$ (b) $x(t) = e^{-j3t}$
(c) $x(t) = \cos(\pi t)\sin(3\pi t)$ (d) $x(t) = u(t+3) - u(t-5)$
(e) $x(t) = te^{-3t}\,u(t)$ (f) $x(n) = \text{Cos}\left(\frac{\pi n}{4}\right)$
(g) $x(n) = (-1)^n$ (h) $x(n) = n\,u(n-2)$
(i) $x(n) = A\delta(n)$

P1.6 6. (a) Find the energy in x (t)
(b) Find the energy in $y(t) = \frac{d\,x(t)}{dt}$



P1.6.7. Find whether the following signals are periodic (or) not, if periodic find the period?

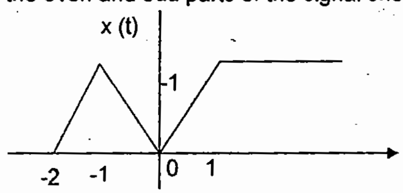
(a) $x(t) = 2e^{j(t+\pi/4)}u(t)$ (b) $x(t) = \cos^2(2\pi t)$

(c) $x(t) = \sum_{K=-\infty}^{+\infty}(-1)^K \delta(t-2k)$ (d) $y(t) = \cos(2\pi t) + \sin(3\pi t) + \cos(5\pi t - 3\pi/4)$

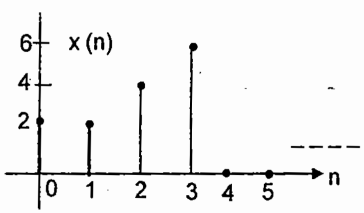
(e) $x(n) = \cos\left(\frac{n}{8} - \pi\right)$ (f) $x(n) = \cos\left(\frac{\pi n^2}{4}\right)$

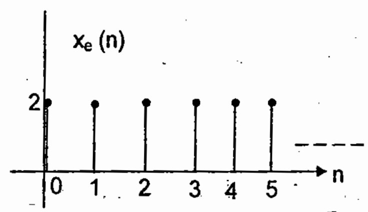
(g) $x(n) = \cos\left(\frac{\pi n}{5}\right)\sin\left(\frac{\pi n}{3}\right)$ (h) $x(n) = e^{j5\pi n/7}$

P1.6.8. Sketch the even and odd parts of the signal shown in figure?



P1.6.9. Given in figure are the parts of a signal x (n) and its even part $x_e(n)$ only for $n \geq 0$ and $x_e(n) = 2$; $n \geq 0$ complete the plots of x(n) and $x_e(n)$ and give a plot of the odd part, $x_o(n)$ of the signal.





P1.6.10. Sketch the wave forms of the following signals?

(a) $x(t) = u(t) - u(t-2)$

(b) $x(t) = r(t+1) - r(t) + r(t-2)$

(c) $x(t) = u(-t+2) + r(t+1) - r(t-1)$

(d) $y(t) = 3u(t+3) - r(t+2) + 2r(t) - 2u(t-2) - r(t-3) - 2u(t-4)$

P1.6.11. The signal

$$x(n) = \begin{cases} 1; & n = 1 \\ -1; & n = -1 \\ 0; & n = 0 \ \& \ |n| > 1 \end{cases}$$ Find the signal $y(n) = x(n) + x(-n)$

P1.6.12. The following input – output Pairs have been observed during the operation of a linear system

$x_1(n) = \{-1, \underset{\uparrow}{2}, 1\} \xleftrightarrow{T} y_1(n) = \{1, \underset{\uparrow}{2}, -1, 0, 1\}$

$x_2(n) = \{1, \underset{\uparrow}{-1}, -1\} \xleftrightarrow{T} y_2(n) = \{-1, \underset{\uparrow}{1}, 0, 2\}$

$x_3(n) = \{0, \underset{\uparrow}{1}, 1\} \xleftrightarrow{T} y_3(n) = \{\underset{\uparrow}{1}, 2, 1\}$

Can you draw any conclusions about the time – invariance of the system?

Hint : $x_3(n) = x_1(n) + x_2(n) \longrightarrow y_3(n) = y_1(n) + y_2(n)$

P1.6.13. Consider 2 system (i) $y(t) = x(\alpha t)$ (ii) $y(t) = x(t+\alpha)$

For what values of 'α' each system is linear, T.I, causal and memory less?

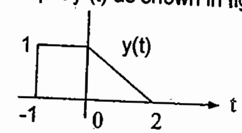
P1.6.14 Consider a system with input x (n) & output y(n) that satisfy the D.E. $y(n) = ny(n-1) + x(n)$. The system is causal

(i) If the Input is $x(n) = \delta(n)$, find y (n) (ii) Is the system LTI?

P1.6.15. For an LTIC system, an Input x (t) produces an output y (t) as shown in figure. Sketch the outputs for the following inputs?

(a) 5x (t) (b) $x(t+1) - x(t-1)$

(c) $\frac{dx(t)}{dt} + 3x(t)$



P1.6.16. Consider a D.T. system with Input x (n) and output y (n). The input – output relationship for system is $y(n) = x(n)\,x(n-2)$

(a) Is the system memoryless?

(b) Determine y (n), When $x(n) = A\,\delta(n)$?

(c) Is the system invertible?

P1.6.17. Determine which of the following system is

(1) memoryless (2) T.I. (3) Linear (4) Causal (5) stable

(a) $y(t) = \int_{-\infty}^{2t} x(\tau)\,d\tau$ (b) $y(t) = \begin{cases} 0 & ; t < 0 \\ x(t) + x(t-2) & ; t \geq 0 \end{cases}$

(c) $y(t) = x(t/2)$ (d) $y(t) = d/dt\,\{e^{-t}x(t)\}$

(e) $y(n) = \log_{10}|x(n)|$ (f) $y(n) = \begin{cases} x[n] & ; n \geq 1 \\ 0 & ; n = 0 \\ x[n+1] & ; n \leq -1 \end{cases}$

(g) $y(n) = \cos[2\pi x[n+1]] + x(n)$ (h) $y(n) = x[4n+1]$

(i) $y(n) = \begin{cases} x(n) & \text{if } x(n) \geq 0 \\ 0 & \text{if } x(n) < 0 \end{cases}$ (j) $y(n) = \text{sign}[x(n)]$.

Ans: (b) linear, causal, stable (f) linear, stable
(g) memory, stable, no causal, nonlinear, T.I.. (i) memoryless, N.L., T.I, C,S.

P1.6.18. Find whether the following systems are invertible (or) not?

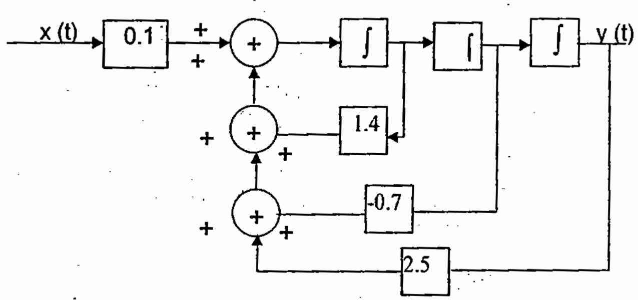
(1) $y(t) = u\{x(t)\}$

For many values of x (t), response is zero and there are many values of x (t), that all cause y (t) of one. ∴Non invertible

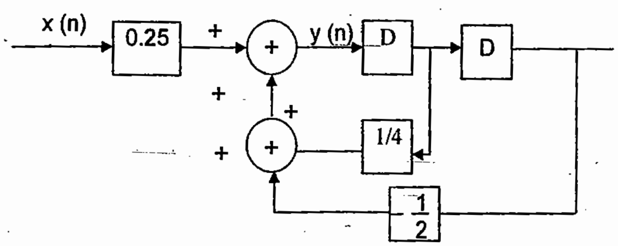
(2) $y(t) = x(t-5) - x(3-t) \rightarrow$ non invertible

(3) $y(t) = x(t)\cos(2\pi t) \rightarrow$ non invertible

P1.6.19. Show that the system shown in figure is linear, T.I unstable, dynamic, invertible?



20. Show that the system is linear, T.I., stable, causal; dynamic, invertible?



USEFUL MATHEMATICAL RELATIONS:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

$$\sum_{K=0}^{N-1} \alpha^n = \frac{1-\alpha^N}{1-\alpha}$$

$$\int \text{Sin}(ax)\,dx = \frac{-\text{Cos}\,ax}{a}$$

$$\sum_{K=0}^{+\infty} \alpha^k = \frac{1}{1-\alpha},\ |\alpha| < 1$$

$$\int \text{Cos}(ax)\,dx = \frac{\text{Sin}\,ax}{a}$$

$$\sum_{K=1}^{+\infty} K\alpha^k = \frac{\alpha}{(1-\alpha)^2},\ |\alpha| < 1$$

$$\int \frac{1}{a^2+b^2x^2}\,dx = \frac{1}{ab}\,\text{Tan}^{-1}\left(\frac{bx}{a}\right)$$

$$\sum_{K=1}^{N} K = \frac{N(N+1)}{2}$$

$$\int x\,\text{Sin}(ax)\,dx = \frac{\text{Sin}(ax) - ax\,\text{Cos}(ax)}{a^2}$$

$$\sum_{K=1}^{N} K^2 = \frac{N(N+1)(2N+1)}{6}$$

$$\int x\cos(ax)\,dx = \frac{\text{Cos}(ax) + ax\,\text{Sin}(ax)}{a^2}$$

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}},\ a > 0$$

$$\int_0^\infty \text{sinc}\,x\,dx = \int_0^\infty \text{sinc}^2 x\,dx = \frac{1}{2}$$

REVIEW NOTES:

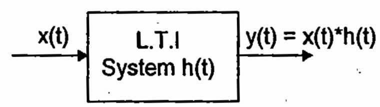
CHAPTER 2. L.T.I (L.S.I) SYSTEMS

→ An L.T.I System is always considered w.r.t impulse response denoted as h(t) or h(n).
→ If the Input is impulse, then the output is impulse response.
→ SIFTING property states that any signal can be produced as a combination of impulses.
→ Convolution may be regarded as a method of finding the zero – state response of a relaxed LTI system.
→ Any DT signal is the sum of scaled and shifted unit impulses.

$$\boxed{x[n] = \sum_{k=-\infty}^{+\infty} x[k]\,\delta[n-k]}$$

→ Convolution may be treated as flip – shift – multiply – time – area method.

CONTINUOUS CONVOLUTION



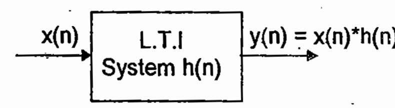
$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\,h(t-\tau)\,d\tau$$

$$= \int_{-\infty}^{+\infty} x(t-\tau)\,h(\tau)\,d\tau$$

Steps: -

1. $x(t) \rightarrow x(\tau)$, $h(t) \rightarrow h(\tau)$
2. Folding: $x(-\tau)$, $h(-\tau)$
3. Shifting: $x(t-\tau)$, $h(t-\tau)$
4. Multiplication: $x(t-\tau)h(\tau)$, $h(t-\tau)x(\tau)$
5. Integration

DISCRETE CONVOLUTION



$$y(n) = \sum_{K=-\infty}^{+\infty} x[k]\,h[n-k]$$

$$= \sum_{K=-\infty}^{+\infty} x[n-k]\,h[k]$$

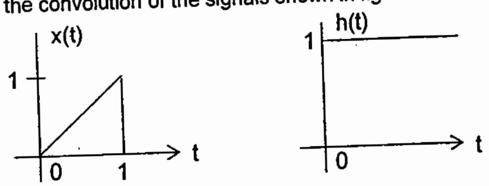
Steps: -

1. $x[n] \rightarrow x[k]$, $h(n) \rightarrow h[k]$
2. Folding: $x[-k]$, $h[-k]$
3. Shifting: $x[n-k]$, $h[n-k]$
4. Multiplication: $x[k]\,h[n-k]$, $x[n-k]\,h[k]$
5. Summation

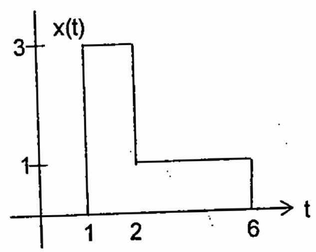
→ In the convolution integral time "t" determines relative location of $h(t-\tau)$ w. r. t. $x(\tau)$. The convolution will yield a non zero result only for those value of "t" over which $h(t-\tau)$ & $x(\tau)$ overlap.

P2.1.1 Find the convolution of the signals $x(t) = e^{-3t}u(t)$ & $h(t) = u(t-2)$

P2.1.2 Find the convolution of the signals shown in figure?

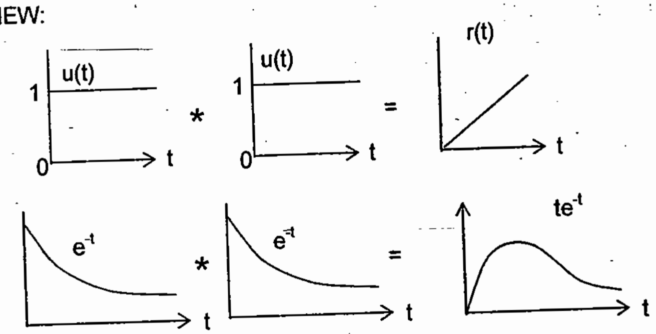


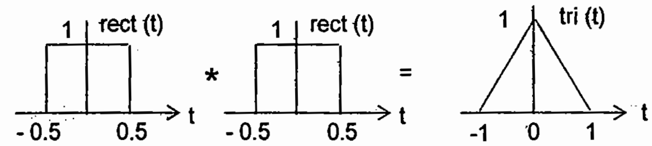
P2.1.3 An L.T.I system is having impulse response $h(t) = u(-t-1)$ for which the Input signal applied is shown in figure. Find the output at $t = 4$ & $t = 0.5$?



P2.1.4. Suppose $z(t) = \int_{-\infty}^{+\infty} x(-\tau + a)\, h(t+\tau)\, d\tau$. Express $z(t)$ in terms of $y(t) = x(t) * h(t)$.

REVIEW:





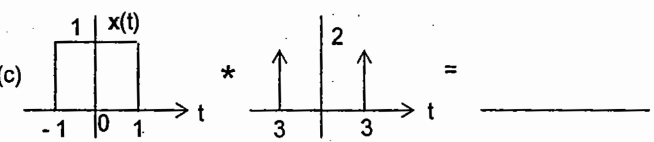
Convolution Property Of Continuous Impluse: - $\boxed{x(t) * \delta(t-t_0) = x(t-t_0)}$

P2.1.5 Find the following terms

(a) $x(t+5) * \delta(t-7) =$ ________

(b) $x(t) * \delta(at+b) =$ ________

(c)



P2.1.6. Explain the difference between each of the following operations ?

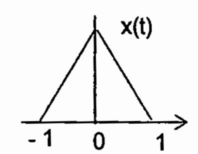
(a) $[e^{-t}u(t)]\,\delta(t-1)$

(b) $\int_{-\infty}^{+\infty} e^{-t}u(t)\,\delta(t-1)\,dt$

(c) $e^{-t}u(t) * \delta(t-1)$

P2.1.7. Let $x(t) = u(t-3) - u(t-5)$ & $h(t) = e^{-3t}u(t)$. Find $\frac{dx(t)}{dt} * h(t)$?

P2.1.8. An Input signal x(t) shown in figure is applied to the system with impulse response $h(t) = \sum_{n=-\infty}^{\infty} \delta(t-3n)$. Find the Output.



P2.1.9. Suppose the Input signal is $x(t) = u(t+0.5) - u(t-0.5)$ & the impulse response is $h(t) = e^{j\omega_0 t}$. If $y(t) = x(t) * h(t)$, find a value of ω_0 which ensures that $y(0) = 0$.

Discrete Convolution.

P2.1.10. Consider the signal $h(n) = \left(\frac{1}{2}\right)^{n-1}\{u(n+3) - u[n-10)\}$

Such that $h(n-k) = \begin{cases} \left(\frac{1}{2}\right)^{n-k-1} & ; A \le K \le B \\ 0 & ; \text{elsewhere} \end{cases}$ Find A & B?

P2.1.11 A linear system with Input x(n) & output y(n) related as

$$y(n) = \sum_{K=-\infty}^{+\infty} x(k)\; g(n-2k) \quad \text{where } g(n) = u(n) - u(n-4). \text{ Find } y(n) \text{ when } x(n) = \delta(n-2)$$

P2.1.12. Find the convolution of $x(n) = \left(\frac{1}{2}\right)^{n-2} u(n-2)$ & $h(n) = u(n+2)$

CONVOLUTION PROPERTY OF DISCRETE IMPULSE: - $\boxed{x(n) * \delta(n - n_0) = x(n - n_0)}$

Sliding strip method: -

Eg. Find the convolution of x(n) = {4, 1, 3} & h(n) = {2, 5, 0, 4}

h(k) **2** 5 0 4
x(- k) 3 1 **4**
y(0) = 8

⇒

2 **5** 0 4
3 **1** **4**
y(1) = 2 + 20 = **22**

⇒

2 **5** **0** 4
3 1 4
y(2) = 6 + 5 + 0= **11**

2 **5** **0** **4**
3 **1** 4
y(3) = 15 + 0 +16 = **31**

⇒

2 5 **0** 4
3 **1** 4
y(4) = 0 + 4 = 4

⇒

2 5 0 **4**
3
y(5) = 12

∴ y(n) = { 8, 22, 11, 31, 4, 12}

P2.1.13 Find the convolution of x(n) = {1, 2, 3, 4} & h(n) = {1, 2, 1, - 1}

P2.1.14. If the output of a system is y(n) = {1, 4, 8, 11, 9, 1, - 4} & Input signal is x(n) = {1, 2, 3, 4} find the impulse response?

P.2.15. Find the periodic convolution of x(n) = {1, 2, 3, 4} & h(n) = {1, 2, 1, -1}

2.2 PROPERTIES OF L.T.I SYSTEM: -

Causality: -

$h(t) = 0;\ t < 0$

$h(n) = 0;\ n < 0$

Stability: -

$$\int_{-\infty}^{+\infty} |h(\tau)|\, d\tau < \infty$$

$$\sum_{n=-\infty}^{+\infty} |h(n)| < \infty$$

Memory less : -

$h(t) = 0$ for $t \ne 0$

$h(n) = 0$ for $n \ne 0$

Invertibility & Inverse : -

$h(t) * h_{inv}(t) = \delta(t)$

$h(n) * h_{inv}(n) = \delta(n)$

P2.2.1 Find whether the following systems are causal & stable?

(a) $h(t) = e^{-4t}u(t-2)$ (b) $h(t) = e^{-6|t|}$
(c) $h(t) = e^{2t}u(-1-t)$ (d) $h(n) = 5^n u(n-3)$
(e) $h(n) = u(n+3) - 2u(n-2) + u(n-4)$

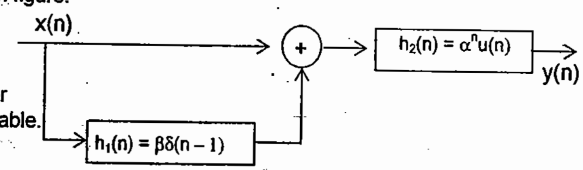
P2.2.2: - Consider a D.T system 'S_1', with I.R $h(n) = (1/5)^n u(n)$

(a) Find 'A' such that $h(n) - Ah(n-1) = \delta(n)$
(b) using result from part (a), determine the I.R g(n) of an LTI system S_2 which is inverse of S_1

P2.2.3: - Consider the system in figure.

(a) Find I.R. of overall system.

(b) Is this system causal? Under what condition the system is stable.



P2.2.4:- The range of a and b for the impulse response $h(n) = \begin{cases} a^n & ; n \ge 0 \\ b^n & ; n < 0 \end{cases}$ to be stable is ________

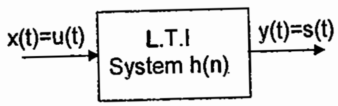
a) |a|<1, |b| < 1 b) |a|>1, |b| < 1
c) |a|>1, |b| > 1 d) |a|<1, |b| > 1

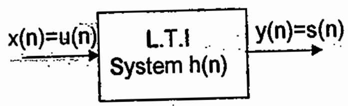
P2.2.5:- Given $h(t) = e^{\alpha t}u(t) + e^{\beta t}u(-t)$. For what values of α and β system is stable?

a) α < 0, β < 0 b) α ≤ 0, β > 0 c) α > 0, β > 0 d) α > 0, β < 0

2.3 STEP RESPONSE OF AN L.T.I SYSTEM: -

→ Step response is the response when the Input is unit step function.





$$s(t) = \int_{-\infty}^{t} h(\tau)\, d\tau \qquad s(n) = \sum_{k=-\infty}^{+n} h(k)$$

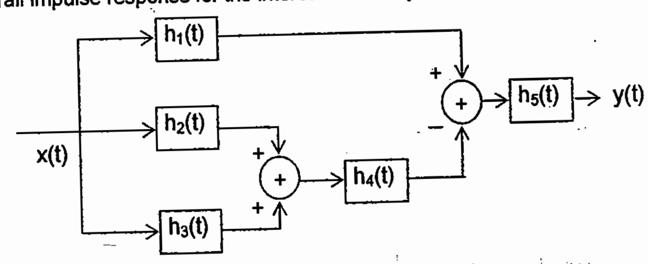
$$h(t) = \frac{d}{dt} s(t) \qquad h(n) = s(n) - s(n-1)$$

P2.3.1 Find the step response of the system if the impulse response is $h(n) = (0.5)^n u(n)$

P2.3.2 Find the impulse response of the system if the step response is $s(t) = \cos \omega_0 t u(t)$

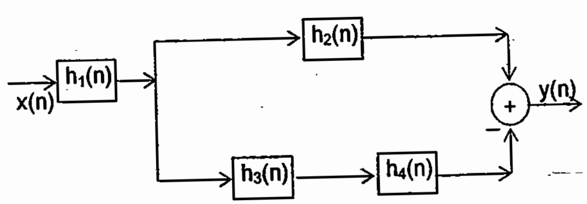
P2.3.3 - An LTI system with Input u(n) produces the output as δ(n), then find the output due to the input nu(n)?

P2.3.4 Find the overall impulse response for the interconnected system shown in figure?



P2.3.5 Consider the inter connection of LTI systems shown in figure. Find h(n) when

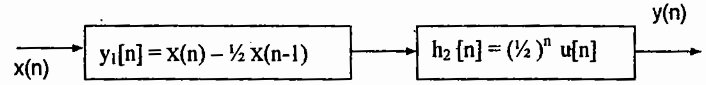
$$h_1(n) = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{2}\right\}$$



$h_2(n) = h_3(n) = u(n + 2)$

$h_4(n) = \delta(n - 2)$

P2.3.6 For the interconnected system shown in Fig. find the overall impulse response.



P2.3.7 Determine whether each of the following statements are TRUE (or) FALSE. Justify your answers

1) The cascade of a non causal LTI system with casual one is necessarily noncausal
2) If an LTI system is causal, it is stable
3) If h(t) is the I.R. of an LTI system which is periodic & nonzero, the system is unstable.

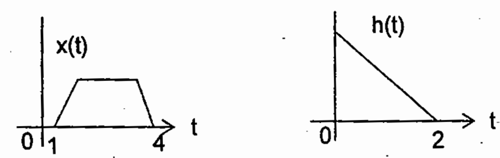
2.4 PREVIOUS QUESTIONS: -

P2.4.1: - Given x = [a, b, c, d] as the Input to an LTI system produces an output y = [x, x, x, x, ... repeated N times]. The impulse response of the system is______ GATE

(A) $\sum_{i=0}^{N-1} \delta[n-4i]$ (B) u(n) – u(n – N)

(C) u(n) – u (n – N – 1) (D) $\sum_{i=0}^{N-1} \delta[n-i]$

P2.4.2 Fig – shows respectively the Input x(t) to a LTI system & the I.R. h(t) of the system GATE



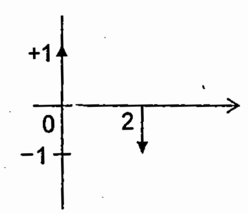
The output of the system is zero every where except for the time interval

(A) 0 < t < 4 (B) 0 < t < 5 (C) 1 < t < 5 (D) 1 < t < 6

P2.4.3 The I.R. of a system is h(t) = δ(t – 0.5). If 2 such systems are cascaded, the I.R. of overall system will be IES

(A) 0.5 δ(t – 0.25) (B) δ (t – 0.25) (C) δ(t – 1) (D) 0.5 δ(t – 1)

P2.4.4 The I.R of a system consists of 2 delta functions as shown in figure. The Input to the system is a unit amplitude square pulse of one unit time duration Find the output IES



P2.4.5 Two rectangular waveforms of duration T_1 & T_2 secs are convolved. What is the shape of the resulting waveform? GATE

(A) Triangular (B) Rectangular
(C) Trapezoidal (D) Semi – circular

P2.4.6 A D.T.L.T.I. system is non causal if its I.R is __________ GATE

(A) $a^n u(n-2)$ (B) $a^{n-2}u(n)$ (C) $a^{n+2}u(n)$ (D) $a^n u(n+2)$

P2.4.7 The output of 2 systems S_1 & S_2 for the same Input $x(n) = e^{jn\pi}$ are 1 and $(-1)^n$ respectively. Which one of the following statement is correct? IES

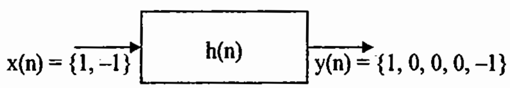
(A) Both S_1 & S_2 are LTI systems (B) S_1 is LTI but S_2 is not LTI
(C) S_1 is not LTI but S_2 is LTI (D) Neither S_1 nor S_2 is LTI

P2.4.8 A D.T. L.S.I system has an I.R. h(n) with h(0) = 1, h(1) = - 1, h(2) = 2 and zero otherwise. The system is given an Input sequence x(n) with x(0) = x(2) = 1 and zero otherwise. The number of nonzero samples in the output sequence y(n) & the value of y(2) are ________ respectively GATE

(A) 5, 2 (B) 6, 2 (C) 6, 1 (D) 5, 3

P2.4.9 Given finite length I/p x[n] & the corresponding finite length o/p y[n] of an LTI system as shown below, the I.R. h[n] of the system is GATE

(a) h[n] = {1, 0, 0, 1}
(b) h[n] = {1, 0, 1}
(c) h[n] = {1, 1, 1, 1}
(d) h[n] = {1, 1, 1}



P2.4.10 2 D.T. systems with I. Rs $h_1[n] = \delta[n-2]$ & $h_2[n] = \delta[n-1]$ are connected in cascade. The overall I.R. of cascaded system is GATE

(a) $\delta[n-1] + \delta[n-2]$ (b) $\delta[n-4]$
(c) $\delta[n-3]$ (d) $\delta[n-1]\,\delta[n-2]$

2.5 Convolution By Differentiation: -

$$x_1(t) * x_2(t) = \int_{-\infty}^{+\infty} x_1(\tau)x_2(t-\tau)d\tau$$

$$x_2(t) * x_1(t) = \int_{-\infty}^{+\infty} x_2(\tau)x_1(t-\tau)d\tau$$

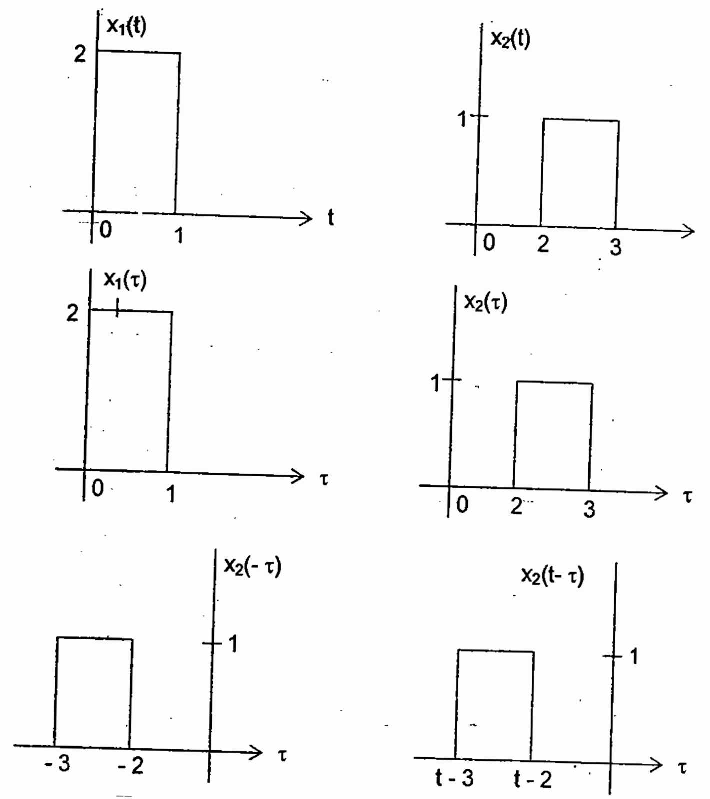
$$x(t) * \delta(t-t_0) = x(t-t_0)$$

$$\frac{d}{dt}[x_1(t) * x_2(t)] = \left(\frac{d}{dt}x_1(t)\right) * x_2(t) = x_1(t) * \frac{d}{dt}x_2(t)$$

$$x_1(t) * x_2(t) = \int_{-\infty}^{t} \left[x_1(t) * \frac{d}{dt}x_2(t)\right] dt$$

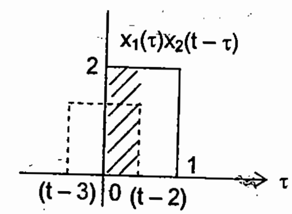
This last formula provides an easier and quicker method for solving convolution problems.

Prob: - Find the convolution of the signals shown in figure?

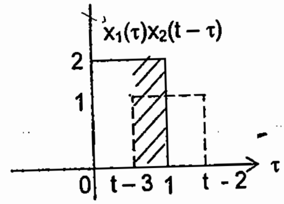


Case 1 : For $0 < t-2 < 1 \Rightarrow 2 < t < 3$

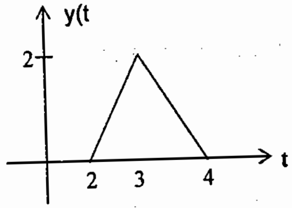
$$Y(t) = x_1(t) * x_2(t) = \int_0^{t-2} (1)(2)d\tau = 2t-4;\ 2 < t < 3$$



Case 2: For $0 < t-3 < 1$
$\Rightarrow 3 < t < 4$

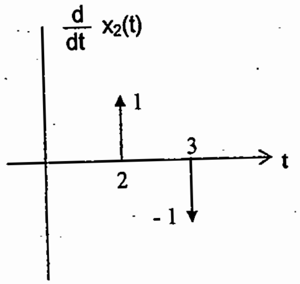


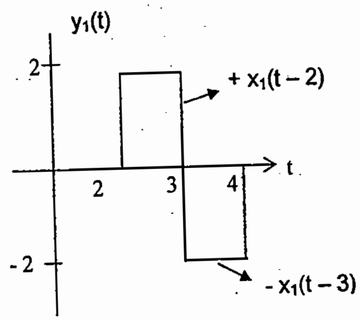
$$Y(t) = x_1(t) * x_2(t) = \int_{t-3}^{1} (2)(1)\, dt = 2(-t+4)\ ;\ 3 < t < 4$$



$$y(t) = \begin{cases} 0 & ; t < 2 \\ 2(t-2) & ; 2 < t < 3 \\ 2(-t+4) & ; 3 < t < 4 \\ 0 & ; t > 4 \end{cases}$$

ALTERNATIVE METHOD: -

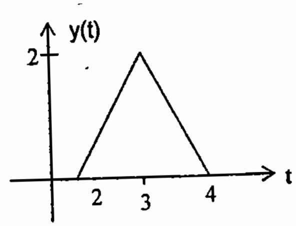




$$\frac{d}{dt} x_2(t) = \delta(t-2) - \delta(t-3)$$

$$y_1(t) = x_1(t) * \frac{d}{dt} x_2(t)$$

$$y(t) = x_1(t) * x_2(t) = \int_{-\infty}^{t} y_1(t)\, dt$$



2.6 PRACTICE PROBLEM SET: -

2.6.1. Let $x(n) = \delta(n) + 2\delta(n-1) - \delta(n-3)$ and $h(n) = 2\delta(n+1) + 2\delta(n-1)$

calculate (a) $y_1(n) = x(n)*h(n)$
(b) $y_2(n) = x(n+2)*h(n)$ (c) $y_3(n) = x(n)*h(n+2)$

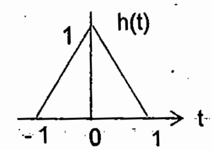
2.6.2. Let $y(t) = e^{-t}u(t) * \sum_{K=-\infty}^{+\infty} \delta(t-3k)$

Show that $y(t) = Ae^{-t}$ for $0 \le t \le 3$, and determine the value of A

2.6.3. Find whether the following LTI systems are causal & stable?
(a) $h_1(t) = e^{-5t}\sin(2\pi t)\, u(t)$
(b) $h_2(t) = \begin{cases} 1 & ; -1 \le t < 0 \\ -1 & ; 0 \le t \le 1 \\ 0 & ; \text{elsewhere} \end{cases}$
(c) $h_3(n) = (-1)^n u(-n)$ (d) $h_4(n) = \sum_{p=-1}^{\infty} \delta(n-2p)$ (e) $h_5(t) = (1-t)\,\text{rect}(t/3)$

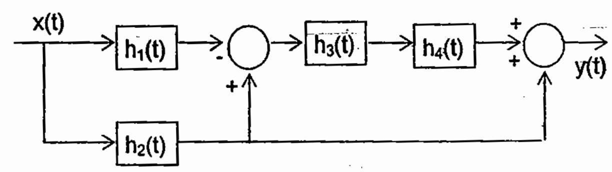
2.6.4. Find the convolution of the following signals?
(a) $x(t) = u(t) - u(t-2)$ & $h(t) = u(t)$
(b) $x(t) = u(t+3) - u(t-1)$ & $h(t) = u(-t+4)$
(c) $x(n) = \alpha^n u(n)$ & $h(n) = \beta^n u(n)$ for i. $\alpha = \beta$ and ii. $\alpha \neq \beta$
(d) $x(n) = \left(\frac{-1}{2}\right)^n u(n-4)$ and $h(n) = 4^n u(2-n)$

2.6.5. An LTI system has the I.R. h(t) shown in figure. Find the output if the input $x(t) = \delta(t-1) + \delta(t-2) + \delta(t-3)$



2.6.6. For the interconnection system shown in fig, if the impulse responses are
$h_1(t) = \delta(t-1)$
$h_2(t) = e^{-2t}\, u(t)$ $h_3(t) = \delta(t-1)$
$h_4(t) = e^{-3(t+2)}\, u(t+2)$

Find the overall impulse response?



2.6.7. Consider an LTI system with input& output related by $y(t) = \int_0^t e^{-\tau} x(t-\tau)\, d\tau$.

(a) Find I.R (b) Is this system causal? Why?
(c) Determine the system output for the input $x(t) = u(t+1)$

2.6.8. Let $x(n) = \{2, 4, 6, 8\}$
(a) Find $y(n) = x(n)*x(n)$ (b) Find $y_1(n) = x(2n) * x(2n)$
(c) Find $y_2(n) = x(n/2)*x(n/2)$

2.6.9. Find the periodic convolution of $x(n) = \{1, 2, 0, 1\}$ & $h(n) = \{2, 2, 3, 0\}$

2.6.10. Consider 2 systems described by $h_1(n) = \delta(n) + \alpha\,\delta(n-1)$ & $h_2(n) = (0.5)^n u(n)$
Find the response to the input $x(n) = (0.5)^n u(n)$ if
(a) 2 systems are connected in parallel with $\alpha = 0.5, -0.5$
(b) 2 systems are connected in cascade with $\alpha = 0.5, -0.5$

2.6.11. Given 3 signals h(n), g(n) & r(n), express g(n) in terms of r(n)

If $g(n) = \sum_{k=-\infty}^{+\infty} h(n+k)\, h(5-k)$ $\quad r(n) = \sum_{k=-\infty}^{+\infty} h(n-k)\, h(k)$

Ans: $g(n) = r(n+5)$

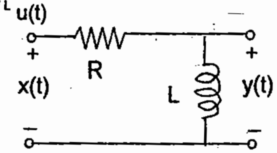
2.6.12. If $h(n) = A\,\delta(n) + \left(\frac{1}{3}\right)^n u(n)$ is the unit sample response of a LSI system, and s(n) is the step response, find 'A' so that $\lim_{n \to \infty} s(n) = 0$?

2.6.13. The I.R of a D.T LTI system is given by $h(n) = (0.5)^n\, u(n)$ of the input is

$x(n) = 2\,\delta(n) + \delta(n-3)$. Find the output at n = 1 & n = 4?

2.6.14. An FIR system is characterised by
$Y(n) = 0.2x(n) - 0.5\,x(n-2) + 0.4\,x(n-3)$
Given the input sequence { - 1, 1, 0, -1 } is applied to this system, find the output.

2.6.15. Given that the RL filter has I.R $h(t) = \delta(t) - \frac{R}{L} e^{-Rt/L} u(t)$
Find the step & ramp response?



REVIEW NOTES:

CHAPTER 3. FOURIER SERIES

→ Representing CT signals as superposition of complex exponentials leads to frequency – domain characterizations. eg :- A human ear is sensitive to audio signals within the frequency range 20Hz to 20 kHz. Typically, musical note occupies a much wider frequency range. Therefore the human ear processor frequency components within the audible range & rejects other frequency components. In such applications, frequency-domain analysis provides a convenient means of solving for the response of L.T.I. systems to arbitrary input.

→ By using F.S, a nonsinusoidal periodic function can be expressed as an infinite sum of sinusoidal function.

→ Sinusoidal signals arise in describing motion of planets & periodic behavior of earth's climate. A.C. sources generate sinusoidal voltages & currents.

→ There are 2 reasons for evaluating the F.S.

1. To obtain an expression for *f (t)* that applies everywhere, rather than only over a single period.
2. To obtain phasors, which indirectly tell how much power is available at each harmonic of the waveform.

3.1 ANALOGY BETWEEN VECTORS & SIGNALS :-

→ Signals are not just like vectors.
A vector can be represented as a sum of its components, depending on the choice of coordinate system. A signal can also be represented as a sum of its components.

→ We know that an arbitrary M-dimensional vector can be represented in terms of M orthogonal co-ordinates.
A vector is specified by its magnitude & direction.

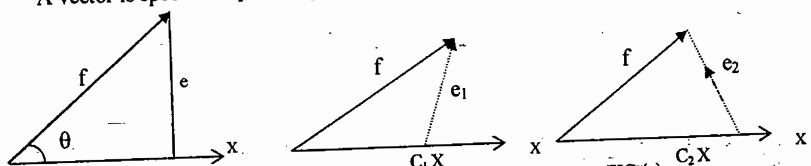


FIG (a) FIG (b) FIG (c)

Consider 2 vectors $\vec{f}$ & $\vec{X}$ as shown in figure. Let the component of f along X be CX.
(Geometrically the component f along x is the projection of f on X)

From Fig (a) $\vec{f} = C\vec{X} + \vec{e}$

From Fig (b) & (c) $\vec{f} = C_1\vec{X} + \vec{e_1} = C_2\vec{X} + \vec{e_2}$

If we approximate f by CX, $\vec{f} \cong C\vec{X}$

Error in the approximation e = f - CX

Minimize the error vector, such that $\vec{f}$ and $\vec{X}$ are approximated

length of the component $\vec{f}$ along $\vec{X} = C|\vec{X}| = |\vec{f}|\cos\theta$

$$C|\vec{X}|^2 = |\vec{f}|.|\vec{X}|\cos\theta$$

$$= \vec{f}\,.\,\vec{X}$$

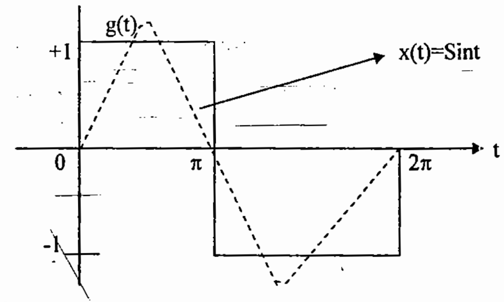
$$\boxed{C = \frac{\vec{f}\cdot\vec{X}}{|\vec{X}|^2}}$$

→ 2 vectors $\vec{f}$ & $\vec{X}$ are orthogonal if inner (or) scalar product $\vec{f}\,.\vec{X} = 0$

→ If we consider 2 basis vectors $\vec{i}$ & $\vec{j}$

$$\left.\begin{array}{ll}\text{orthogonality} & \vec{i}\,.\,\vec{j} = \vec{j}\,.\,\vec{i} = 0 \\ \text{unit magnitude} & \vec{i}\,.\,\vec{i} = \vec{j}\,.\,\vec{j} = 1\end{array}\right\}\text{Orthonormal Property}$$

Component Of A Signal



$g(t) \cong Cx(t);\ t_1 < t < t_2$

$$C = \frac{\int_{t_1}^{t_2} g(t)\,x(t)\,dt}{\int_{t_1}^{t_2} x^2(t)\,dt}$$

Note :-
2 signals g(t) & x(t) are said to be orthogonal over the interval (t_1, t_2) if $\int_{t_1}^{t_2} g(t)x^*(t)dt = \int_{t_1}^{t_2} x(t)g^*(t)dt = 0$

They are also said to be orthonormal if they satisfies $\int_{t_1}^{t_2} x(t)x^*(t)dt = \int_{t_1}^{t_2} g(t)g^*(t)dt = 1$ (Unit magnitude)

P3.1. For the three continuous functions shown in figure 3.1

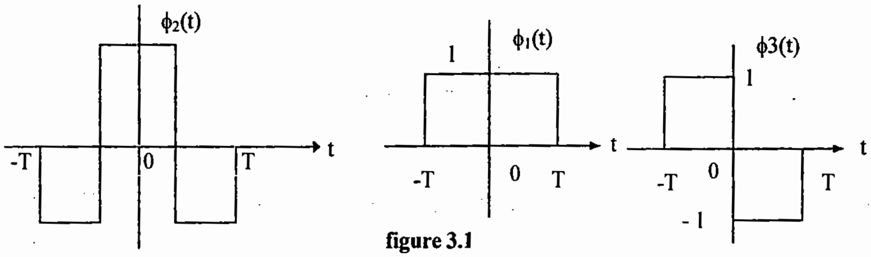


figure 3.1

a) Show that the functions form an orthogonal set
b) Find the value of T that makes 3 functions orthonormal
c) Express the signal $x(t) = \begin{cases} A \text{ for } 0 \le t \le T \\ 0 \text{ elsewhere} \end{cases}$ in terms of orthogonal set determined in (a)?

Sol:- a) For all the 3 signals $\underbrace{\int_{-T}^{T} |\varnothing_1(t)|^2 dt = \int_{-T}^{T} |\varnothing_2(t)|^2 dt = \int_{-T}^{T} |\varnothing_3(t)|^2 dt = 2T}_{\text{Unit – Magnitude property}}$

Orthogonality $\int_{-T}^{T} \varnothing_1(t)\varnothing_2(t) = \int_{-T}^{T} \varnothing_2(t)\varnothing_3(t) = \int_{-T}^{T} \varnothing_1(t)\varnothing_3(t) = 0$

b) For orthonormality $2T = 1 \Rightarrow T = ½$

c) $x(t) = C_1\varnothing_1(t) + C_2\varnothing_2(t) + C_3\varnothing_3(t)$

$$C_i = \frac{1}{E_\varnothing}\int_{t1}^{t2} x(t)\,\varnothing_i^*(t)dt \qquad \text{but } E_\varnothing = 2T$$

$$C_1 = \frac{1}{2T}\int_0^T A(1) = A/2 \qquad C_2 = \frac{1}{2T}\left(\int_0^{T/2} Adt - \int_{T/2}^{T} Adt\right) = 0$$

Similarly $C_3 = - A/2$

$\therefore\ x(t) = A/2\ [\ \varnothing_1(t) - \varnothing_3(t)\ ]$

3.2.1 TRIGONOMETRIC F.S. (T.F.S)

→ Any practical periodic function of frequency ω_0 can be expressed as an infinite sum of sine (or) cosine functions that are integral multiples of ω_0

$$g(t) = a_0 + a_1\cos\omega_0 t + a_2\cos 2\omega_0 t + \text{-----}$$
$$+ b_1\sin\omega_0 t + b_2\sin 2\omega_0 t + \text{-----}$$

$$\boxed{g(t) = \underset{\text{d.c.}}{\underbrace{a_0}} + \sum_{n=1}^{\infty} \underbrace{a_n \cos n\omega_0 t + b_n \sin n\omega_0 t}_{\text{a.c}}}$$

ω_0 → Fundamental frequency

a_0, a_n, b_n → T.F.S. coefficients

$a_0 = 1/T\int_0^T g(t)dt$ → d.c. (or) Average value

$a_n = 2/T\int_0^T g(t)\cos n\omega_0 t dt$

$b_n = 2/T\int_0^T g(t)\sin n\omega_0 t dt$

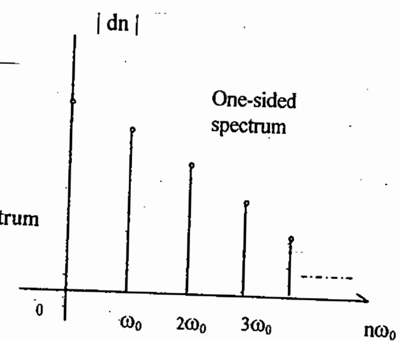
Polar form of T.F.S:-

$$g(t) = d_0 + \sum_{n=1}^{\infty} d_n\cos(n\omega_0 t \pm \theta_n)$$

$d_0 = a_0$

$|d_n| = \sqrt{a_n^2 + b_n^2}$ ⟶ Magnitude spectrum

$\theta_n = \text{Tan}^{-1}(-bn/an)$ ⟶ Phase spectrum

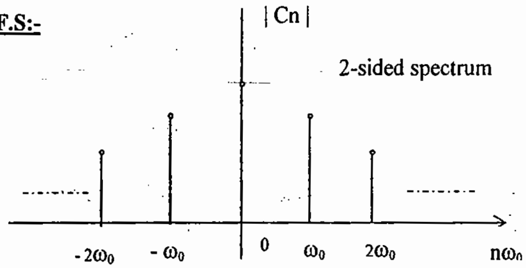


Effect of symmetry on Fourier coefficients:-

Symmetry	Condition	a_0	a_n	b_n
Even	$g(t) = g(-t)$	?	?	0
Odd	$g(t) = -g(-t)$	0	0	?
Half-wave	$g(t) = -g(t \pm T/2)$	0	$= 0$; n even $= ?$; n odd	$= 0$; n even $= ?$; n odd

3.2.2 EXPONENTIAL (OR) COMPLEX F.S:-

$$\boxed{g(t) = \sum_{n=-\infty}^{\infty} C_n e^{-jn\omega_0 t}}$$



where $C_n = \frac{1}{T}\int_0^T g(t)\, e^{-jn\omega_0 t}\, dt$

exponential F.S. coefficient

In terms of T.F.S. coefficients $C_0 = a_0$; $C_n = \frac{a_n - jb_n}{2}$; $C_{-n} = \frac{a_n + jb_n}{2}$

P3.2.1. A periodic signal is given by $x(t)=3\sin(4t+30^\circ) - 4\cos(12t-60^\circ)$ find the amplitude of second harmonic?

P3.2.2. Which of the following signal is not the representation of F.S.?

a) $\cos 3t + \sin 12t$ b) $1+\sin \pi t$

c) e^{j6t} d) $\cos 2\pi t + \sin 6t$

P3.2.3. Find the T.F.S representation of the periodic signal x(t) shown in fig.3.2.3?

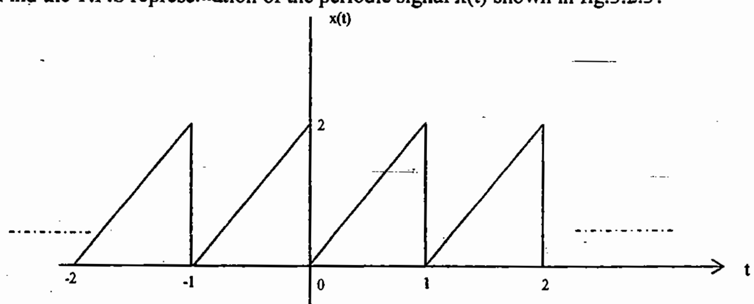


fig.3.2.3

P3.2.4 :- Find the symmetry of the signals shown in figure 3.2.4?

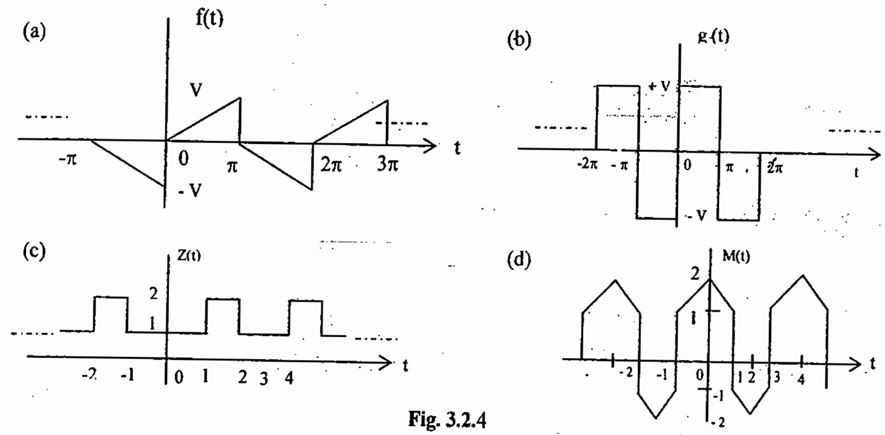
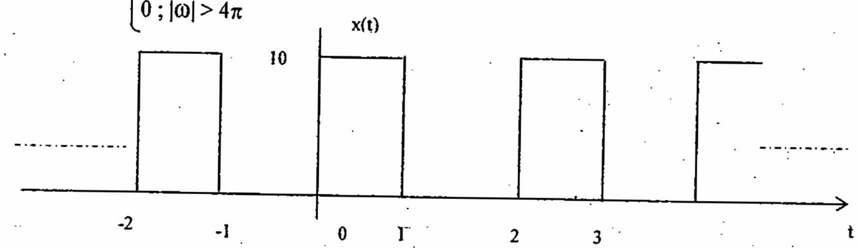


Fig. 3.2.4

P3.2.5. A periodic input signal x(t) shown below is applied to an L.T.I. system with frequency response

$$H(\omega) = \begin{cases} 1 ; |\omega| < 4\pi \\ 0 ; |\omega| > 4\pi \end{cases}$$ find the output?



P3.2.6. Obtain the T.F.S. & E.F.S representations of the periodic signal x(t) shown in fig.3.2.6?

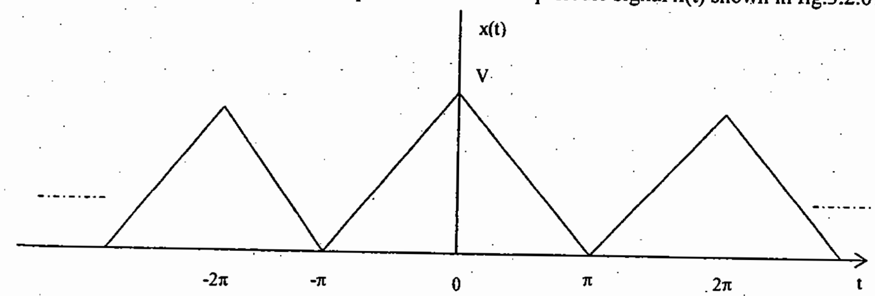
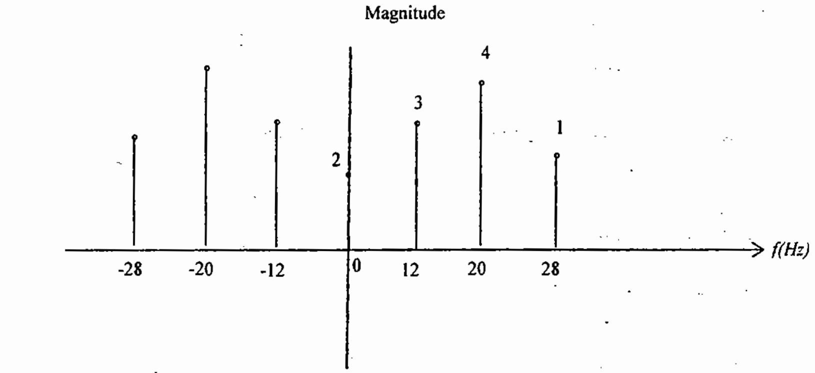


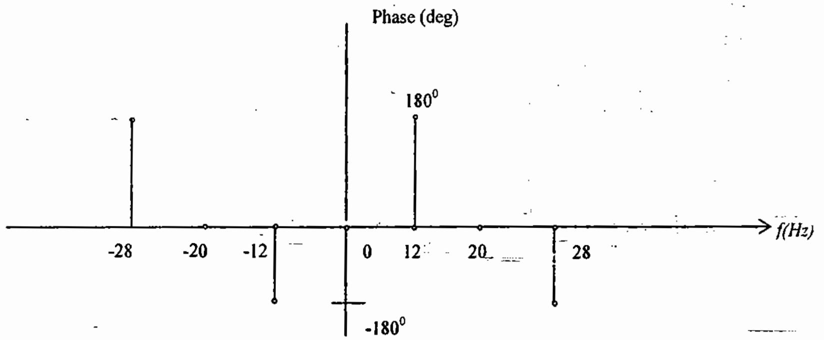
Fig 3.2.6

P3.2.7. For the periodic signal $x(t)=2+\cos\left(\frac{2\pi t}{3}\right)+4\sin\left(\frac{5\pi t}{3}\right)$, find the E.F.S. coefficients?

P3.2.8. Consider an ideal LPF with frequency response $H(\omega) = 1, |\omega| < 100$
$0, |\omega| > 100.$
When the output to this filter is a signal x(t) with $T=\pi/6$ & F.S. Coefficient C_n, it is found that $x(t)\rightarrow y(t)=x(t)$, for what values of "n" it is guaranteed that $C_n=0$?

P3.2.9. Consider the two-sided signal spectrum shown in figure for signal x(t), find x(t)?





Convergence of F.S:- (Dirichlet Conditions)

1) x(t) is absolutely integrable i.e., $\int_0^T |x(t)|dt < \infty$
2) x(t) has only a finite number of maxima & minima
3) The number of discontinuities in x(t) must be finite .

These conditions are sufficient, but not necessary.

3.3 Properties Of F.S.

1. Linearity :- $x(t) \longleftrightarrow C_n$, $y(t) \longleftrightarrow d_n$ } Period = T

Then $\alpha x(t) + \beta y(t) \longleftrightarrow \alpha C_n + \beta d_n$

2. Time-shift :- $x(t) \longleftrightarrow C_n$ **then** $x(t-t_0) \longleftrightarrow C_n e^{-jn\omega_0 t_0}$

when we shift in the time-domain, it changes the phase of each harmonic in proportion to its frequency $n\omega_0$.

P3.3.1:- The F.S. coefficient of the signal x(t) shown in fig (a) are $C_0=1/\pi$, $C_1=-j0.25$, $C_n=1/\pi(1-n^2)$ (n even) Find F.S. coefficients of y(t), f(t) and g(t)?

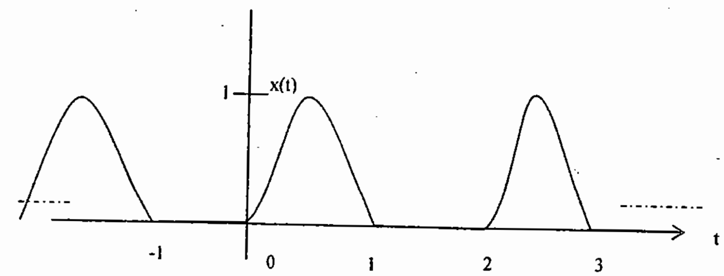
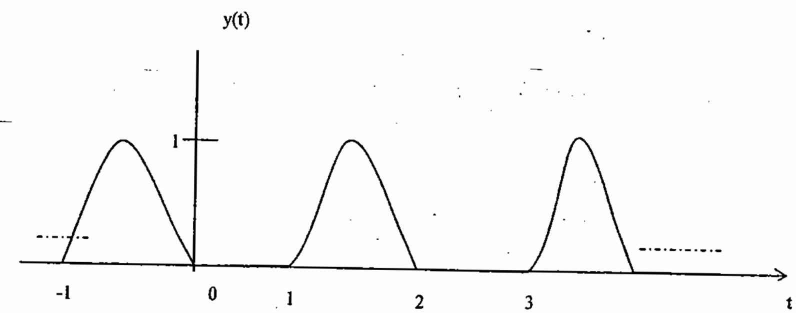
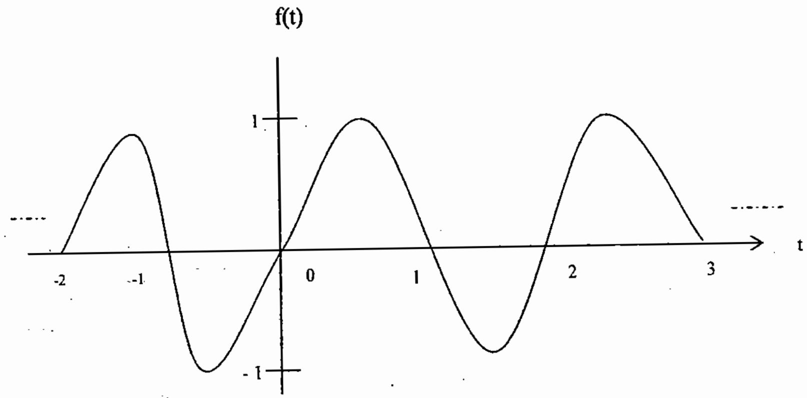
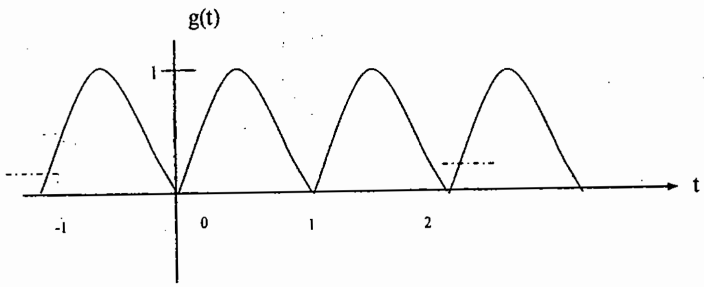


FIG (a)







P3.3.2:- Let x(t) be a periodic signal with period T and F.S coefficient Cn
Let $y(t)=x(t-t_0)+x(t+t_0)$. The F.S. coefficient of y(t) is d_n.
If $d_n = 0\ \forall$ odd n then t_0 can be
a) T/8 b) T/4 c) T/2 d) 2T

3) Frequency Shift :- $x(t) \longleftrightarrow C_n$ then $x(t)e^{j\omega_0 Mt} \longleftrightarrow C_{n-M}$

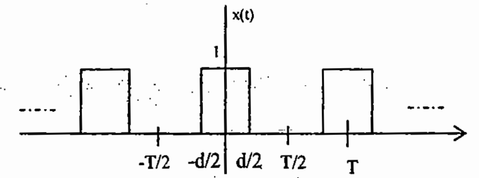
4) Time-Scaling:- $x(t) \longleftrightarrow C_n$
$x(\alpha t) \longleftrightarrow C_n$

Time-Compressing by α changes frequency from ω_0 to $\alpha\,\omega_0$

5) Differentiation in time:- $x(t) \longleftrightarrow C_n$

$$\frac{d}{dt}x(t) \longleftrightarrow (jn\omega_0)C_n$$

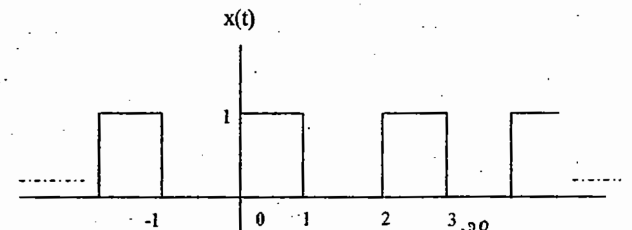
P3.3.3:- By using derivative method, find F.S. coefficient of the signal shown in figure?



6) Parseval's Power theorem:-

$x(t) \longleftrightarrow C_n$ then $\frac{1}{T}\int_0^T |x(t)|^2 dt = \sum_{n=-\infty}^{+\infty} |C_n|^2$

P3.3.4:- Find the power upto II harmonic for the periodic signal shown in figure?



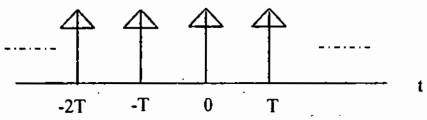
P3.3.5:- A periodic signal has the F.S. representation $x(t) \xleftrightarrow{FS,\ \pi/10} C_n = -n\,2^{-|n|}$ Without finding x(t), find the F.S. representation [d_n & ω_0] if

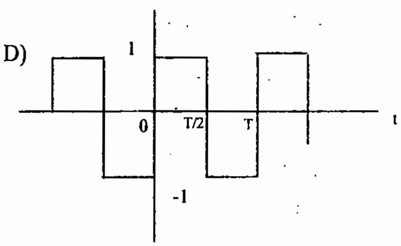
(a) $y(t)=x(3t)$ (b) $y(t)=\frac{d}{dt}x(t)$

(c) $y(t) = x(t-1)$ (d) $y(t)=\mathrm{Re}\{x(t)\}$ (e) $y(t) = \cos 4\pi t\ x(t)$

P3.3.6:- Match the following

List – I (Periodic fuction)	List-II (Properties of spectrum)
A) Impulse train	1) Only even harmonics are present
B) Full-wave rectified sine wave	2) Impulse train with strength 1/T
C) $2\sin(2\pi t/6)\cos(4\pi t/6)$	3) $C_3 = 1/2j$ $C_{-3} = -1/2j$ $C_1 = -1/2j$ $C_{-1} = 1/2j$
D)	4) Only odd harmonics are present
	5) Both even & odd harmonics are present





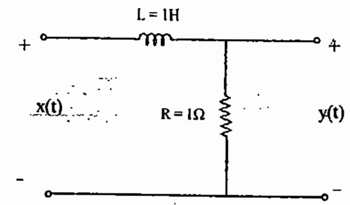
3.4 Systems With Periodic Inputs :-

→ If we apply an Input of the form $x(t)=e^{j\omega t}$, then the output is $y(t)=e^{j\omega t}H(\omega)$, where $H(\omega)$ is known as frequency response of the system.

→ Knowing $H(\omega)$, we can determine whether the system amplifies or attenuates a given sinusoidal component of the Input & how much of a phase shift adds to that particular component.

→ The response y(t) of an LTI system to the periodic Input x(t) is $y(t) = \sum_{n=-\infty}^{+\infty} C_n H(n\omega_0)e^{jn\omega_0 t}$

P3.4.1 Find the Output voltage of the system shown in figure, if the Input voltage is $x(t) = 4\cos 2t$

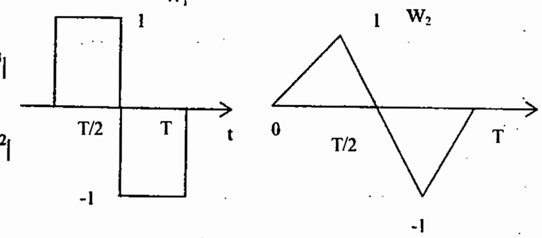


3.5 Previous Questions : -

1) One period (O,T) each of 2 periodic waveforms w_1 & w_2 for $n\geq1$, n odd are respectively F.S. coefficient is proportional to **GATE**

a) $|n^{-3}|$ & $|n^{-2}|$ b) $|n^{-2}|$ & $|n^{-3}|$

c) $|n^{-1}|$ & $|n^{-2}|$ d) $|n^{-4}|$ & $|n^{-2}|$



2) Choose the function f(t) ;$-\infty<t<\infty$ for which F.S. can't be defined **GATE**
a) $3\sin(25t)$ b) $4\cos(20t+3)+2\sin(10t)$ c) $e^{-|t|}\sin(25t)$ d) 1

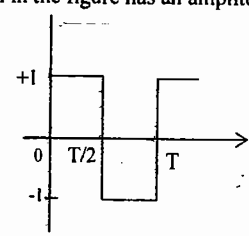
3) For the signal x(t), E.F.S. coefficient is C_n. one of the coefficient is observed to be $C_3 = 2 + j3$, then C_{-3} is ________ **GATE**

a) -2-j3 b) 2-j3 c) 3+j2 d) 3-j2

4) Consider the signal $x(t)=10\cos(10\pi t+\pi/7) + 4\sin(30\pi t+\pi/8)$. It's power lying within the frequency band 10Hz to 20Hz is ___________ **DRDO**
a) 4W b) 8W c) 50W d) 58W

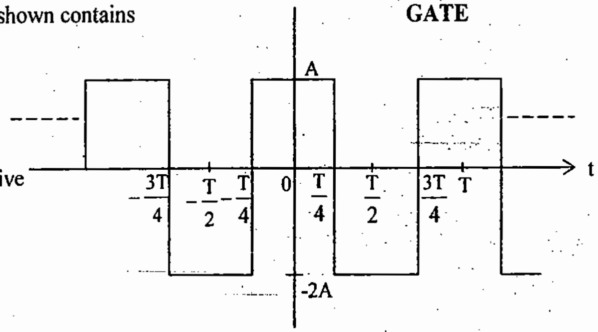
5) Consider the trigonometric series, which holds true ∀t, given by **DRDO**
$x(t)=\sin\omega_0 t+1/3\sin3\omega_0 t+1/5\sin5\omega_0 t+1/7\sin7\omega_0 t+$ ---------- At $\omega_0 t = \pi/2$ the series converges to
a) 0.5 b) $\pi/4$ c) $\pi/2$ d) 2

6) The II harmonic component of the periodic waveform given in the figure has an amplitude of
(a) 0 (b) 1
(c) $2/\pi$ (d) $\sqrt{5}$



7) The TFS for the waveform f(t) shown contains GATE

(a) only cosine terms with zero d.c. component
(b) only cosine terms with positive d.c. component
(c) only cosine terms with negative d.c. component
(d) only sine terms & negative d.c. component

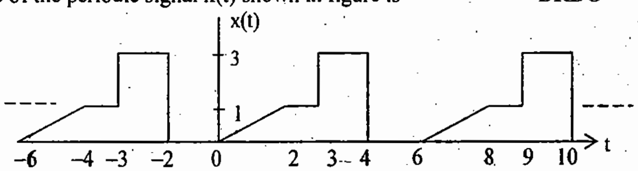


8) A function is given by $f(t) = \sin^2 t + \cos 2t$. Which of the following is TRUE? **GATE**
(a) f has frequency components at 0 and $1/2\pi$ Hz
(b) f has frequency components at 0 and $1/\pi$ Hz
(c) f has frequency components at $1/2\pi$ and $1/\pi$ Hz
(d) f has frequency components at 0, $1/2\pi$ & $1/\pi$ Hz

9) The fundamental frequency of the composite signal
$x(t) = 2 + 3\cos(0.2t) + \cos(0.25t+\pi/2) + 4\cos(0.3t - \pi)$ is **DRDO**
(a) 0.05 rad/sec (b) 0.1 rad/sec (c) 0.2 rad/sec (d) 0.25 rad/sec

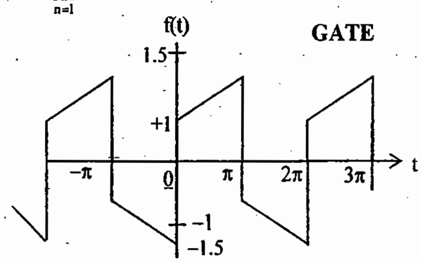
10) The average value of the periodic signal x(t) shown in figure is **DRDO**

(a) 5/6
(b) 1
(c) 5
(d) 6



11) f(t), shown in figure is represented by $f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + b_n \sin nt$ The value of a_0 is **GATE**

(a) 0 (b) $\pi/2$
(c) π (d) 2π



3.6 PRACTICE PROBLEM SET :-

P3.6.1 .Obtain the F.S expression for the ware form shown in fig.1?

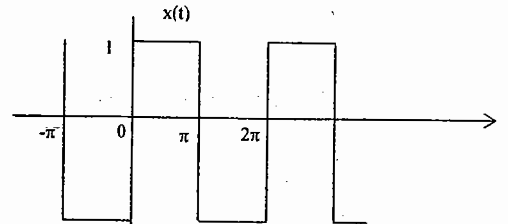


Fig (1)

P3.6.2 Calculate the F.S expression for the function shown in figure (2), (3)?

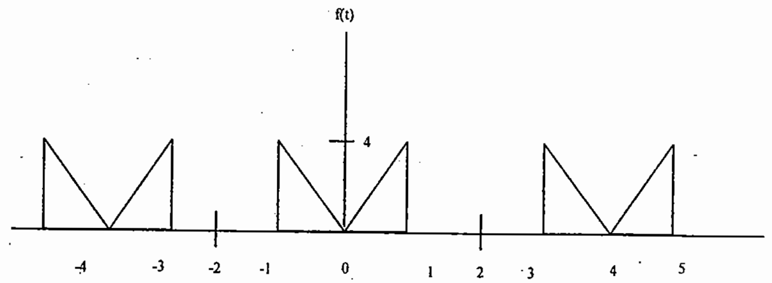


Fig (2)

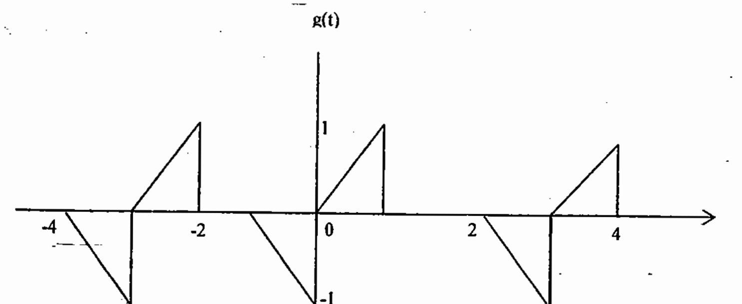
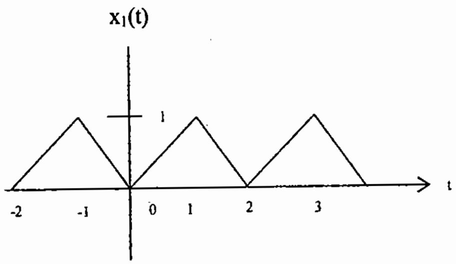
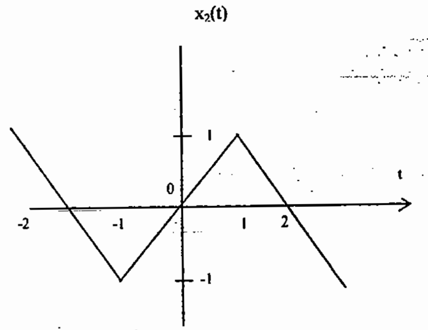


Fig (3)

3. The F.S representation of $x_1(t)$ shown in figure is given by
$x_1(t)=1/2-4/\pi^2[\cos\pi t+1/9\cos3\pi t+1/25\cos5\pi t+$ - - - - - - - -] obtain the F.S. of $x_2(t)$?



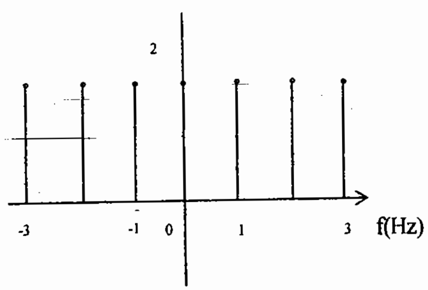


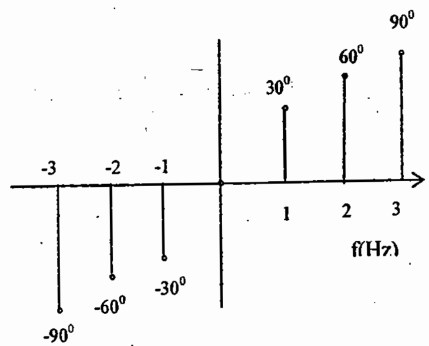
4) Determine whether the functions given can be represented by a Fourier series?

a) $x(t)=\cos6t+\sin8t+e^{j2t}$
b) $x(t)=\cos t+\sin\pi t$
c) $x_3(t)=x_1(t)+x_2(3t)$ where $x_1(t)=\sin(\pi t/6)$ & $x_2(t)=\sin(\pi t/9)$

5) Find the F.S. coefficients of $x(t)=\sum_{\substack{K=-\infty \\ K \text{ even}}}^{+\infty} u(t-K)-u(t-1-K)$

6) The magnitude & phase spectra of a periodic signal x(t) are shown in figure





a) Find the TFS representation?

b) Draw magnitude & phase spectrum of
i) $f(t)=x(2t)$ ii) $g(t)=x(t-1/6)$ iii)h(t)= d

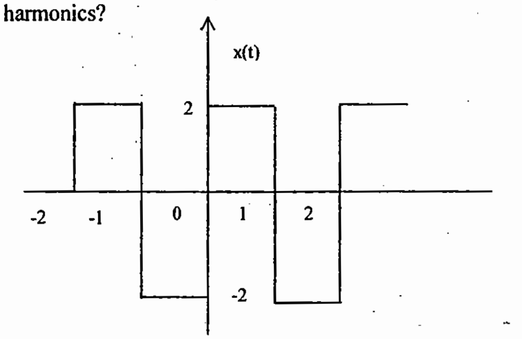
7) Let X[K] be the E.F.S coefficient of a periodic signal x(t), find the F.S. coefficients of the following?
a) $f(t)=x(2t)$ b) $g(t)=x(-t)$
c) $h(t)=x(-2t)$ d) $y(t)=2+x(2t)$

8) The periodic signal $x(t)=|\sin250\pi t|$ is applied to an ideal filler as shown.
Sketch the Output spectrum of the filter if

a) Ideal filter blocks all frequencies post 200Hz
b) Ideal filter blocks all frequencies post 400Hz
c) Ideal filter passes only frequencies in the range 200-400Hz

9) Let $x(t) \longleftrightarrow C_n$ with period T. Find F.S. coefficients of following signals in terms of "C_n"
a) $x(t-t_o)+x(t+t_o)$ b) Ev{x(t)} c) $d^2/dt^2\ x(t)$ d) $x(3t-1)$ e) $x(-t)$

10) The power in the first 4 harmonics of the signal shown in figure is 1.28 joules, what is the power in rest of the harmonics?



11) Consider an L.T.I. system with impulse response $h(t)=e^{-2t}u(t)$. Find F.S. representation of output if Input is $x(t)=\sum_{n=-\infty}^{+\infty}\delta(t-n)$

12) The F.W.R. sinusoidal voltage is fig(a) is applied to LPF of fig(b) obtain the output voltage $\upsilon_0(t)$ of the filter?

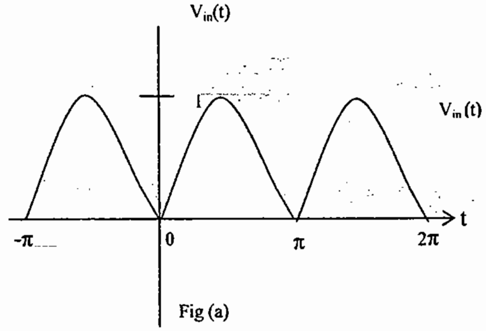


Fig (a)

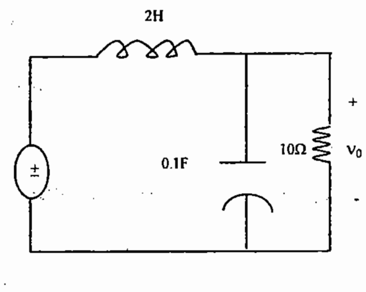


Fig (b)

13) Given the set of functions shown in fig(a) show that this is an orthogonal set & that each member of the set is normalized.

(i) If $x(t) = \cos(2\pi t)$, $0<t<1$, find y(t) expressed as $y(t) = \sum_{K=1}^{N} d_n \varnothing_n(t)$

(ii) Repeat part (i) for $x(t)=\sin(2\pi t)$, $0<t<1$

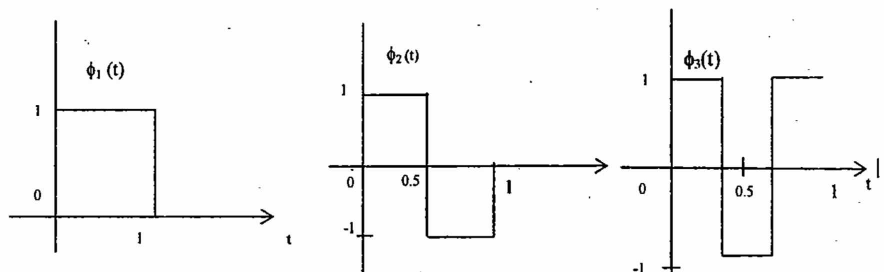


Fig (a)

HINT:- Use $d_n = \int_a^b x(t)\, \varnothing_n(t)dt$

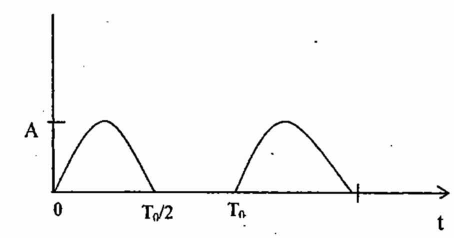
14) Consider basis functions o the form $y_1(t)=\sqrt{2}\; e^{-t}$; $t>0$ & second one of the form $y_2(t)=Ae^{-2t}+Be^{-t}$; $t>0$. Find A & B such that $y_1(t)$&$y_2(t)$ are orthogonal over $(0, \infty)$.

15) The E.F.S. representation of a signal x(t) over (0,T) is $x(t) = \sum_{n=\infty}^{\infty} \frac{3}{4+(n\pi)^2} e^{jn\pi t}$

a) Find T?
b) One of the components of x(t) is $A\cos(3\pi t)$. Find A?

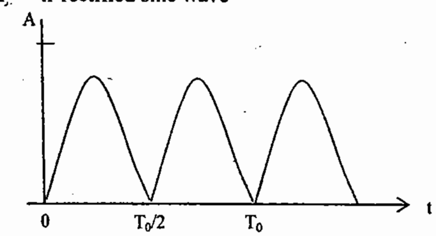
E.F.S coefficients of several signals:

i) Half-rectified sine wave



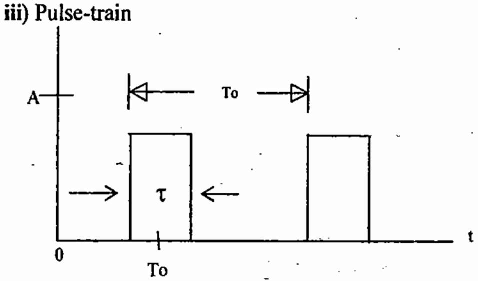
$$C_n = \begin{cases} \dfrac{A}{\pi(1-n^2)} & , n=0, \pm2, \pm4,.. \\ 0 & , n \text{ odd } \& \; n \neq \pm1 \\ \dfrac{-jnA}{4} & , n = \pm1 \end{cases}$$

ii) Full-rectified sine wave



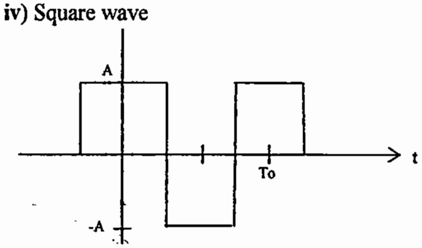
$$C_n = \begin{cases} \dfrac{2A}{\pi(1-n^2)} & , n \text{ even} \\ 0 & , n \text{ odd} \end{cases}$$

iii) Pulse-train



$$C_n = \frac{A\tau}{T_0} \,\text{sinc}(nf_0\tau)e^{-j2\pi nf_0t_0}$$

iv) Square wave



$$C_n = \begin{cases} \dfrac{2A}{|n|\,\pi} & , n=\pm1, \pm5, \ldots \\ \dfrac{-2A}{|n|\,\pi} & , n=\pm3, \pm7, \ldots \\ 0 & , n \text{ even} \end{cases}$$

Chapter 4 : Fourier Transform (F.T.)

4.1 INTRODUCTION:

→ Fourier Transform (F.T.) provides a frequency domain description of time domain signals and is extension of F.S to non-periodic signals.
→ CTFT expresses signals as linear combination of complex Sinusoids
→ Transformation makes the analysis of signal much easier because certain features which may be obscure in one form may be obvious in other form
→ Spectrum of F.T. is continuous whereas spectrum of F.S is discrete.

F.T (or) spectrum of a signal $x(t) = X(\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt$ ------------ (1)

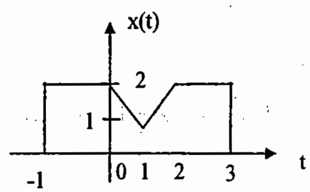
Inverse F.T. (I.F.T) is $x(t) = \dfrac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega)\, e^{j\omega t}\, d\omega$ ------------ (2)

→ $X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t}\, dt$ | $x(t) = \int_{-\infty}^{+\infty} X(f)\, e^{j2\pi f t}\, df$

$x(t) \longleftrightarrow X(\omega)$
$\longleftrightarrow X(f)$
$\longleftrightarrow X(j\omega)$

P4.1.1 If x(t) is a voltage waveform, then what are the units of $X(f)$?

P4.1.2 For the signal x(t) shown in figure, find
a) $X(0)$
b) $\int_{-\infty}^{+\infty} X(\omega)\, d\omega$



Convergence of F.T.:-

1) F.T. is defined for all stable signals i.e., $\int_{-\infty}^{+\infty} |x(t)|\, dt < \infty$

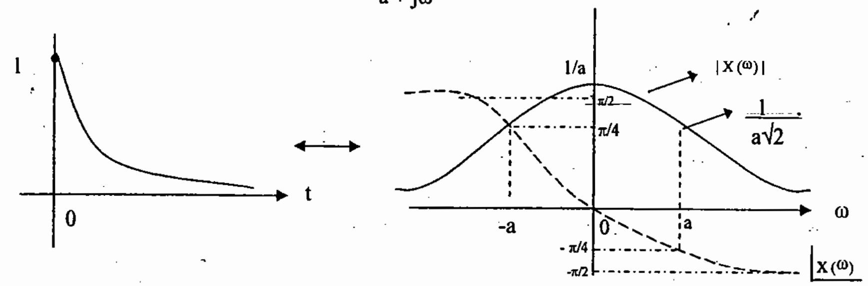
2) Periodic signals, which are neither absolutely integrable nor square integrable over an infinite interval, can be considered to have F.T. if impulse functions are permitted in the transform.

3) x(t) have a finite number of discontinuities and finite number of maxima and minima within any finite interval.

F.T. Of Standard Signals:

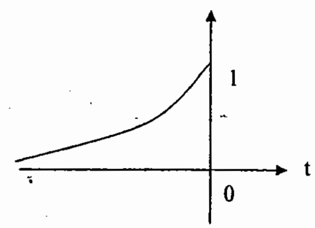
1) Decaying exponential

$x(t) = e^{-at} u(t),\ a > 0 \longleftrightarrow \dfrac{1}{a + j\omega}$

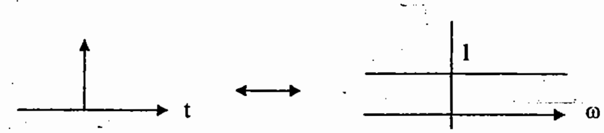


2) Increasing exponential

$e^{at}u(-t) \longleftrightarrow \dfrac{1}{a - j\omega}$

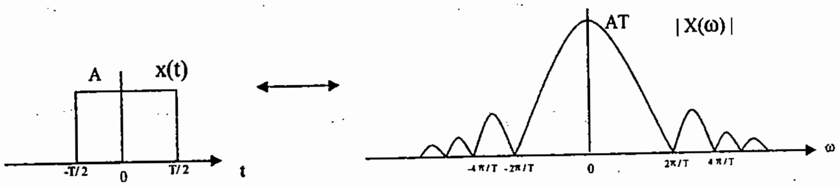


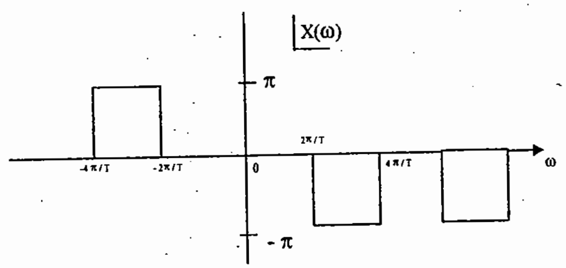
3) C.T. impulse function $\delta(t) \longleftrightarrow 1$



4) Rectangular (or) Gate function:-

$$x(t) = A\ \text{rect.}(t/T)\ \text{(or)}\ A\,\pi(t/T) \longleftrightarrow A\,T\,\text{SinC}(\omega T/2)$$





4.2 Properties of F.T.:

1) Linearity:- $x_1(t) \longleftrightarrow X_1(\omega)$ and $x_2(t) \longleftrightarrow X_2(\omega)$

then $a\,x_1(t) + b\,x_2(t) \longleftrightarrow aX_1(\omega) + bX_2(\omega)$

P4.2.1 Find the F.T of the signals (i) $x(t) = e^{-a|t|}$ [Two sided exponential]

(ii) $x(t) = \text{Sgn}(t)$ [Signum function]

2) Time – Scaling:- $x(t) \longleftrightarrow X(\omega)$ then $x(at) \longleftrightarrow \frac{1}{|a|} X(\omega/a)$

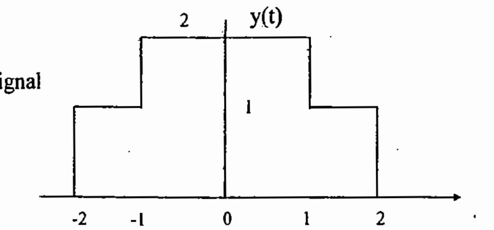
Expansion ⇔ Compression

Linear scaling in time by a factor of "a" corresponds to linear scaling in frequency by a factor of "1 / a"

P4.2.2. Find the F.T. of the signals

(i) $x_1(t) = A\ \text{rect.}(t/2T)$ (ii) $x_2(t) = A\ \text{rect.}(2t/T)$

P4.2.3. Find the F.T of the signal shown in figure?



3) Duality (or) Symmetry:-

$$x(t) \longleftrightarrow X(\omega) \text{ then } X(t) \longleftrightarrow 2\pi\, x(-\omega)$$

P4.2.4. Find F.T. of the following signals:

(i) $x(t) = 1$ (ii) $x(t) = 1/(a + jt)$ (iii) $x(t) = 2a/(a^2 + t^2)$ (iv) $x(t) = 1/\pi t$

P4.2.5 Find the F.T. of $x(t) = u(t)$

4) Time – Shift:- $x(t) \longleftrightarrow X(\omega)$ then $x(t - t_0) \longleftrightarrow e^{-j\omega t_0} X(\omega)$

Time – delay in a signal causes a linear phase shift in its spectrum

P4.2.6 Find the F.T. of the signals

(i) $x(t) = e^{-3t} u(t-1)$ (ii) $z(t) = \pi[(t-1)/2]$ (iii) $y(t) = e^{-2|t-2|}$

5) Frequency – shift (or) Modulation: -

If $x(t) \longleftrightarrow X(\omega)$, then $x(t)\, e^{j\omega_0 t} \longleftrightarrow X(\omega - \omega_0)$

Modulation spreads the signal spectrum to higher frequencies

P4.2.7 Find the F.T. of the following signals ?

(i) $\text{Cos}\,\omega_0 t$ (ii) $\text{Sin}\,\omega_0 t$ (iii) $e^{-at}\,\text{Sin}\,\omega_c t u(t)$ (iv) $A\ \text{rect.}(t/T)\ \text{Cos}\,\omega_0 t$

P4.2.8. Find the F.T. of $y(t) = \dfrac{4\,\text{Cos}\,2t}{t^2 + 1}$ ________

6) Time – reversal: - $x(t) \longleftrightarrow X(\omega)$ then $x(-t) \longleftrightarrow X(-\omega)$

Only phase spectrum changes

P4.2.9. Let $x(t) = \text{rect.}(t - ½)$ { where $\text{rect.}(x) = 1$ for $-½ \le x \le ½$ } then if $\text{Sinc}(x) = \dfrac{\text{Sin}\,\pi x}{\pi x}$. F.T. of $x(t) + x(-t)$ will be given by ________

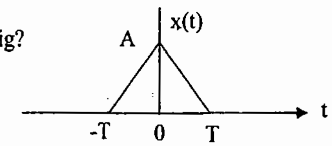
7) Differentiation in time:-

$x(t) \longleftrightarrow X(\omega)$, then $\dfrac{dx(t)}{dt} \longleftrightarrow j\omega X(\omega)$

Differentiation accentuates the HF components of the signal

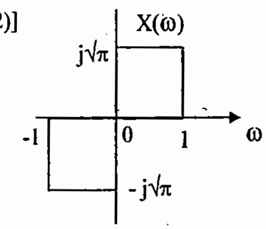
Differentiation destroys any d.c. component of x(t) i.e., F.T. of the differentiated signal at $\omega = 0$ is zero

P4.2.10 Find the F.T. of the signal x(t) shown in fig?



P4.2.11 Find the F.T. of the signal $y(t) = \dfrac{d}{dt}\,[u(t-2) + u(-t-2)]$

P4.2.12. For the spectrum $X(\omega)$ shown in figure, find $\dfrac{d}{dt}x(t)$ at $t = 0$?



8) Frequency Differentiation:-

$$x(t) \longleftrightarrow X(\omega) \text{ then } -jt\,x(t) \longleftrightarrow \frac{d}{d\omega}X(\omega)$$

P4.2.13. Find the F.T. of $y(t) = te^{-at}u(t)$

P4.2.14 Find the F.T. of $t\,e^{-|t|}$, hence find the transform of $\dfrac{4t}{(1+t^2)^2}$

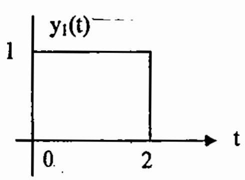
P4.2.15 Given $x(t) \longleftrightarrow X(\omega)$, express the F.T. of the following signals in terms of $X(\omega)$?

(i) $x_1(t) = x(2-t) + x(-t-2)$ (ii) $x_2(t) = x(3t-6)$

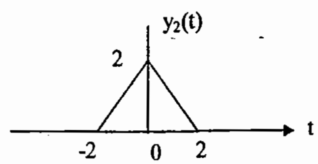
(iii) $x_3(t) = \dfrac{d^2}{dt}x(t-3)$ (iv) $x_4(t) = t\,\dfrac{d\,x(t)}{dt}$

P4.2.16 Given $x(t) = \begin{cases} 1\,;\ |t| < 1 \\ 0\,;\ \text{elsewhere} \end{cases} \longleftrightarrow \dfrac{2\,\text{Sin}\,\omega}{\omega}$ Find the F. T of the following signals?

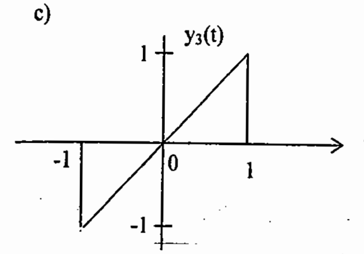
a)



b)



c)



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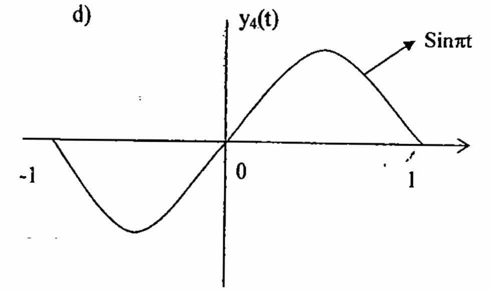
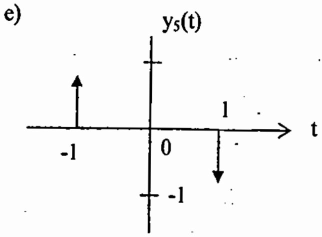
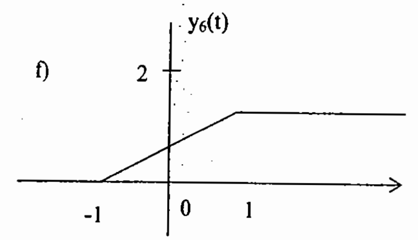


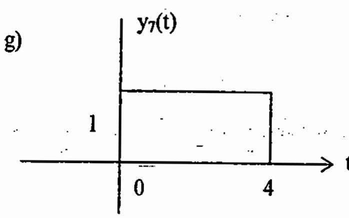
e)



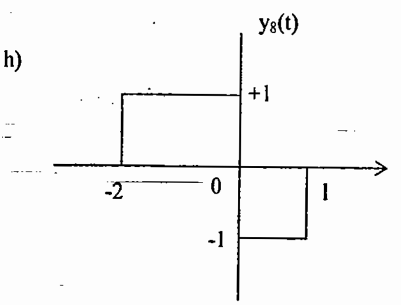
f)



g)



h)



9) Convolution in time:

$x(t) \longleftrightarrow X(\omega)$, $h(t) \longleftrightarrow H(w)$, then $x(t) * h(t) \longleftrightarrow X(\omega)\,H(\omega)$

→ Convolution in one-domain corresponds to multiplication in frequency domain

→ F.T. of impulse response, h(t) is known as frequency response, $H(\omega)$

→ Since h(t) completely characterizes an LTI system, then so must $H(\omega)$.

P4.2.17 Given $y(t) = x(t) * h(t)$ and $g(t) = x(3t) * h(3t)$ such that $g(t) = Ay(Bt)$, find A & B?

P4.2.18 An L.T.I system is having I.R. $h(t) = \frac{\sin 4t}{\pi t}$ for which the input applied is

$x(t) = \cos 2t + \sin 6t$, find the output ?

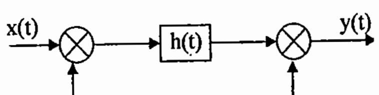
P4.2.19. Consider a causal LTI system with frequency response

$H(\omega) = \frac{1}{j\omega + 3}$ Find the input such that the output is $y(t) = e^{-3t}u(t) - e^{-4t}u(t)$

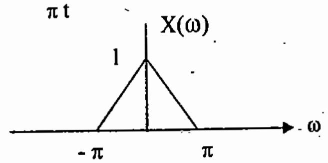
P4.2.20. A causal & stable LTI system has the frequency response $H(\omega) = \frac{j\omega+4}{6-\omega^2+5j\omega}$

a. Find the impulse response?
b. What is the o/p when the I/p applied is $x(t) = e^{-4t}\,u(t) - t\,e^{-4t}\,u(t)$?

P4.2.21 Consider the system shown in fig(a). The F.T. of the input signal shown in fig (b).
Find F.T. of y(t) given $w(t) = \cos(5\pi t)$ and $h(t) = \frac{\sin(6\pi t)}{\pi t}$



Fig(a)



Fig(b)

10) Frequency Convolution:-

$x_1(t) \longleftrightarrow X_1(\omega)$ and $x_2(t) \longleftrightarrow X_2(\omega)$ then $x_1(t)\,x_2(t) \longleftrightarrow \frac{1}{2\pi}[X_1(\omega) * X_2(\omega)]$

P4.2.22 Find the F.T. of $y(t) = x(t)\cos\omega_0 t$?

11) Integration in Time:-

If $x(t) \longleftrightarrow X(\omega)$, then $\int_{-\infty}^{t} x(\tau)\,d\tau \longleftrightarrow \frac{X(\omega)}{j\omega}$ if $X(0) = 0$

$\longleftrightarrow \frac{X(\omega)}{j\omega} + \pi X(0)\,\delta(\omega)$ if $X(0) \neq 0$

P4.2.23 Find the F.T. of $\int_{-\infty}^{t} \frac{\sin(2\pi t)}{\pi t}\,dt$

12. Rayleigh's Energy theorem (or) Parseval's Power theorem:-

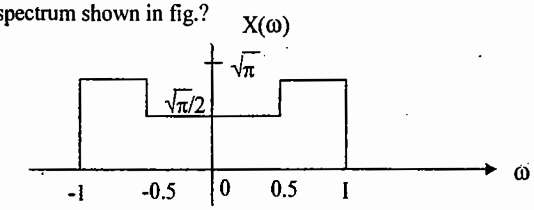
It states that the energy (or) power in the time domain representation of a signal is equal to the energy (or) power in the frequency domain.

→ The power (or) energy spectrum of a signal is defined as the square of magnitude spectrum

If $x(t) \longleftrightarrow X(\omega)$, then $\int_{-\infty}^{+\infty} |x(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \underbrace{|X(\omega)|^2}_{\text{E.S.D. (or) P.S.D}}\,d\omega$

P4.2.24 Find the energy in the signal $x(t) = \frac{\sin at}{\pi t}$?

P4.2.25 Find the energy in the spectrum shown in fig.?



P4.2.26 An input signal $x(t) = e^{-2t}\,u(t)$ is applied to an ideal L.P.F with frequency response characteristics $H(w) = 1;\ |\omega| < \omega_c$
$0;\ |\omega| > \omega_c$

Find ω_c, such that energy in the output is half that of Input energy?

P4.2.27 Consider $x(t) \longleftrightarrow X(\omega)$. suppose we are given the following facts

i) x(t) is real and non negative

ii) $F^{-1}\{(1 + j\omega)X(\omega)\} = Ae^{-2t}u(t)$, where 'A' is independent of t

iii) $\int_{-\infty}^{+\infty} |X(\omega)|^2\,d\omega = 2\pi$, find a closed-form expression for x(t).

P4.2.28 Find the value of the integral $\int_{-\infty}^{+\infty} \frac{8}{(\omega^2+4)^2}\,d\omega$

Fourier Transform Of Useful Signals:

Signal, x(t)	X(f) form	X(w) form
$e^{-at}u(t)$, a>0	$\frac{1}{a+j2\pi f}$	$\frac{1}{a+j\omega}$
$e^{at}u(-t)$, a>0	$\frac{1}{a-j2\pi f}$	$\frac{1}{a-j\omega}$
$\delta(t)$	1	1
A, Constant	$A\,\delta(f)$	$2\pi A\delta(\omega)$
A rect(t/T)	ATSinc(fT)	ATSinc(ωT/2)
Sinc(t)	Rect.(f)	Rect.(ω /2π)
$e^{-a\|t\|}$	$\frac{2a}{a^2+4\pi^2 f^2}$	$\frac{2a}{a^2+\omega^2}$
Sgn(t)	$\frac{1}{j\pi f}$	$2/j\omega$
u(t)	$\frac{1}{j2\pi f}+0.5\,\delta(f)$	$\frac{1}{j\omega}+\pi\delta(\omega)$
$\cos\omega_o t$	$\frac{\delta(f-f_o)+\delta(f+f_o)}{2}$	$\pi[\delta(\omega-\omega_o)+\delta(\omega+\omega_o)]$
$\sin\omega_o t$	$\frac{\delta(f-f_o)-\delta(f+f_o)}{2j}$	$\pi/j[\delta(\omega-\omega_o)-\delta(\omega+\omega_o)]$
e^{-at^2}	$\sqrt{\frac{\pi}{a}}\;e^{-\pi^2 f^2/a}$	$\sqrt{\frac{\pi}{a}}\;e^{-\omega^2/4a}$

Fourier Transform Properties :

PROPERTY	X(f) - form	X(ω) - form
Linearity $ax_1(t)+bx_2(t)$	$aX_1(f)+bX_2(f)$	$aX_1(\omega)+bX_2(\omega)$
Time-Scaling $x(at)$	$1/\|a\|X(f/a)$	$1/\|a\|X(\omega/a)$
Time-reversal $x(-t)$	$X(-f)$	$X(-\omega)$
Time-Shift $x(t-t_0)$	$e^{-j2\pi f t_0}X(f)$	$e^{-j\omega t_0}X(\omega)$
Frequency Shift $x(t)e^{j\omega_0 t}$	$X(f-f_0)$	$X(\omega-\omega_0)$
Differentiation in time	$\frac{d}{dt}x(t) \longleftrightarrow j2\pi f X(f)$	$\frac{d}{dt}x(t) \longleftrightarrow j\omega X(\omega)$
Frequency Differentiation	$-j2\pi t x(t) \longleftrightarrow \frac{d}{df}X(f)$	$-jtx(t) \longleftrightarrow \frac{d}{d\omega}X(\omega)$
Convolution in time $x(t)*h(t)$	$X(f)H(f)$	$X(\omega)H(\omega)$
Frequency convolution $x_1(t) * x_2(t)$	$X_1(f)* X_2(f)$	$\frac{1}{2\pi}[X_1(\omega)* X_2(\omega)]$
Integration $\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{X(f)}{j2\pi f}+0.5\,X(0)\,\delta(f)$	$\frac{X(\omega)}{j\omega}+\pi X(0)\,\delta(\omega)$
Parseval's theorem $\int_{-\infty}^{+\infty} x^2(t)dt$	$\int_{-\infty}^{+\infty} \|X(f)\|^2 df$	$1/2\pi \int_{-\infty}^{+\infty} \|X(\omega)\|^2 d\omega$

4.3 DISTORTIONLESS TRANSMISSION:

In several application such as signal amplification or message signal transmission over communication channel, we require that the output waveform be a replica of the Input waveform

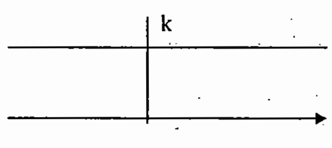
→ Transmission is said to be distortionless if the input and output have identical waveshapes within a multiplicative constant (or) a delayed output that retains the input waveform is considered to be distortionless.

→ For distortionless transmission, the input x(t) and output y(t) satisfies the condition

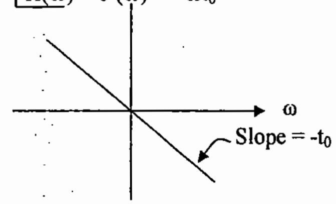
$y(t) = k\,x(t - t_0)$

$H(\omega) = \dfrac{Y(\omega)}{X(\omega)} = Ke^{-j\omega t_0}$

$|H(\omega)| = k$



$\angle H(\omega) = \theta(\omega) = -\omega t_0$



→ For distortionless transmission, magnitude response must be a constant, phase response must be a linear function of ω with slope $-t_0$, where t_0 is delay in output with respective to input.

→ If the gain is not constant over the required frequency range, we have amplitude distortion. If the phase shift is not linear with frequency, we have phase distortion as the signal undergoes different delays for different frequencies.

→ Ideal filters are noncausal, unstable and physically unrealizable.

→ For a physically realizable system, h(t) must be causal i.e., ~~h(t) = 0~~ for t < 0. In the frequency domain, this condition is known as **Paley – Wiener Criterion** which states that the necessary and sufficient condition for the amplitude response | H (ω)| to be realizable is

$$\int_{-\infty}^{+\infty} \left| \frac{\ln |H(\omega)|}{1+\omega^2} \right| d\omega < \infty$$

→ Phase delay $t_p(\omega)$ is the delay occurring at a Single frequency. As the signal propagates from source to destination the amount of delay caused is known as phase delay.

$$\boxed{t_p(\omega) = -\frac{\theta(\omega)}{\omega}}$$

→ Phase delay is not necessarily the true signal delay. A steady Sinusoidal signal doesn't carry information. Information can be transmitted only by applying some appropriate change to Sinusoidal wave. Suppose that a slowly varying signal is multiplied by a Sinusoidal wave so that resulting modulated wave consists of a narrow group of frequencies. When this modulated wave is transmitted through the channel, we find that there is a delay between envelope of Input and received signal. This is known as envelope (or) group delay (True signal delay)

$$\boxed{t_g(\omega) = -d\theta(\omega)/d\omega}$$

P4.3.1 For a linear phase channel, what is t_p & t_g ?

P4.3.2 The system under consideration is an RC LPF with R = 1 k Ω & C = 1 μ F

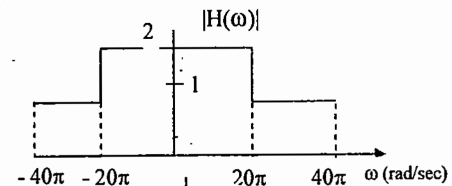
a) Let H (f) denote the frequency response of RC LPF. Let f_1 be the highest frequency component such that $0 \le |f| \le f_1$, $\dfrac{|H(f_1)|}{|H(0)|} \ge 0.95$ then f_1 (in Hz) is ________

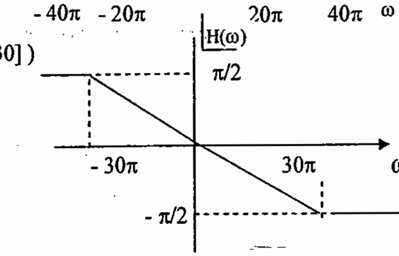
a) 327.8 b) 163.9 c) 52.2 d) 104.4

b) Let $t_g(f)$ denote the group delay of RC LPF and f_2 = 100 Hz, then $t_g(f_2)$ in msec, is ________

a) 0.717 b) 7.17 c) 71.7 d) 4.505

P4.3.3 Consider a distortionless transmission system H(ω) with magnitude and phase response as shown in figure. If an input signal x(t) = 2Cos 10πt + Sin 26 π t is given to the system the output will be ________





a) 4 Cos 10 πt + Sin 26π t

b) 8 Cos 10 πt + Sin 26π t

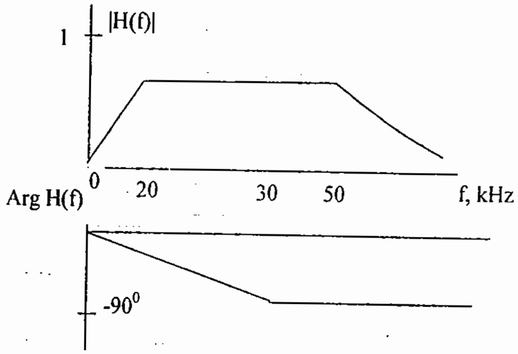
c) 4 Cos (10 πt - π/6) + Sin (26π t – [13 π/ 30])

d) 8 Cos (10 πt - π/2) + Sin (26π t – π/2)

P4.3.4 An RC LPF with a time constant of 16 msec is excited by a modulated signal x(t) = Sin (2πt) Cos(20πt). Find the phase and group delays at 10 Hz?

Ans: 12.54 msec, 7.95 msec

P4.3.5 Suppose a transmission system has the frequency response as shown in figure. For what range of frequency there is no distortion ?

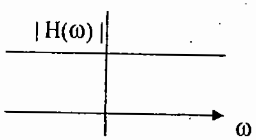


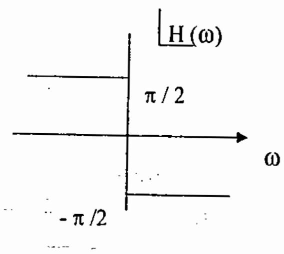
4.4. HILBERT TRANSFORM: (H.T)

The Hilbert transform is an operation that shifts the phase of x(t) by $-\pi/2$, while the amplitude spectrum of the signal remains unaltered. An ideal H.T. is an all pass 90^0 phase shifter. H.T. is used in number of application such as representation of band pass signals, phase shift modulators, generation of SSB.

$x(t) \longrightarrow \boxed{\dfrac{1}{\pi t}} \longrightarrow \hat{x}(t) = x(t) * 1/\pi t$

Frequency response of the H.T. = $-j\text{sgn}(\omega)$





Properties of H.T:

1) H.T. doesn't change the domain of a signal
2) H.T. doesn't alter the amplitude spectrum of a signal
3) If $\hat{x}(t)$ is H.T. of x(t), then H.T. of $\hat{x}(t)$ is $-x(t)$
4) x(t) and $\hat{x}(t)$ are orthogonal to each other.

P4.4.1 Find H.T. of (1) $x(t) = \cos\omega_0 t$ (2) $x(t) = \sin\omega_0 t$

(3) $x(t) = \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$

(4) $x(t) \cos(2\pi f_c t)$, where x(t) represents a signal band limited to B, $f_c > B$

P4.4.2 For the system shown in figure if x(t) = Cos t and h(t) = $(1/\pi t)$, then the output y(t) is __

$x(t) \longrightarrow \boxed{h(t)} \longrightarrow \boxed{h(t)} \longrightarrow \boxed{h(t)} \longrightarrow y(t)$

a) Cos t b) – Cos t c) Sin t d) – Sin t

4.5 CORRELATION:

→ It provides a measure of the similarity between 2 waveforms as the function of search parameter.

→ An application of correlation to signal detection in a radar, where a signal pulse is transmitted in order to detect a suspect target. If a target is present, the pulse will be reflected by it. If the target is not present, there will be no reflection pulse, just noise. By detecting the presence or absence of the reflected pulse we confirm the presence or absence of the target.

→ In digital communication, the important thing is that 1s and 0s in the data stream be distinguishable from each other so the receiver can reproduce the bit pattern that was transmitted.

→ Auto correlation function of an energy signal x(t) is

$$\boxed{R_x(\tau) = \int_{-\infty}^{+\infty} x(t)\, x(t-\tau)\, dt = \int_{-\infty}^{+\infty} x(t+\tau)\, x(t)\, dt}$$

ACF of power signals is

$$R_x(\tau) = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} x(t)\,x(t-\tau)\,dt = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} x(t+\tau)\, x(t)\,dt$$

Properties of ACF:-

1) ACF is an even function of τ i.e., $R_x(\tau) = R_x(-\tau)$
2) ACF at origin indicates either energy (or) power in the signal
3) Maximum value of ACF occurs at origin i.e., $|R_x(\tau)| \le |R_x(0)| \ \forall \tau$
4) $R_x(\tau) = x(\tau) * x(-\tau)$
5) F.T. of ACF is known as ESD (or) PSD

$R_x(\tau) \longleftrightarrow S_x(\omega)$ ESD / PSD

6) For an LTI system

$x(t) \longrightarrow \boxed{\text{L.T.I System } h(t)} \longrightarrow y(t)$

$Y(\omega) = X(\omega)\, H(\omega)$

$|Y(\omega)|^2 = |X(\omega)|^2\, |H(\omega)|^2$

$S_Y(\omega) = S_X(\omega)\, |H(\omega)|^2$

output spectral density = [input spectral density] [$|H(\omega)|^2$]

P4.5.1 Find the auto correlation and power in the signal

$$x(t) = 6\cos(6\pi t + \pi/4)$$

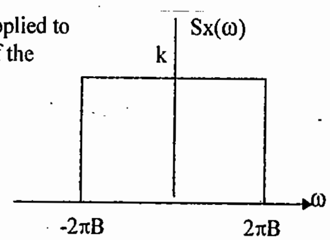
P4.5.2 Find the ACF of $x(t) = e^{-3t}u(t)$

P4.5.3 Consider a filter with $H(\omega) = \dfrac{1}{1 + j\omega}$ and input $x(t) = e^{-2t}u(t)$

a) Find the ESD of the output?

b) Show that total energy in the output is one-third of the input energy?

P4.5.4 A power signal x(t) whose PSD is shown in fig. is applied to an ideal differentiator, find the mean square value of the output of the differentiator.



P4.5.5 A power signal x(t) whose PSD is a constant K is applied to a RC low-pass filter. Find the MSV of output?

P4.5.6 Find the C.C.F of $x(t) = e^{-t}u(t)$ and $y(t) = e^{-3t}u(t)$?

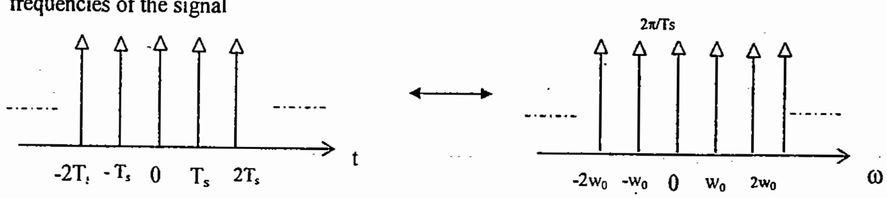
F.T. of Periodic signals:

$$x_p(t) = \sum_{n=-\infty}^{+\infty} C_n e^{jn\omega_0 t}$$

$\downarrow$ **F.T.**

$$X_p(\omega) = 2\pi \sum_{n=-\infty}^{+\infty} C_n\,\delta(\omega - n\omega_0)$$

F.T. of a periodic signal consists of a sequence of equidistant impulses located at harmonic frequencies of the signal

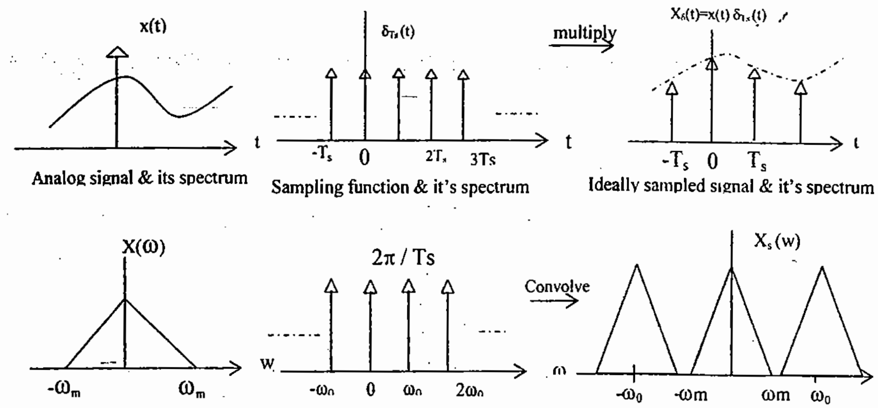


$$\sum_{n=-\infty}^{+\infty} \delta(t - nT_s) \longleftrightarrow \frac{2\pi}{T_s}\sum_{n=-\infty}^{+\infty} \delta(\omega - n\omega_0)$$

4.6 SAMPLING THEOREM:-

Digital transmission of analog signals is possible by virtue of sampling theorem and the sampling operation is performed in accordance with the sampling theorem.

(1) **Ideal sampling:-** describes a sampled signal as a weighted sum of impulses, the weights being equal to the values of the analog signal at the impulse locations.



→ An analog signal band-limited to a frequency ω_m can be sampled without loss of information if the sampling rate $\geq 2\omega_m$

→ To reconstruct the original signal, it is necessary that the B.W. of the original continuous signal being sampled is smaller than half the sampling frequency [**'Nyquist frequency'**]

→ Critical sampling rate is known as Nyquist rate N.R $= 2\omega_m$

→ If $\omega_0 < 2\omega_m$ (under sampling) causes spectral overlap, aliasing & irreversible loss of information

P4.6.1 Find the Nyquist rate & Nyquist interval for each of the following signals?

(a) $x_1(t) = \dfrac{\sin 200\pi t}{\pi t}$ (b) $x_2(t) = \left(\dfrac{\sin 200\pi t}{\pi t}\right)^2$

(c) $x_3(t) = 5\cos 1000\pi t \cos 4000\pi t$ (d) $x_4(t) = e^{-6t}u(t) * \dfrac{\sin at}{\pi t}$

P4.6.2. Let x(t) be a signal with Nyquist rate ω_0. Determine the Nyquist rate for each of the following signals.

(a) $x(t) + x(t-1)$ (b) $\dfrac{d}{dt}x(t)$

(c) $x(3t)$ (d) $x(t)\cos\omega_0 t$

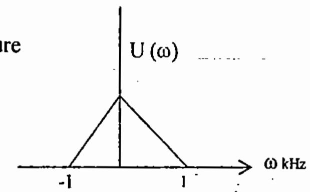
P4.6.3. Two signals $x_1(t)$ & $x_2(t)$ are band limited to 2 kHz & 3 kHz respectively find the Nyquist rae of the following signals?

(a) $x_1(2t)$ (b) $x_2(t-3)$ (c) $x_1(t) + x_2(t)$

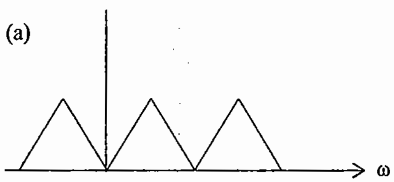
(d) $x_1(t)x_2(t)$ (e) $x_1(t) * x_2(t)$ (f) $x_1(t)\,\text{Cos}(1000\pi t)$

P4.6.4. A Signal $x(t) = 100\,\text{Cos}\,(24\pi \times 10^3 t)$ is ideally sampled with a sampling period of 50 μ secs & then passed through an ideal LPF with a cut-off frequency of 15 kHz. Which frequencies are present at the filter output?

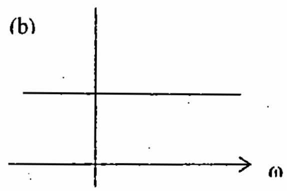
P4.6.5 The frequency spectrum of a signal is shown in figure if this signal is ideally sampled at intervals of 1 msec, then the frequency spectrum of the sampled signal will be ________



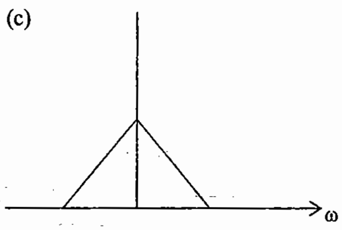
(a)



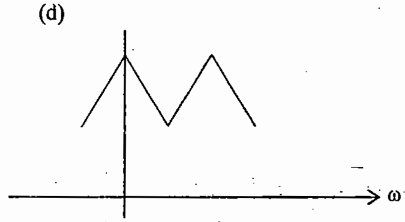
(b)



(c)



(d)



P4.6.6 A signal with 2 frequency components at 6 KHz & 12 KHz is sampled at the rate of 16 KHz and then passed though a LPF having a Cut-off frequency of 16 KHz. The o/p signal of the filter is _______

(a) is an undistorted version of original signal

(b) contains 6 KHz & spurious component of 4 KHz

(c) contains only 6 KHz components

(d) contains both components of original signal and 2 spurious components of 4 KHz & 6 KHz

P4.6.7 A signal $x(t) = 6\,\text{Cos}10\pi t$ is sampled at a rate of 14 Hz to recover the original signal, cut-off frequency of the LPF should be _________

(a) $5 < f_c < 9$ (b) 9 (c) 10 (d) 14

4.7 PREVIOUS QUESTIONS :-

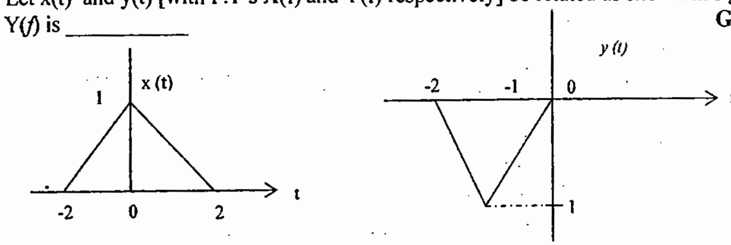
(1). The F.T of the signal $x(t) = e^{-3t^2}$ is of the following form, where A & B are constants GATE

(a) Ae^{-Bf^2} (b) $A + B|f|^2$ (c) Ae^{-Bf} (d) $Ae^{-Bf/2}$

2) Let x(t) be the input to a LTI system. The required output is 4x (t-2). The transfer function of the system should be________ GATE

a) $4\,e^{j4\pi f}$ b) $2\,e^{-j8\pi f}$ c) $4\,e^{-j4\pi f}$ d) $2\,e^{j8\pi f}$

3) Let x(t) and y(t) [with F.T's X(f) and Y(f) respectively] be related as shown in figure. Then Y(*f*) is ___________ GATE



a) $-1/2\,X(f/2)\,e^{-j2\pi f}$ b) $-1/2\,X(f/2)\,e^{j2\pi f}$ c) $-X(f/2)\,e^{j2\pi f}$ d) $-X(f/2)\,e^{-j2\pi f}$

4) The output y(t) of L.T.I. system is related to its input by the following equation GATE

$Y(t) = 0.5x(t - t_d + T) + x(t - t_d) + 0.5x(t - t_d - T)$

The filter transfer function H(ω) of such a system is given by

a) $(1+\text{Cos}\omega T)e^{-j\omega t_d}$ b) $(1+0.5\,\text{Cos}\omega T)e^{-j\omega t_d}$

c) $(1+\text{Cos}\omega T)e^{-j\omega t_d}$ d) $(1+0.5\,\text{Cos}\omega T)e^{j\omega t_d}$

5) For a signal x(t), the F.T. is X(f). Then I.F.T. of X(3f+2) is GATE

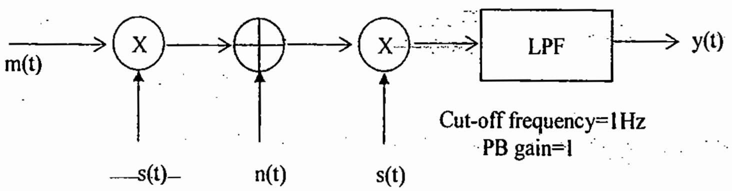
a) $1/2\,x(t/2)e^{j3\pi t}$ b) $1/3\,x(t/3)\,e^{-j4\pi t/3}$

c) $3x(3t)e^{-j4\pi t}$ d) $x(3t+2)$

6) Match the following :- IES

List I (CT function)	List II (CTFT)
A) $e^{-t}u(t)$	1) $e^{-\lvert\omega\rvert}$
B) $x(t)=\begin{cases}1, & \lvert t\rvert<1\\ 0, & \lvert t\rvert>1\end{cases}$	2) $j\omega X(\omega)$
C) $\frac{d}{dt}x(t)$	3) $\frac{1}{1+j\omega}$
D) $\frac{2}{1+t^2}$	4) $\frac{2\text{Sin}\omega}{\omega}$

7) In the figure shown $m(t) = \frac{2\sin 2\pi t}{t}$, $S(t) = \cos 200\pi t$ and $n(t) = \frac{\sin 199\pi t}{t}$. The output y(t) will be _____ **GATE**

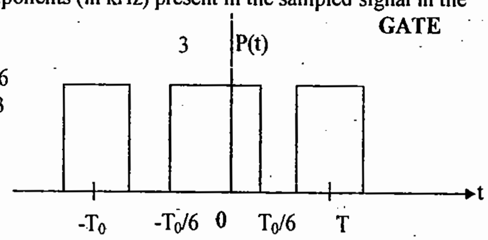


8) Consider a sampled signal $y(t) = 5 \times 10^{-6} x(t) \sum_{n=-\infty}^{+\infty} \delta(t - nT_s)$ **GATE**

where $x(t) = 10\cos(8\pi \times 10^3 t)$ and $T_s = 100\mu sec$. When y(t) is passed through an ideal LPF with a Cut-off frequency of 5 kHz, the output of the filter is_____

9) Let $x(t) = 2\cos(800\pi t) + \cos(1400\pi t)$ and x(t) is sampled with the rectangular pulse train shown in fig. The only spectral components (in kHz) present in the sampled signal in the frequency range 2.5kHz to 3.5kHz. **GATE**

a) 2.7, 3.4 b) 3.3, 3.6
c) 2.6, 2.7, 3.3, 3.4, 3.6 d) 2.7, 3.3



$T = 10^{-3}$ sec

10) Which of the following is /are not a property/properties of PSD? **GATE**

(a) $S_x(\omega)$ is real function of ω b) $S_x(\omega)$ is an even function of ω
c) $S_x(\omega)$ is non +ve i-e., $S_x(\omega) \le 0$ d) All the above

11) Match the following **IES**

List I (Application of signals)	List II (Definition)
(a) Reconstruction	(1) A sampling rate is chosen significantly > the Nyquist rate
(b) Oversampling	(2) A mixture of continuous & discrete Signals
(c) Interpolation	(3) To convert discrete sequence back to continue signal & then resample
(d) Decimation	(4) Assign values between samples & signals.

12) F.T of x(t) is $\frac{2\sin\pi\omega}{\omega}$ then F.T of $x(t)\,e^{j5t}$ is_____________ **IES**

13) What is the I.F.T. of $u(\omega)$? **IES**

14) Match the following **IES**

List I (time function	List II (Fourier spectrum/F.T)
(a) periodic function	(1) Continuous spectrum at all frequencies
(b) aperiodic function	(2) $\delta(\omega)$
(c) $\delta(t)$	(3) Line discrete spectrum
(d) $\sin \omega t$	(4) 1

15) A Signal represented by $x(t) = 5\cos(400\pi t)$ is sampled at a rate of 300 samples / sec. The resulting samples are passed through an ideal LPF with cut-off frequency of 150 HZ. Which of the following will be contained in the output of LPF ? **IES**

(a) 100 Hz (B) 100 Hz, 150 Hz (c) 50 Hz, 100Hz (d) 20,100,150 Hz

16) I.F.T of $X(\omega) = 2\pi\delta(\omega) + \pi\delta(\omega - 4\pi) + \pi\delta(\omega + 4\pi)$ is **ISRO**

(a) $1+\cos 4\pi t$ (b) $\pi(1 - \cos 4\pi t)$ (c) $2\pi(1 - \cos 4\pi t)$ (d) $2\pi(1 + \cos 4\pi t)$

17) If a C.T. signal $x(t) = 2 + \cos(50\pi t)$ is sampled with a periodic pulse function, aliasing occurs when the sampling period is **DRDO**

(a) 0.01 sec (b) 0.015 sec (c) 0.019 sec (d) 0.025 sec

18) If a CT signal $x(t) = 2 + \cos 50\pi t$ is sampled with a period pulse function of period 0.025 sec & then it is passed through an ideal LPF with a response of 1 within the range of $\frac{-\omega s}{2} \le \omega \le \frac{\omega s}{2}$ (ωs is the angular frequency) & 0 otherwise, the reconstructed signal the o/p of the LPF will be _______ **DRDO**

(a) $2 + \cos 30\pi t$ (b) $2+\cos 40\pi t$ (c) $2+\cos 50\pi t$ (d) $2+\cos 80\pi t$

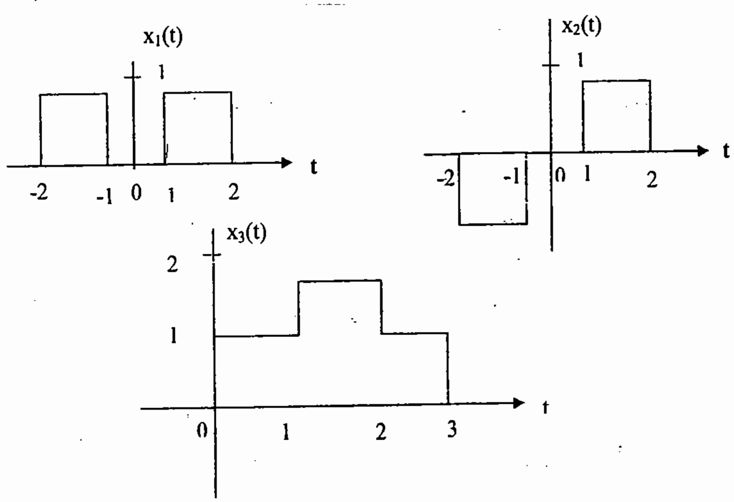
4.8 Practice Problem Set:-

4.8.1) Let $X(\omega) = \text{rect}\left(\frac{\omega - 1}{2}\right)$ Find the F.T. of the following signals?

a) $x(-t)$ b) $t\,x(t)$ c) $x(-2t+4)$

d) $x(2t-1)e^{-j2t}$ e) $\int_{-\infty}^{t} x(\tau)d\tau$

4.8.2) Given $\pi(t/T) \longleftrightarrow T\,\text{Sinc}(fT)$. Obtain F.T. of the following signals.

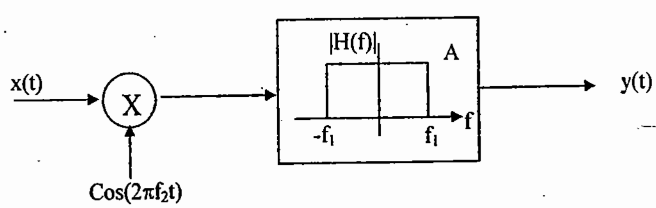


4.8.3) The F.T. of 2 signals x(t) & y(t), are defined as

$X(t) = \text{Cos}(\pi f)\pi(f)$ and $Y(f) = X(f - f_0) + X(f + f_0)$

a) Find closed-form expression for x(t) and y(t)?

b) Design the system shown in the block diagram in terms of choosing the parameters A, f_1 and f_2 so that the output is y(t)



4.8.4) Find the I.F.T of the following :

a) $X_1(f) = \frac{e^{-j4\pi f}}{1+4\pi^2 f^2}$ b) $X_2(f) = \frac{\text{Cos}(\pi f)}{1+j2\pi f} e^{-j\pi f}$

4.8.5) A signal $x(t) = 8 - 8\text{Cos}^2(6\pi t)$ is passed through an ideal LPF. The filter blocks frequencies above 5Hz. Find the output?

4.8.6) The transfer function of a system is $H(\omega) = \frac{2+2j\omega}{4+4j\omega - \omega^2}$

Find the output if input is $x(t) = e^{-t}u(t)$

4.8.7) The signal $x(t) = \text{Sinc}(0.5t) + \text{Sinc}(0.25\,t)$ is applied to a filter whose impulse response is $h(t) = A\,\text{Sinc}(\alpha t)$. For what values of A & α will the filter output equal x(t)?

4.8. 8) Find the frequency response & impulse response of a filter whose input - output relation is described by the following equation $y(t) = x(t) - 2\int_{-\infty}^{t} y(\lambda)e^{-(t-\lambda)}u(t-\lambda)d\lambda$

4.8.9) Find the output y(t) for each cascaded system. Do the 2 systems yield the same output?

a) $x(t) = \text{Cos}(\pi t) \longrightarrow$ [Phaseshift of $-\pi/2$] $\longrightarrow$ [Squarer] $\longrightarrow y(t)$

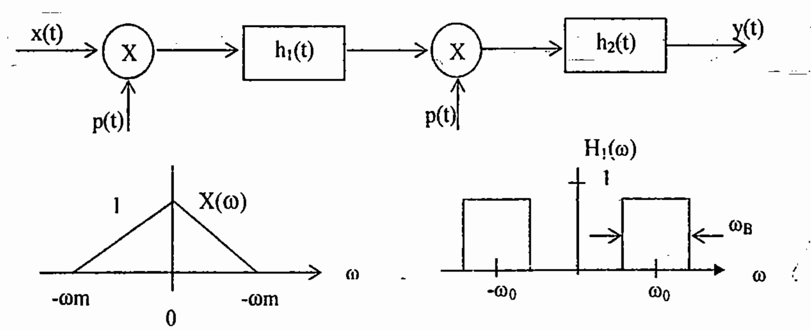
b) $x(t) = \text{Cos}(\pi t) \longrightarrow$ [Squarer] $\longrightarrow$ [Phaseshift of $-\pi/2$] $\longrightarrow y(t)$

4.8.10) The input to the system shown in figure has the spectrum shown.

Let $P(t) = \text{Cos}\,\omega_0 t$, $\omega_0 >> \omega_m$

Find the spectrum $Y(\omega)$ of the output if $h_2(t) = \frac{\text{Sin}\,\omega_B t}{\pi t}$

Consider the cases $\omega_m > \omega_B$ and $\omega_m \leq \omega_B$.

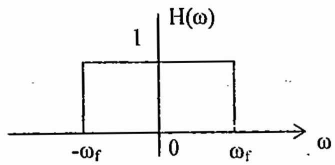


4.8.11) The input signal $x(t) = \frac{\sin\omega_1 t}{t} + \frac{\sin\omega_2 t}{t}$, $\omega_1<\omega_2$ is applied to the system shown in figure.

find y(t) if a) $0<\omega_f<\omega_1$

b) $\omega_1<\omega_f<\omega_2$

c) $\omega_2<\omega_f$



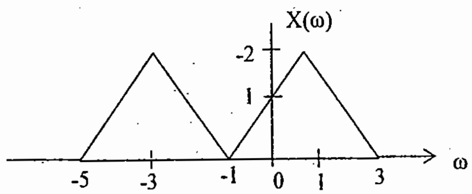
4.8.12) Find x(t) if $X(\omega)=j\frac{d}{d\omega}\left\{\frac{e^{j2\omega}}{1+j(\omega/3)}\right\}$

4.8.13) For the F.T. X(w) shown in figure, evaluate the following quantities without calculating x(t)?

a) $\int_{-\infty}^{+\infty}|x(t)|^2dt$

b) $\int_{-\infty}^{+\infty}x(t)e^{j3t}dt$

c) x(0) d) arg{x(t)}



4.8.14). Find the I.F.T of $X(\omega) = |X(\omega)|e^{j\angle X(\omega)}$, where

$|X(\omega)| = 2[u(\omega+3)- u(\omega-3)]$ and $\angle X(\omega) = \frac{-3\omega+\pi}{2}$

4.8.15). The output & input of a causal LTI system are related by the

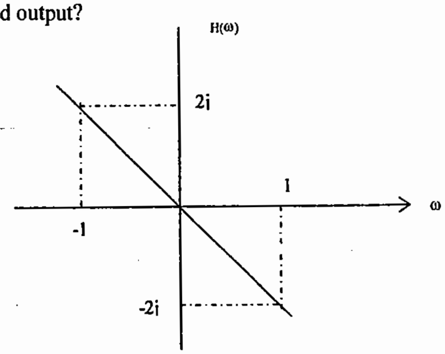
D.E $\frac{d^2y(t)}{dt^2} + \frac{6dy(t)}{dt} + 8y(t)=2x(t)$

(a) Find the I.R.? (b) Find the response if $x(t)= t\,e^{-2t}\,u(t)$

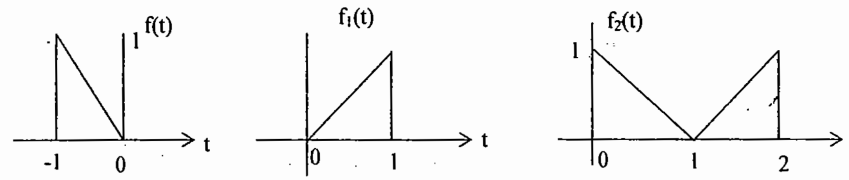
4.8.16) A casual LTI filter has the frequency response H(ω) shown in figure. For each of the input signals given below, find output?

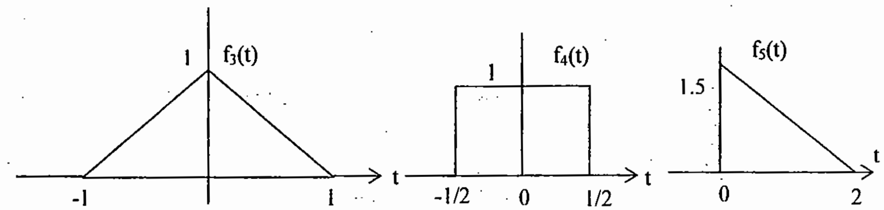
(a) $x(t) = \sin\omega_0 t\, u(t)$

(b) $X(\omega) = \frac{1}{j\omega\,(6+j\omega)}$



4.8.17) The F.T of a triangular pulse $f(t)$ shown in figure is $F(\omega) = \frac{e^{j\omega}-j\omega e^{j\omega}-1}{\omega^2}$ using this find the F.T of the signals shown in figure?





4.8.18) What percentage of the total energy in the signal $f(t) = e^{-t}u(t)$ is contained in the frequency band $|\omega| \le 7$ rad./sec ? Hint: Rayleigh's theorem.

4.8.19) If the input to a system is $\cos 10\pi t + 2\cos 20\pi t$, tell what kinds of distortion, if any, a system introduces if its output is

(a) $\cos(10\pi t - \pi/4) + 5\cos(20\pi t - \pi/2)$

(b) $\cos(10\pi t - \pi/4) + 2\cos(20\pi t - \pi/4)$

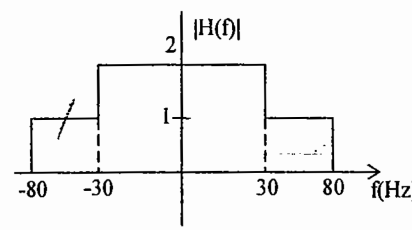
(c) $\cos(10\pi t - \pi/4) + 2\cos(20\pi t - \pi/4)$

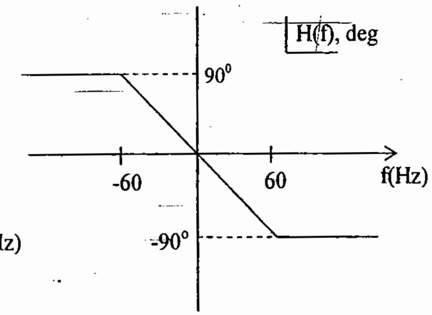
(d) $2\cos(10\pi t - \pi/4) + 4\cos(20\pi t - \pi/8)$

4.8.20) Consider a fixed, liner system with amplitude and phase responses as shown. Obtain the output for the following input & tell what type of distortion, if any results.

(a) $x(t) = \cos 20\pi t + \cos 60\pi t$

(b) $x(t) = \cos 130\pi t + \cos 220\pi t$





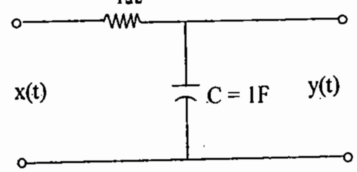
4.8.21) Find the average auto Correlation of $x(t) = \cos 2\pi t + 2\cos 4\pi t$

4.8.22) A signal $x(t) = e^{-2t}u(t)$ is passed through an ideal LPF with cut-off frequency $\omega_m = 1$rad/sec. Find the ratio of output energy to input energy?

4.8.23) Find the average power at the output of the circuit shown in figure, if x(t) is a finite power signal with PSD

(i) $S_x(\omega) = k$

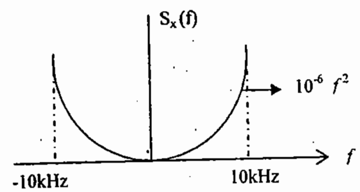
(ii) $S_x(\omega) = \pi[\delta(\omega + 2\pi) + \delta(\omega - 2\pi)]$



4.8.24) The P.S.D of a signal is shown figure

(a) Find the normalized average power of the signal.

(b) Find the amount of power contained in the frequency range 5-10 KHZ.

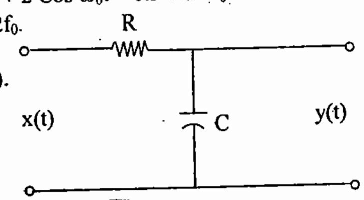


4.8.25) The input to RC N.W shown in figure is $x(t) = 1 + 2\cos\omega_0 t + 0.5\sin\omega_0 t$

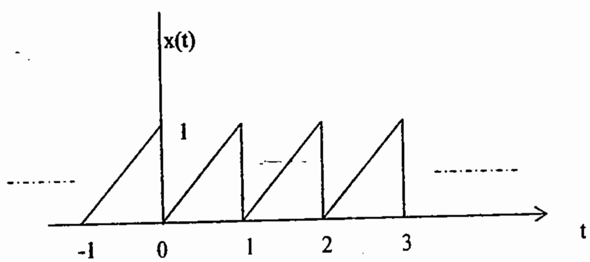
Assume that the 3-dB frequency of the N.W. is $f_c = 2f_0$.

(a) Find the input & output spectral densities

(b) Find the normalized average power content of y(t).



4.8.26) Find the F.T of the periodic signal shown in figure?



4.8. 27) 2 signals $x_1(t) = 10\cos 1000\pi t$ and $x_2(t) = 10\cos 50\pi t$ are both sampled with $f_s = 75$ Hz. Show that the 2 sequences of samples so obtained are identical?

4.8.28) A signal $x(t) = \cos 200\pi t + 2\cos 320\pi t$ is ideally sampled at f_s=300Hz of the sampled signals is passed through an ideal LPF with a cut-off frequency of 250Hz, What frequency components will appear in the output?

Ans :- 100, 140, 160 & 200 Hz components.

4.8.29) A signal $x(t) = \sin(150\pi t)$ is ideally sampled at 80 Hz. Describe the signal y(t) that is recovered if the sampled signal is passed through

(a) An ideal LPF with cut-off $f_c = 100$Hz.

(b) An ideal BPF with PB between 60HZ & 80 HZ.

4.8.30) Determine whether (or) not the C.T. systems with the following frequency response are causal ?

(a) $H(f) = \text{Sinc}(f)$ (b) $H(f) = Ae^{j2\pi f}$ (c) $H(f) = \text{Sinc}(f)\,e^{-j\pi f}$

REVIEW NOTES:

Chapter 5. LAPLACE TRANSFORM

- → L.T expresses signals as linear combination of complex exponentials, which are eigen functions of D.E which describe continuous – time L.T.I systems.
- → The primary role of the L.T in engineering is the transient & stability analysis of Causal L.T.I systems.
- → L.T provides a broader characterization of systems & their interaction with signals than is possible with F.T.
- → In addition to its simplicity, many design techniques in circuits, filters & Control systems have been developed in L.T. domain.
- → Consider applying an INPUT of the form $x(t)=e^{st}$ (where $s = \sigma + j\omega$ then the output is $y(t)=e^{st}H(s)$ where H(s) is transfer function of the system.
- → L.T. of a general signal $x(t)=X(s)=\int_{-\infty}^{+\infty} x(t)e^{-st}dt \longrightarrow$ (1)

$$= F\{x(t)e^{-\sigma t}\}$$

- → $e^{-\sigma t}$ may be decaying (or) growing depending on whether 'σ' is +ve (or) –ve.

$$x(t) \longleftrightarrow X(s)$$

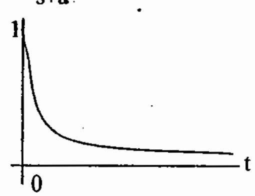
- → **Region of Convergence (R.O.C) of L.T:-**

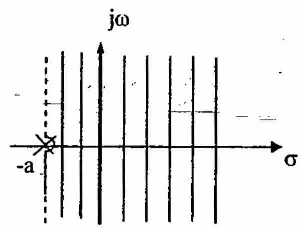
The range of values of 'S' for which eq(1) is satisfied i.e., $\int_{-\infty}^{+\infty} |x(t)\, e^{-\sigma t}|\, dt < \infty$ is known as R.O.C of L.T.

- → L.T. calculated on the jω-axis (σ = 0) is F.T.

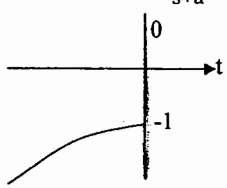
5.1 L.T. OF STANDARD SIGNALS:

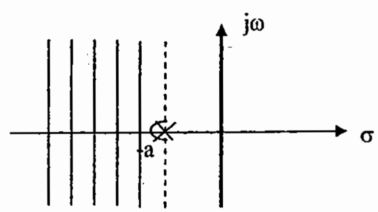
1) $x(t)=e^{-at}u(t), a>0 \longleftrightarrow \frac{1}{s+a}$; Re{s} > -a



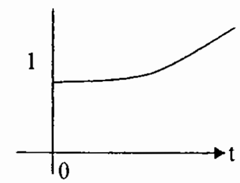


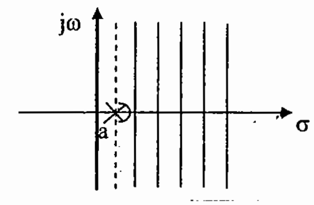
2) $x(t) = -e^{-at}u(-t) \longleftrightarrow \frac{1}{s+a}$; Re{s} < -a



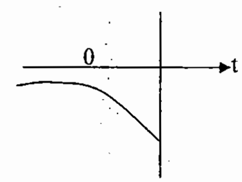


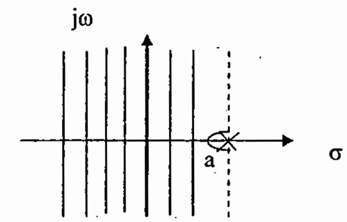
3) $x(t)=e^{at}u(t) \longleftrightarrow \frac{1}{s-a}$; Re{s} > a





4) $x(t) = -e^{at}u(-t) \longleftrightarrow \frac{1}{s-a}$; Re{s} < a





5) unit impulse $\delta(t) \longleftrightarrow 1$; ROC: entire s-plane

6) unit step function $u(t) \longleftrightarrow \frac{1}{s}$; Re{s}>0

Note: If the L.T. X(s) of x(t) is rational, then if x(t) is right sided the ROC is the region in the s-plane to the right of the rightmost pole and if x(t) is left sided, ROC is left of the left most pole.

5.1 PROPERTIES OF L.T

1) **Linearity**: - **If $x_1(t) \longleftrightarrow X_1(s)$ with ROC=R_1**
$x_2(t) \longleftrightarrow X_2(s)$ with ROC=R_2
then $ax_1(t)+bx_2(t) \longleftrightarrow aX_1(s)+bX_2(s)$ with ROC=$R_1 \cap R_2$

P5.1.1 Given $X(s) = \frac{2s+5}{S^2+5s+6}$, find all the time-domain signals?

P5.1.2. Find the L.T. of the following signals with R.O.C?

1) $x_1(t) = e^{-t}u(t)+e^{-3t}u(t)$

2) $x_2(t) = e^{-2t}u(t)+e^{4t}u(-t)$

3) $x_3(t) = e^{-t}u(-t)+e^{5t}u(t)$

4) $x_4(t) = 1 \quad \forall \quad t$

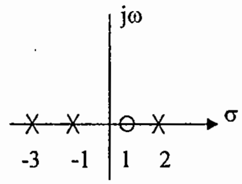
5) $x_5(t) = sgn(t)$

6) $x_6(t) = \delta(3t) + u(2t)$

7) $x_7(t) = e^{-\alpha|t|}$ $0 < \alpha < 1$

P5.1.3 Consider the signal $x(t)=e^{-5t}+e^{-\beta t}u(t)$ & its L.T. is X(s). What are the constraints placed on the real & imaginary parts of β if the R.O.C of X(s) is Re {s} > -3 ?

P51.4 How many possible ROCs are there for the pole-zero plot shown in fig(1)?



Fig(1)

2) **Time-shifting**: - $x(t) \longleftrightarrow X(s)$, ROC=R
Then $x(t-t_0) \longleftrightarrow e^{-st_0} X(s)$, with ROC=R

P5.1.5 Find the L.T. of the following signals?
a) unit ramp starting at t=a b) $x(t) = u(t-5)$ c) $y(t) = e^{5t}u(-t+3)$

P5.1.6 Find the I.L.T. of $Y(s) = \dfrac{e^{-3s}}{(s+1)(s+2)}$, $\sigma > -1$

P5.1.7 Consider the signal $x(t)=e^{-5t}u(t-1)$ with L.T. X(s)
a) Find X(s) with R.O.C.?
b) Find the values of 'A' &'t_0' such that the L.T. G(s) of $g(t) = Ae^{-5t}u(-t-t_0)$ has same algebraic form as X(s). What is the R.O.C corresponding to G(s)?

3) **Shift in S-domain**:- $x(t) \longleftrightarrow X(s)$ with ROC = R
then $e^{S_0 t}x(t) \longleftrightarrow X(s-s_0)$ with ROC = R + Re{S_0}

P5.1.8. Find the L.T. of 1) $x_1(t) = \text{Cos}\omega_0 t u(t)$ 2) $x_2(t) = te^{-3t}u(t)$ 3) $x_3(t) = e^{-at}\,\text{Sin}\omega_0 t u(t)$

P5.1.9 Let x(t) be a signal that has a rational L.T. with exactly 2 poles located at s=-1 and s = - 3 If $g(t)=e^{2t}x(t)$ & G(ω) converges, determine whether g (t) is
a) Left-sided (b) right-sided (c) two-sided (d) finite-duration.

4) **Time-reversal**:- $x(t) \longleftrightarrow X(s)$ then $x(-t) \leftrightarrow X(-s)$, ROC = -R

P5.1.10 Let $g(t) = x(t) + \alpha x(-t)$ where $x(t) = \beta e^{-t}u(t)$ &$G(s) = \dfrac{S}{S^2-1}$ $-1 < \text{Re}\{s\} < 1$,
Find α & β?

Differentiation in time:- $x(t) \leftrightarrow X(s)$ with ROC = R then $\dfrac{dx(t)}{dt} \leftrightarrow SX(S)$ with ROC = R

P5.1.11. Consider 2 right-sided signals x(t) & y(t) related through the equation
$\dfrac{dx(t)}{dt} = -2y(t)+\delta(t)$ & $\dfrac{dy(t)}{dt} = 2x(t)$ Find X(s) & Y(s) with ROCs?

P5.1.12. Find the LT of following signals with ROC?

(a) $x_1(t) = \dfrac{d^2}{dt^2}[e^{-3(t-2)}u(t-2)]$

(b) $x_2(t) = e^{-t}\dfrac{d}{dt}[e^{-(t+1)}u(t+1)]$

Differentiation in S-domain:-

$x(t) \longleftrightarrow X(s)$ RoC=R, then $tx(t) \leftrightarrow \dfrac{-d}{ds}X(s)$ RoC=R

P5.1.13. Find the L.T. of (1) $y(x) = t^2u(t-1)$ (2) $y(t) = te^{-at}u(t)$

P5.1.14. Find the I.L.T of $X(s) = \log\left[\dfrac{S+5}{S+6}\right]$?

P5.1.15. Find the ILT of

(a) $X(s) = \dfrac{4}{(s+2)(s+1)^3}$ (b) $X(s) = e^{-2s}\dfrac{d}{ds}\left(\dfrac{1}{(s+1)^2}\right)$

Convolution in time:-

If $x(t) \longleftrightarrow X(s)$ with ROC = R_1 & $h(t) \longleftrightarrow H(s)$ with ROC=R_2
Then $x(t) * h(t) \longleftrightarrow X(s)H(s)$ ROC=$R_1 \cap R_2$

L.T. of impulse response is known as system (or) transfer function

P5.1.16. Solve the following D.E. $y(t) + \int_0^\infty y(\tau)x(t-\tau)d\tau = x(t) + \delta(t)$?

P5.1.17. Consider a signal $y(t) = x_1(t-2) * x_2(-t+3)$ where $x_1(t)=e^{-2t}u(t)$&$x_2(t)=e^{-3t}u(t)$.
Find Y(s) with ROC?

P5.1.18. Find the impulse response of a linear causal system describe by the equation.
$\dfrac{dy(t)}{dt} + 4y(t) + 3\int_{-\infty}^{t} y(\tau)d\tau = x(t)$ Also determine response to an excitation $x(t)=u(t)+\delta(t)$?

P5.1.19. Find the transfer function & impulse response of a filter whose input –output relation is described by $y(t) = x(t) + \int_{-\infty}^{t} y(\lambda)\, e^{-3(t-\lambda)} u(t-\lambda) d\lambda$.

Frequency Integration:- $\frac{x(t)}{t} \longleftrightarrow \int_{s}^{\infty} X(s) ds$.

P5.1.20. Find the L.T. of $\frac{\sin\omega_0 t}{t}\, u(t)$?

Integration in time:- $\int_0^t x(\tau) d\tau \longleftrightarrow \frac{X(s)}{s}$

5.2 UNILATERAL L.T.:- $\chi(s) = \int_0^{\infty} x(t) e^{-st} dt$.

P5.2.1 Find the U.L.T. of the following signals & find the Roc?
(a) $x(t) = e^{-3t} u(t+2)$ (b) $x(t) = \delta(t+2)$

Differentiation in time:- $\frac{d}{dx} x(t) \longleftrightarrow S\chi(s) - x(0)$

$$\frac{d^2}{dt^2} x(t) \longleftrightarrow S^2\chi(s) - Sx(0) - x^1(0)$$

P5.2.2 Solve the differential equation

$$y^{11}(t)+5y^1(t)+6y(t)=2x^1(t)+x(t), \quad x(t) = u(t),\ y(0)=1; y^1(0)=2$$

P5.2.3 Consider a system with transfer function $H(s) = \frac{S-2}{s^2+4s+4}$ Find the steady-state response when the input applied is 8cos2t?

10) **Initial & final value theorem:-**

$$x(0) = \underset{s\to\infty}{Lt}\, SX(s) \qquad x(\infty) = \underset{s\to 0}{Lt}\, SX(s)$$

P5.2.4 Find the initial & final values for the following L.T.?

a) $X(s) = \frac{2S+5}{s^2+5s+6}$ b) $X(s) = \frac{4S+5}{2s+1}$ c) $X(s) = \frac{12(s+2)}{S(s^2+4)}$

P5.2.5 A LTI, Causal continuous time system has a rational transfer function with simple poles at $S = -2$ and $S = -4$ and one of the simple zero at $S = -1$. A unit step u(t) is applied as the input of the system. At steady state, the output has a constant value of 1. Find the impulse response?

P5.2.6 Find the final value of the output a system whose transfer function is $H(s) = \frac{S+3}{s^2+4s+5}$ when system is excited by unit step input?

P5.2.7 Find the S.S.R of a system $\frac{Y(s)}{X(s)} = \frac{1}{s^2+3s+2}$ when unit step input is applied?

A) 0.5 B) 1 C) 1.5 D) 0

P5.2.8 Consider a system described by the transfer function. $G(s) = \frac{2s+3}{s^2+2s+5}$ when it is subjected to an input of 10u(t), the initial & final values of the response are given by

5.3. CAUSALITY & STABILITY:-

→ For a Causal system h(t)=0;t<0 and thus is right-sided & the ROC associated with the system function for a Causal system is a right-half plane.
→ An LTI system is stable if and only if the ROC of the system function H(s) includes jω axis.

P5.3.1 Given $H(s) = \frac{S-1}{(s+1)(s-2)}$. Find h(t) for each of the following Cases

(i) stable (ii) Causal (iii) neither Causal nor stable.

P5.3.2 Given $X(s) = \frac{5-S}{S^2-S-6}$ & F.T. of the signal is defined then x(t) is ________

P5.3.3 Consider an LTI system for which we are given the following information

$X(s) = \frac{S+2}{S-2}$ and $x(t) = 0, t>0$ and output is $y(t) = \frac{-2}{3} e^{2t} u(-t) + \frac{1}{3} e^{-t} u(t)$

a) Find T.F & R.O.C.?
b) Find the output if the input is $x(t) = e^{3t}\ \forall\ t$?

P5.3.4 Consider an LTI system with input x(t) and output y(t) related as

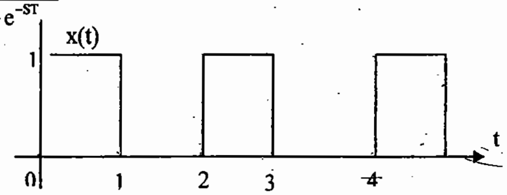
$$\frac{dy(t)}{dt} + 3y(t) = \frac{d^2x(t)}{dt^2} + \frac{dx(t)}{dt} - 2x(t)$$

Find the T.F. of inverse system. Does a stable & Causal inverse system exist?

L.T. of periodic signals:-

$$X_p(s) = \frac{\int_0^T x(t) e^{-st} dt}{1 - e^{-ST}}$$

Eg. Find the L.T. of the periodic signal shown in fig.?



5.4 PREVIOUS QUESTIONS:-

1) $L\{f(t)\} = \dfrac{s+2}{s^2+1}$, $L\{g(t)\} = \dfrac{s^2+1}{(s+2)(s+3)}$ then $L\left[\int_0^t f(\tau)g(t-\tau)d\tau\right]$ is ______ **GATE**

2) T.F. of a system is $H(s) = \dfrac{1}{s^2(s-2)}$ then I.R. of the system is ________ **GATE**

A) $t^2 * e^{-2t}u(t)$ B) $(t*e^{2t})\,u(t)$ C) $te^{-2t}u(t)$ D) $te^{2t}u(t)$

3) What does the T.F. of a system describe for the system? **IES**

A) Only Z.I.R B) Only Z.S.R
C) Both Z.I.R & Z.S.R D) neither Z.I.R nor Z.S.R.

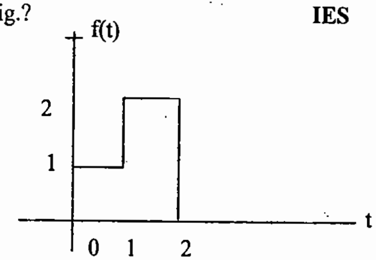
4) What is the L.T. of the waveform shown in Fig.? **IES**

A) $F(s) = \dfrac{1}{s} + \dfrac{e^{-s}}{s} - \dfrac{2}{s}e^{-2s}$

B) $F(s) = \dfrac{1}{s} - \dfrac{e^{-s}}{s} + \dfrac{2}{s}e^{-2s}$

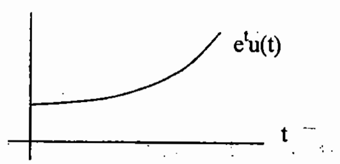
C) $F(s) = \dfrac{1}{s} + \dfrac{e^{s}}{s} + \dfrac{2}{s}e^{2s}$

D) $F(s) = \dfrac{1}{s} - \dfrac{e^{-2s}}{s} - \dfrac{2}{s}e^{-s}$



5) For the signal shown **IES**

A) Only F.T. exists
B) Only L.T. exists
C) Both F.T. & L.T. exists
D) Neither L.T. nor F.T. exists



6) An LTI system is having transfer function $\dfrac{s^2+1}{s^2+2s+1}$ & input x(t)=sin(t+1) is in steady state. The output is sampled at ω_s rad/sec to obtain final output {y(k)} which of the following is true? **GATE**

A) $y(\bullet) = 0$ for all ω_s (B) $y(\bullet) \neq 0$ for all ω_s
C) $y(\bullet) \neq 0$ for $\omega_s > 2$ but zero for $\omega_s < 2$ (D)) $y(\bullet) = 0$ for $\omega_s > 2$ but nonzero for $\omega_s < 2$

7) What is the output as $t \longrightarrow \infty$ for a system that has T.F. $G(s) = \dfrac{2}{s^2-s-2}$ when subjected to a step input? **IES**

A) -1 B) 1 C) 2 D) unbounded

8) L.T. of δ(t-1) is ______ **IES**

9) L.T. of $x(t) = -e^{2t}u(t) * tu(t)$ is ______ **IES**

10) L.T. of the waveform shown in fig10 is $\dfrac{1}{s^2}\left(1 + Ae^{-s} + Be^{-4s} + Ce^{-6s} + De^{-8s}\right)$ then D is ________ **IES**

A) -0.5 B) -1.5 C) 0.5 D) 2

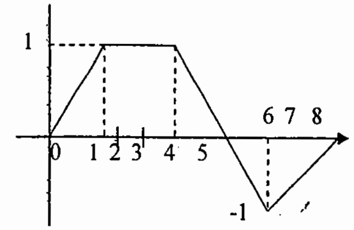


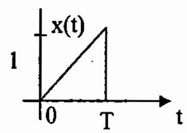
Fig. (10)

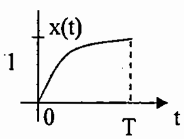
11) The I.L.T. of is $X(s) = \dfrac{8s+10}{(s+1)(s-2)^3}$ ______ **IES**

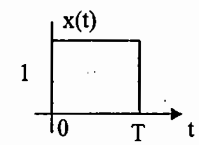
12) Given $X(s) = e^{-s}\left[\dfrac{-2}{s(s+2)}\right]$ then initial & final values of x(t) are respectively ________

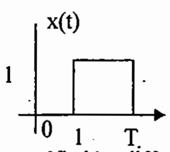
A) 0 & 1 B) 0 & -1 C) 1 & 1 D) -1 & 0 **IES**

13) The L.T. X(s) of a signal is $X(s) = \dfrac{1-e^{-ST}}{s}$, then waveshape of x(t) is ________ **IES**

A) 

B) 

C) 

D) 

14) What is the impulse response h(t) for a system specified by differential equation $y^{11}(t)+5y^{1}(t)+6y(t)=x^{1}(t)+x(t)$ **IAS**

15) The response of a system to a unit ramp input is $\dfrac{tu(t)}{2} - \dfrac{1}{8}u(t) + \dfrac{1}{8}e^{-4t}u(t)$. Then the unit impulse response of the system is ________ **IES**

16) Let a signal $a_1\sin(\omega_1 t + \phi_1)$ be applied to a stable LTI system. Let the corresponding s.s output be represented as $a_2\,F(\omega_2 t + \phi_2)$. Then which of the following statement is TRUE?

A) F is not necessarily a "sine" or "Cosine" function but must be peroidic & $\omega_1 = \omega_2$
B) F must be "Sine" or "Cosine" with $a_1 = a_2$ **GATE**
C) F must be "Sine", $\omega_1 = \omega_2$, $a_1 \neq a_2$
D) F must be "Sine" (or) "Cosine" functions with $\omega_1 = \omega_2$.

17) In what range should Re{s} remain so that the L.T. of the function $e^{(a+2)t+5}u(t)$ exits? **GATE**

A) $Re\{s\} > a+2$ B) $Re\{s\} > a+7$ C) $Re\{s\} < 2$ D) $Re\{s\} > a+5$

5.5 PRACTICE PROBLEM SET:-

P5.5.1 Find the L.T. & associated R.O.C of the following signals?

a) $x(t)=e^{-2t}u(t)+e^{-3t}u(t)$

b) $x(t)=e^{2t}u(-t)+e^{3t}u(-t)$

c) $x(t)=e^{-2|t|}$

d) $x(t)=e^{-t}u(t)+e^{3t}u(-t)$

P5.5.2 Find the corresponding signal for each of the following L.T & their associated ROC?

a) $\dfrac{s}{s^2+9}, \mathrm{Re}\{s\}<0$ (b) $\dfrac{(s+1)^2}{s^2-s+1}, \mathrm{Re}\{s\}>\dfrac{1}{2}$

c) $\dfrac{s+1}{s^2+5s+6}, -3<\mathrm{Re}\{s\}<-2$

P5.5.3 Given the transform pair $\cos 2t\,u(t) \longleftrightarrow X(s)$, find the time signals corresponding to the following L.T.

a) $(s+1)X(s)$ (b) $X(3s)$ (c) $X(s+2)$

d) $s^{-2}X(s)$ (e) $\dfrac{d}{ds}\{e^{-3s}X(s)\}$

P5.5.4 Consider an LTI system for which the system function H(s) has the pole –zero pattern shown in fig.(5.5.4)?

a) Indicate all possible RoCs

b) For each RoC of part (a), specify whether the system is stable and/or Causal

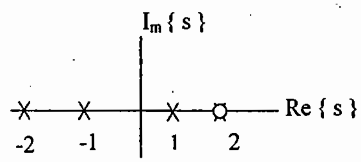


Fig5.5.4

P5.5.5 Find the I.L.T. of $X(s)=\dfrac{s^2+2s+5}{(s+3)(s+5)^2}, \mathrm{Re}\{s\}>-3$

P5.5.6 Solve the differential equation

$$\frac{d^2y(t)}{dt^2}+5\frac{dy(t)}{dt}+6y(t)=-4x(t)-3\frac{dx(t)}{dt}$$

Given $y(0^-)=-1, y^1(0^-)=5, x(t)=e^{-t}u(t)$

P5.5.7 A non Causal signal $x(t)=e^{-3t}u(t)+e^{-t}u(-t)$ is the excitation of a filter whose impulse response is $h(t)=\delta(t)-e^{-2|t|}$ .Find the response?

P5.5.8 The step response of C.T. LTI system is given by $(1-e^{-t})u(t)$. For a certain unknown input x(t), the output y(t) is observed to be $(2-3e^{-t}+e^{-3t})u(t)$. Find the input x(t)?

P5.5.9 Determine the initial & final values of *f(t),* if they exist, given that

a) $F(s)=\dfrac{s^2+10s+16}{s(s+1)^2(s+2)}$ (b) $F(s)=\dfrac{s^2-2s+1}{(s-2)(s^2+2s+4)}$

P5.5.10 Find the U.L.T. of the periodic signals shown in fig(5.5.10)?

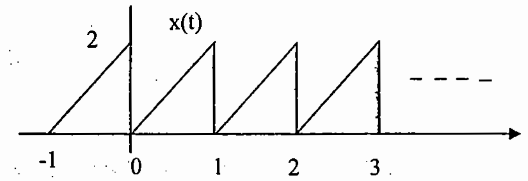


Fig5.5.10

P5.5.11 Consider a C.T.L.T.I. system for which the input x(t) & output y(t) are related by the D.E.

$\dfrac{d^2y(t)}{dt^2}-\dfrac{dy(t)}{dt}-2y(t)=x(t)$ Find impulse response, h(t) under the following conditions

a) Stable (b) Causal (c) neither stable nor Causal

P5.512 The signal $y(t)=e^{-2t}u(t)$ is the output of a Causal all-pass system for which the system function is . $H(s)=\dfrac{s-1}{s+1}$

a) Find atleast 2 possible input's x(t) that could produce y(t)?

b) What is the input x(t) if it is a stable signal?

P5.5.13 The relation between the input x(t) & output y(t) of a system is described by D.E.

$$\frac{d^2y(t)}{dt^2}+\frac{dy(t)}{dt}+5y(t)=\frac{d^2x(t)}{dt^2}-2\frac{dx(t)}{dt}+x(t)$$

a) Does this system have a stable & Causal inverse?

b) Find D.E. of the inverse system?

P5.5.14 A Causal LTI system has the B.D. shown in fig.5.5.14. Find the D.E. relating input x(t) & output y(t) of the system, is the system stable?

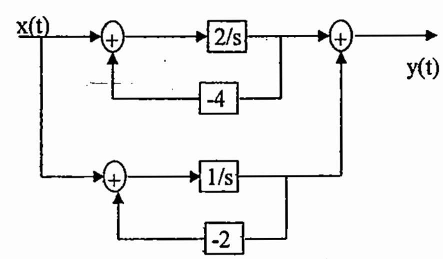


Fig. 5.5.14

P5.5.15 A Causal LTI system with I.R. h(t) has the following properties.

i) When the input is $x(t)=e^{2t}\forall t$, the output is $y(t)=\frac{1}{6}e^{2t}\ \forall t$

ii) The I.R. h(t) satisfies the differential equation , $\frac{dh(t)}{dt}+2h(t)=e^{-4t}u(t)+bu(t)$ where b is unknown constant. Find the transfer function? {constant b should not appear in the answer}

REVIEW NOTES:

Chapter 6. DTFT

- The DTFT describes the spectrum of discrete signals & formalizes that discrete signals have periodic spectra. The frequency range for a discrete signal is unique over $(-\pi, +\pi)$ (or) $(0,2\pi)$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$x(n) = \frac{1}{2\pi}\int_{<2\pi>} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$x(n) \rightleftharpoons X(e^{j\omega})$ (or) $X(\omega)$

- $X(e^{j\omega})$ is decomposition of x(n) into its frequency components.

Convergence of DTFT

A sufficient condition for existence of DTFT is $\sum_{n=-\infty}^{+\infty} |x(n)| < \infty$

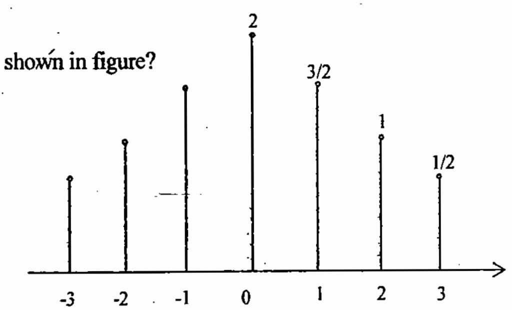
- Some sequences are not absolutely summable, but they are square summable.
- There are signals that are neither absolutely summable nor have finite energy, but still have DTFT.

$$a^n u(n),\ |a| < 1 \longleftrightarrow \frac{1}{1-ae^{-j\omega}}$$

$$\delta(n) \longleftrightarrow 1$$

Periodicity property:- $X(e^{j(\omega+2\pi)}) = X(e^{j\omega})$

P6.1.1 Find the F.T. of the signal shown in figure?



P6.1.2 Let $x(n) = (1/2)^n u(n)$, $y(n) = x^2(n)$ & $Y(e^{j\omega})$ be the F.T of y(n). Then $Y(e^{j\omega})$ is ___

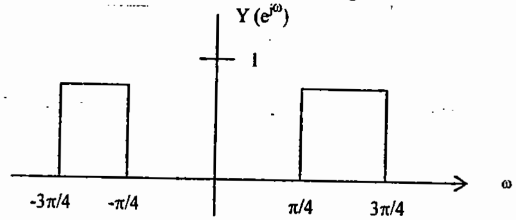
Time – Shift: $x(n) \longleftrightarrow X(e^{j\omega})$ then $x(n-n_0) \longleftrightarrow (e^{-j\omega n_0}) X(e^{j\omega})$

P6.1.3 Find the F.T of (i) $y_1(n) = (1/4)^n\, u(n-3)$

(ii) $y_2(n) = \delta(n - n_0)$

Frequency –shift :- $x(n) \longleftrightarrow X(e^{j\omega})$ then $x(n)\, e^{j\omega_0 n} \longleftrightarrow X\left(e^{j(\omega-\omega_0)}\right)$

P6.1.4 Find the signal corresponding to the spectrum shown in figure?



P6.1.5 Given $X(e^{j\omega}) = \cos^3(3\omega)$, then find the sum $S = \sum_{n=-\infty}^{\infty} (-1)^n\, x(n)$

Time –Scaling :- $x(n) \longleftrightarrow X(e^{j\omega})$, then $x[n/k] \longleftrightarrow X(e^{j\omega k})$

P6.1.6 Find the I.F.T of $Y(e^{j\omega}) = \dfrac{1}{1-1/2\, e^{-j10\omega}}$?

P6.1.7 If the DTFT of $x(n) = (1/5)^n u(n+2)$ is $X(e^{j\omega})$, find the sequence that has a DTFT Given $Y(e^{j\omega}) = X(e^{j2\omega})$

Frequency Differentiation :- $-jn\, x(n) \longleftrightarrow \dfrac{d}{d\omega} X(e^{j\omega})$

P6.1.8 Find the F.T. of $y(n) = na^n\, u(n)$?

P6.1.9 Find the value of $\sum_{n=0}^{\infty} n\,(1/2)^n$?

P6.1.10 Find the F.T of $x(n) = n\, e^{\frac{jn\pi}{8}}\, \alpha^{n-3}\, u(n-3)$

Convolution in time : - $x(n) \longleftrightarrow X(e^{j\omega})$, $h(n) \longleftrightarrow H(e^{j\omega})$ then $x(n) * h(n) \longleftrightarrow X(e^{j\omega})\, H(e^{j\omega})$

F.T. of impulse response is known as frequency response

P6.1.11 Consider $x(n) = \sin\left(\frac{\pi n}{8}\right) - 2\cos\left(\frac{\pi n}{4}\right)$ find the output if the impulse response is $h(n) = \dfrac{\sin(\pi n/6)}{\pi n}$

P6.1.12 An L.T.I system is having impulse response

$$h(n) = \begin{cases} 4\sqrt{2}\ ; & n=2,-2 \\ -2\sqrt{2}\ ; & n=1,-1 \\ 0\ ; & \text{elsewhere} \end{cases}$$

Find the output when input applied is $x(n) = e^{jn\pi/4}$?

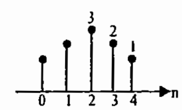
P6.1.13 Let $h(n) = \{\alpha, \underset{\uparrow}{\beta}, \alpha\}$ & the magnitude response blocks the frequency $f = 1/3$ & passes the frequency $f = 1/8$ with unity again. What is the D.C gain of the filter?

P6.1.14 What type of filter the following difference equation indicates?
(1) $y(n) = x(n)-x(n-1)$ (2) $y(n) = x(n) + x(n-1)$

P6.1.15 Consider the system described by the equation $y(n) = ay(n-1)+ bx(n) + x(n-1)$, where 'a' & 'b' are real, find a and b such that $|H(e^{j\omega})| = 1\ \forall\omega$?

P6.1.16 An input x(n) with length 3 is applied to a LTI system having an impulse response h(n) of length 5, and $Y(\omega)$ is the DTFT of the output y(n) of the system. If $|h(n)| \le L$ & $|x(n)| \le B\ \forall n$, the maximum value of $Y(0)$ can be ....
(a) 15 LB (b) 12 LB (c) 8 LB (d) 7 LB

P6.1.17 Consider a filter with I. R shown in figure. Find the group delay of the filler?



Parseval's relation ;- $x(n) \longleftrightarrow X(e^{j\omega})$

$$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi} \int_{<2\pi>} |X(e^{j\omega})|^2\, d\omega$$

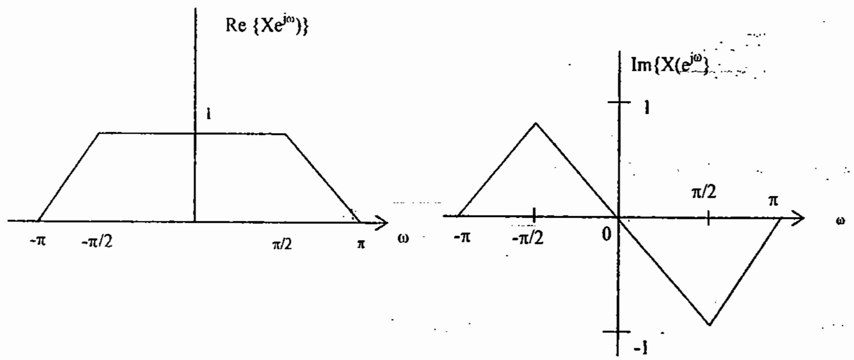
Parseval's relation is known as conservation of energy theorem, because DTFT operator preserves energy when going from time domain to frequency domain.

P6.1.17 Find the energy in the signal $x(n) = \dfrac{\sin \omega_c n}{\pi n}$?

P6.1.18 Find the value of $\displaystyle\sum_{n=-\infty}^{+\infty} \frac{\sin\left(\frac{n\pi}{4}\right)}{2\pi n} \cdot \frac{\sin\left(\frac{n\pi}{3}\right)}{5\pi n}$

P6.1.19 The centre of gravity of a sequence x(n) is defined as $C = \dfrac{\sum_{n=-\infty}^{\infty} n\, x(n)}{\sum_{n=-\infty}^{\infty} x(n)}$

And it used to measure time delay of the sequence find the value of ' C ' for the spectrum shown in figure ?



P6.1.20 For the signal shown in fig,6.1.20, find the following quantities without calculating DTFT?

(a) $X(e^{j0})$

b) $X(e^{j\pi})$

(c) $\int_{-\pi}^{\pi} X(e^{j\omega})\, d\omega$

d) $\int_{-\pi}^{\pi} X(e^{j\omega})e^{j2\omega} d\omega$

(e) $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$

f) $\int_{-\pi}^{\pi} \left|\frac{d}{d\omega} X(e^{j\omega})\right|^2 d\omega$

g) $\angle X(e^{j\omega})$

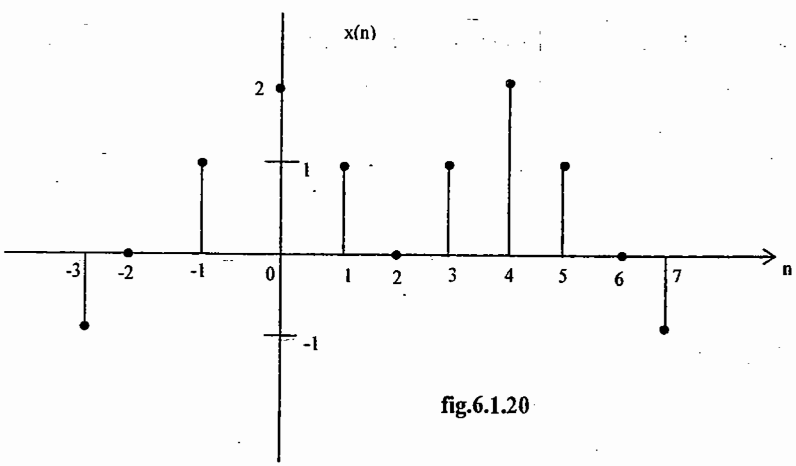


fig.6.1.20

P6.1.21 Find the autocorrelation of x(n) = { 1,2,3,4 } ?

P6.1.22 A continuous time signal x(t) is to be filtered to remove frequency component in the range 5kHz ≤ f ≤ 10 kHz. The maximum frequency present in x(t) is 20KHZ. Find the minimum sampling frequency & find frequency response of ideal digital filter that will remove the desired frequencies from x (t)?

P6.1.23 A signal $x(n) = \sin(\omega_0 n + \Phi)$ is the input to a LTI system having frequency response $H(e^{jw})$. If the output the system is $A\,x(n-n_0)$, then the most general form of $\angle H(e^{jw})$ will be ___________

(a) $-n_o\omega_0 + \beta$ for any arbitrary real β

(b) $-n_o\omega_0 + 2\pi k$ any arbitrary inter k

(c) $n_o\omega_0 + 2\pi k$ for any arbitrary integer k

(d) $-n_o\omega_0\, \phi$

P6.1.24 A 5 point sequence x(n) is given by

x (-3) =1, x(-2) =1, x(-1) = 0, x(o) = 5, x(1) = 1

Let $x(n) \longleftrightarrow X(e^{j\omega})$, then the value of $\int_{-\pi}^{\pi} X(e^{j\omega})\, d\omega$ is ____________

P6.1.25 An LTI system is having Impulse Response $h(n)=(1/3)^n u(n)$. Find the response when the input applied is $x(n)=2+\cos(\pi n+\pi/3)$.

P6.1.26 $y(n) = x(n) - 2x(n-1) + x(n-2)$ is a good approximation to

(A) HPF (B) BPF passing $\pi/8 \le |\omega| \le \pi/4$

(C) LPF (D) BSF passing $\pi/8 \le |\omega| \le \pi/4$

6.2. PRACTICE PROBLEM SET:-

P6.2.1 Compute F.T of the following signals ?

(a) $x(n) = u(n-2) - u(n-6)$ (b) $x(n) = (1/2)^{-n} u(-n-1)$

(c) $x(n) = \frac{\sin(\pi n/5)}{\pi n} \cos(7\pi n/2)$ (d) $x(n) = \{ -2,-1,\underset{\uparrow}{0},1,2 \}$

(e) $x(n) = 2^n u(-n)$

P6.2.2 Determine the signal corresponding to following transforms ?

(a) $X(e^{j\omega}) = \cos^2\omega$ (b) $X(e^{j\omega}) = \begin{cases} 0, & 0 \le |\omega| \le \omega_0 \\ 1, & \omega_0 \le |\omega| < \pi \end{cases}$

(c) $X(e^{j\omega}) = e^{-j\omega/2}$ for $-\pi \le \omega \le \pi$ (d) $X(e^{j\omega}) = \cos 4\omega \left(\frac{\sin 3\omega/2}{\sin \omega/2}\right)$

(e) $X(e^{j\omega}) = \frac{1 - 1/3\, e^{-j\omega}}{1 - 1/4\, e^{-j\omega} - 1/8\, e^{-j2\omega}}$

P6.2.3 Find & sketch the magnitude & phase responses of the following systems ?

(a) $y(n) = 0.5\,[x(n+1) - x(n-1)]$ (b) $y(n) = 2x(n-1) - x(n-2)$
(c) $y(n) = x(n) - x(n-8)$

P6.2.4 A signal $x(n)$ has the F.T $X(e^{j\omega}) = \dfrac{1}{1-ae^{-j\omega}}$

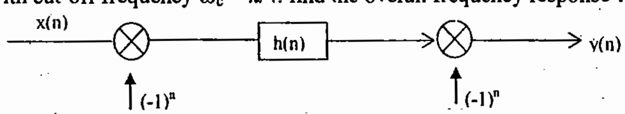
Find the F.T of the following signals ?

(a) $x(2n+1)$ (b) $e^{j\pi n/2}\, x(n+2)$ (c) $x(n)\cos(0.3\pi n)$ (d) $x(n) * x(n-1)$

P6.2.5 Consider the filter $y(n) = 0.9y(n-1) + bx(n)$

(a) Find 'b' so that $|H(0)| = 1$
(b) Find the frequency at which $|H(\omega)| = 1/\sqrt{2}$

P6.2.6 For the system shown in figure, let h(n) be the unit sample response of an ideal LPF with cut-off frequency $\omega_c = \pi/4$. find the overall frequency response ?



P6.2.7 Find the group delay of the system that has a frequency response

$$H(e^{j\omega}) = \frac{1 - \frac{1}{2} e^{-j\omega}}{1 + 1/4\, e^{-j\omega}}$$

P6.2.8 What type of filter has a unit sample response

$$h(n) = \delta(n) - \frac{\sin(\pi n/3)}{\pi n}$$

P6.2.9 Consider a D.T ideal HPF whose frequency response is specified as

$$H(e^{j\omega}) = \begin{cases} 1\,;\ \pi - \omega_c \le |\omega| \le \pi \\ 0\,;\ |\omega| < \pi - \omega_c \end{cases}$$

If h(n) is the impulse response of this filter, determine a function g(n) such that

$$h(n) = \left\{ \frac{\sin \omega_c n}{\pi n} \right\} g(n)$$

P6.2.10 Find the I.F.T of $X(e^{j\omega}) = |X(e^{j\omega})|\, e^{j\,\angle X(e^{j\omega})}$

Where $|X(e^{j\omega})| = \begin{cases} 1\,;\ 0 \le |\omega| < \pi/4 \\ 0\,;\ \pi/4 \le |\omega| \le \pi \end{cases}$ $\quad \angle X(e^{j\omega}) = -3\omega/2.$

P6.2.11 An LTI system with impulse response $h_1(n) = (1/3)^n\, u(n)$ is connected in parallel with another causal LTI system with I.R. $h_2(n)$. The resulting parallel

interconnection has the frequency response

$$H(e^{j\omega}) = \frac{-12 + 5e^{-j\omega}}{12 - 7e^{-j\omega} + e^{-j2\omega}}$$, Find $h_2(n)$?

P6.2.12 Find the I.R corresponding to the frequency response

$$H(e^{j\omega}) = \begin{cases} -j\,;\ 0<\omega<\pi \\ j\,;\ -\pi<\omega<0 \end{cases}$$

Ans ;- $h(n) = \begin{cases} \mathbf{2/n\pi\ ;\ odd\ n} \\ \mathbf{0\ ;\ even\ n} \end{cases}$

P6.2.13 Let $x(n) = 2\delta(n+2) - \delta(n+1) + 3\delta(n) - \delta(n-1) + 2\delta(n-2)$ evaluate the following quantities without solving DTFT

(a) $X(e^{j0})$ (b) $\angle X(e^{j\omega})$ (c) $\int_{-\pi}^{\pi} X(e^{j\omega})\,d\omega$

(d) $X(e^{j\pi})$ (e) $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2\, d\omega$

P6.2.14 For the DTFT spectrum shown in figure, find the value of centre of gravity of the signal x (n)

$$'C' = \frac{\sum_{n=-\infty}^{+\infty} nx(n)}{\sum_{n=-\infty}^{+\infty} x(n)}$$

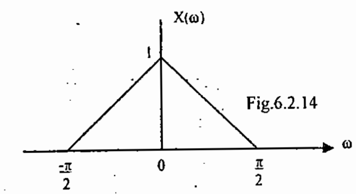


Fig.6.2.14

P6.2.15 Calculate the output of a system for which input is $x(n) = (-1)^n\ \forall\ n$ & the impulse response is $h(n) = (1/2)^n u(n)$

P6.2.16 Find the autocorrelation of the sequences

(i) $x(n) = \{1,2,1,1\}$ (ii) $y(n) = \{1,1,2,1\}$

Is there exists any relation between auto correlations?

P6.2.17 Find the cross correlation of the sequences

$x(n) = \{1,3,1,2\}$ & $y(n) = \{1,2,1,3\}$

P6.2.18. Find the IDFT of spectrum shown in. fig 6.2.18?

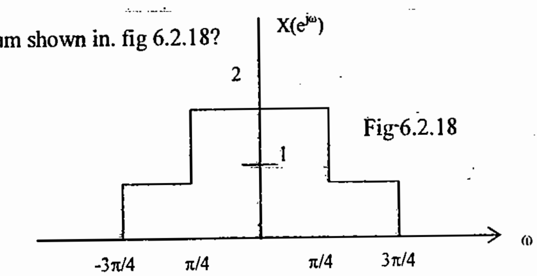


Fig 6.2.18

P6.2.19 The DTFT of a sequence is $X(e^{j\omega}) = \frac{3}{(1 - 0.8\,e^{-j\omega})^5}$

Evaluate $\sum_{n=-\infty}^{+\infty} x(n)$?

P6.2.20 Evaluate the integral $\int_{-\pi}^{\pi} \frac{e^{j\omega}}{1 - 0.3\,e^{-j\omega}}\, d\omega$?

P6.2.21 Given that the frequency response of a system is given by $H(e^{j\omega}) = \frac{1}{1 + 3\,e^{-j\omega}}$

Find & sketch

(1) $Re\{H(e^{j\omega})\}$ (ii) $|H(e^{j\omega})|$ (iii) $Im\{H(e^{j\omega})\}$ (iv) $\underline{|H(e^{j\omega})}$

P6.2.22 Determine whether or not the DT systems with these frequency response are causal?

(a) $H(\omega) = \frac{\sin(7\omega/2)}{\sin(\omega/2)}$ (b) $H(\omega) = \frac{\sin(3\omega/2)}{\sin(\omega/2)}\, e^{-j\omega}$

(c) $H(\omega) = e^{-j3\omega} + e^{+j2\omega}$

REVIEW NOTES:

Chapter 7. Z – TRANSFORM

7.1 Introduction To Z-Transform

→ Discrete-time counterpart of L.T. is Z.T.

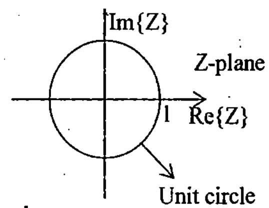
→ For a D.T.L.T.I system with impulse response h(n), the response y(n) of the system to a complex exponential Input of the form Z^n is $y(n)=Z^nH(Z)$ where H(Z) is known as transfer function of the system.

$$\boxed{\text{Z.T. of a general D.T. signal } x(n) \text{ is } \quad X(Z)=\sum_{n=-\infty}^{+\infty} x(n)Z^{-n} \longrightarrow (1)}$$

Where $Z=re^{j\omega} \longrightarrow$ Complex Variable $\qquad x(n) \xleftrightarrow{Z} X(Z)$

$X(Z) = F\{x(n)r^{-n}\}$

→ Z.T. calculated on the unit circle is D.T.F.T.



→ The range of values of 'Z' for which eq (1) is defined

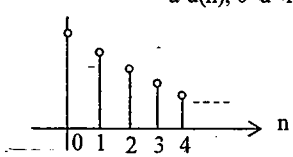
$\left[\sum_{n=-\infty}^{+\infty}|x(n)r^{-n}|<\infty\right]$ is R.O.C. of Z.T.

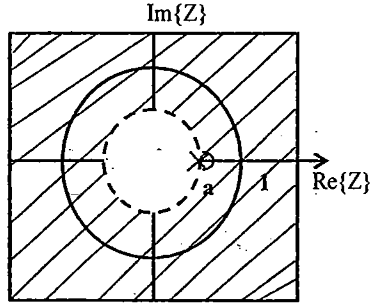
→ DTFT is defined only for stable signals where as Z.T. is defined for unstable signals also.

→ The primary role of Z.T. in engineering are the study of system characteristics & the derivation of computational structures for implementing discrete systems on computers.

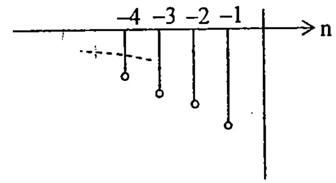
Z.T. of standard signals :-

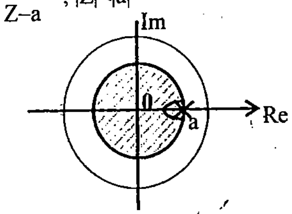
$a^nu(n),\ 0<a<1 \longleftrightarrow \dfrac{1}{1-aZ^{-1}}$ (or) $\dfrac{Z}{Z-a}$; $|Z|>|a|$





$-a^nu(-n-1) \longleftrightarrow \dfrac{1}{1-aZ^{-1}}$ (or) $\dfrac{Z}{Z-a}$; $|Z|<|a|$





$\delta(n) \longleftrightarrow 1$; ROC : entire Z-plane

$u(n) \longleftrightarrow 1/1-Z^{-1}$; $|Z|>1$

Note :

1) For a finite length signal, ROC is entire Z-plane, except perhaps for Z=0 and /or Z=∞
2) For a right-sided signal ROC is outside a circle whose radius is largest pole in magnitude.
3) For a left sided signal, ROC is inside a circle whose radius is smallest pole in magnitude.
4) For a two-sided signal, ROC is annulus bounded by largest & smallest pole radius.

P7.1.1 Determine all possible signals x(n) associated with the Z.T.?

$$X(Z)=\frac{1}{(1+1/2\,Z^{-1})(1-3Z^{-1})}$$

P7.1.2 Find the Z.T. of the following signals with ROC?

(a) $x_1(n) = (1/2)^nu(n)+(1/3)^nu(n)$ (b) $x_2(n) = (1/2)^nu(n)+(2/3)^nu(-n-1)$

(c) $x(n) = (1/3)^nu(-n-1)+(2)^nu(n)$

P7.1.3 Let $x(n)=(-1)^nu(n)+\alpha^nu(-n-n_0)$. Determine the constraints on "α" & "n_0", given that the ROC of X(Z) is $1<|Z|<2$

P7.1.4 Find the ROC of the following signals without finding Z.T.

(a) $x_1(n) = \{1,\underset{\uparrow}{2},3,-1\}$ (b) $x_2(n) = (1/2)^n[u(n) - u(n-10)]$

(c) $x_3(n) = \{(1/2)^n+(3/4)^n\}\,[u(n-10)]$

7.2 PROPERTIES OF THE Z.T. :-

1) Linearity :-
$$\left\{\begin{array}{l} x_1(n) \longleftrightarrow X_1(Z),\ ROC=R_1 \\ x_2(n) \longleftrightarrow X_2(Z),\ ROC=R_2 \\ \text{then } ax_1(n)+bx_2(n) \longleftrightarrow aX_1(Z)+bX_2(Z),\ ROC=R_1 \cap R_2 \end{array}\right.$$

P7.2.1 Find the Z.T. of $y(n) = \alpha^{|n|}$, $|\alpha| < 1$

P7.2.2 The ZT X(z) of a real & right-sided sequence x(n) has exactly 2 poles and one of them is at $Z = e^{j\pi/2}$ and there are 2 zeros at the origin. If X(1) = 1, which one of the following is TRUE ?

(a) $X(z)=\dfrac{2z^2}{(z-1)^2+2};\ \dfrac{1}{2}<|z|<1$ (b) $X(z)=\dfrac{2z^2}{z^2+1};\ |z|>\dfrac{1}{2}$

(c) $X(z)=\dfrac{2z^2}{(z-1)^2+2};\ |z|>1$ (D) $X(z)=\dfrac{2z^2}{z^2+1};\ |z|>1$

2) Time-shift :- $x(n) \Leftrightarrow X(Z)$, ROC = R

Then $x(n-n_0) \Leftrightarrow Z^{-n_0} X(Z)$, ROC = R except for the possible addition (or) deletion of 0 (or) ∞

P7.2.3 Find the Z.T. of $y(n) = (1/4)^n u(n+2)$

P7.2.4 Consider the signal $x(n) = \begin{cases} 1 \,;\, 0 \le n \le 5 \\ 0 \,;\, \text{elsewhere} \end{cases}$

Let $g(n)=x(n)-x(n-1)$. Find G(Z) with ROC?

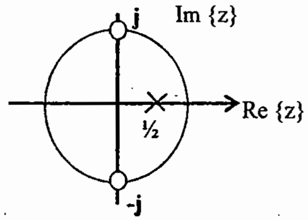
P7.2.5:-Given $X(Z) = \dfrac{Z^3 - 2Z}{Z-2}$ & x(n) is left-sided, find x(n)?

3) Exponential multiplication (or) Scaling in Z-domain:-

$x(n) \longleftrightarrow X(Z)$ with ROC = R then $a^n x(n) \longleftrightarrow X(Z/a)$ with ROC = |a| R

P7.2.6:- Find the Z.T. of $y(n) = \cos\omega_o n u(n)$

P7.2.7:- The pole-zero plot shown in fig. for a signal x(n). Find the Z.T. of $y(n) = (1/2)^n x(n)$?



4) Time-reversal :- $x(n) \Leftrightarrow X(Z)$, then $x(-n) \Leftrightarrow X(Z^{-1})$, ROC = 1/R

P7.2.8:- Find the Z.T. of $y(n)=2^n u(-n)$?

5) Differentiation in Z-domain :- $nx(n) \longleftrightarrow -Z\dfrac{d}{dZ}X(Z)$, ROC = R

P7.2.9:- Find the I.Z.T. of $X(Z) = \log(1+aZ^{-1})$, $|Z|>|a|$?

P7.2.10:- Find the Z.T. of $y(n) = na^n u(n-1)$?

6) Convolution in time :-
$x(n) \Leftrightarrow X(Z)$ with ROC=R_1 and $h(n) \Leftrightarrow H(Z)$ with ROC=R_2
then $x(n)*h(n) \Leftrightarrow X(Z)H(Z)$ with ROC=$R_1 \cap R_2$

Z.T. of impulse response is known as system (or) transfer function

P7.2.11 A sequence $x(n) \longleftrightarrow X(Z) = Z^4 + Z^2 - 2Z + 2 - 3Z^{-4}$ is applied as an Input to a LTI system with impulse response $h(n)=2\delta(n-3)$. The Output at n = 4 is________

P7.2.12 Consider a signal $y(n) = x_1(n+3) * x_2(-n+1)$
Where $x_1(n) = (1/2)^n u(n)$ & $x_2(n) = (1/3)^n u(n)$ Find Y(Z) with ROC?

P7.2.13 Find the response of the system $y(n) = 5/6\, y(n-1) - 1/6\, y(n-2) + x(n)$ to the Input signal, $x(n) = \delta(n) - 1/3\delta(n-1)$

P7.2.14 Find the impulse response & step response of the system described by the D.E. $y(n) + 1/4\; y(n-1) - 1/8\; y(n-2) = x(n) - x(n-1)$.

P7.2.15 $G(Z) = \alpha Z^{-1} + \beta Z^{-3}$ represents a digital LPF with linear phase if and only if
A) $\alpha = \beta$ B) $\alpha = \beta^{1/3}$ C) $\alpha = -\beta$ D) $\alpha = -\beta^{1/3}$

P7.2.16 The following is known about a D.T.L.T.I system with Input x(n) & output y(n)
1. If $x(n) = (-2)^n\ \forall n$, then $y(n)=0\ \forall n$
2. If $x(n) = (1/2)^n u(n)\ \forall n$, then $y(n)=\delta(n)+a(1/4)^n u(n)\ \forall n$ Where 'a' is a constant.
a) Find the value of 'a' b) Find the response y(n) if the Input is $x(n)=1\ \forall n$.

P7.2.17 A system with T.F. H(Z) has I.R. h(n) defined as h(2) = 1, h(3) = -1 and h(k) = 0 otherwise, consider the following statements
S1: H(Z) is a LPF
S2: H(Z) is a FIR filter
a) only S2 is true
b) both are false
c) both are true and S2 is the reason for S1
d) both are true but S2 is not a reason for S1

7. Accumulation :- $x(n) \Leftrightarrow X(Z)$, ROC = R

Then $\sum_{k=-\infty}^{n} x(k) \longleftrightarrow \dfrac{X(Z)}{1-Z^{-1}}$, ROC = $R \cap |Z|>1$

<u>UNILATERAL Z.T.</u>:- $\boxed{\text{(U.Z.T)}\ \chi(Z)=\sum_{n=0}^{\infty} x(n)Z^{-n}}$

P7.2.18 Find the U.Z.T of the following signals?
(a) $x_1(n)=2^n u(-n)$ (b) $x_2(n)=(1/4)^n u(n+5)$
(c) $x_3(n)=3^n u(-n-1)$ (d) $x_4(n)=\{2,4,\underset{\uparrow}{5},7,0,1\}$

8. Left-Shift :- $x(n+1) \longleftrightarrow ZX(Z) - Zx(0)$

9. Right-Shift:- $x(n-1) \longleftrightarrow Z^{-1}X(Z)+x(-1)$
$x(n-2) \longleftrightarrow Z^{-2}X(Z)+Z^{-1}x(-1)+x(-2)$

10. Initial Value Theorem: $x(0) = \underset{Z\to\infty}{Lt} X(z)$

11. Final Value theorem. $x(\infty) = \underset{Z\to 1}{Lt}(1-Z^{-1})X(Z)$ (or) $\underset{Z\to 1}{Lt}(Z-1)X(Z)$

P7.2.19 Find initial & final values of the transform

$$X(Z) = \frac{1+Z^{-1}}{(1-1/2\,Z^{-1})(1+3Z^{-1})}$$

P.7.2.20. Given $X(Z) = \frac{0.5}{1-2Z^{-1}}$. It is given that the ROC of X(Z) includes the unit circle. Then x(0) is ________

P7.2.21 Apply F.V.T. for $x(n) = \begin{cases} 1 ; n \text{ even} \\ 0 ; n \text{ odd} \end{cases}$. Assume the signal is right-sided?

P7.2.22 Consider a system whose Input x(n) & Output y(n) are related by $y(n-1)+2y(n)=x(n)$. Find the Output of the system if $x(n)=(1/4)^n u(n)$ & $y(-1)=2$.

7.3 CAUSALITY & STABILITY :-

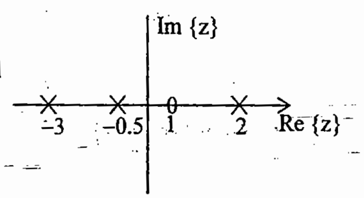
→ A D.T. LTI system with rational system function H(Z) is causal if and only if (a) ROC is exterior of a circle outside the outermost pole (b) with H(Z) expressed as a ratio of polynomials in Z, order of Nr. Can't be greater than order of Dr.

→ An LTI system is stable if and only if the ROC of H(Z) includes the unit circle.

P7.3.1 Find the corresponding impulse response for the given Z.T. $H(Z) = \frac{3}{1-10/3\,Z^{-1}+Z^{-2}}$ if the system is
i) Causal ii) Stable

P7.3.2 The step response of an LTI system is $S(n)=(1/3)^{n-2}u(n+2)$.
a) Find the system function & impulse response?
b) Check if the system is causal & stable?

P7.3.3 Consider an LTI system whose pole-zero pattern is shown in figure
a) Find the ROC of system function, if it is known to be stable?
b) Is it possible for the given pole-zero plot to a causal & stable system?
c) How many possible ROC's are there?



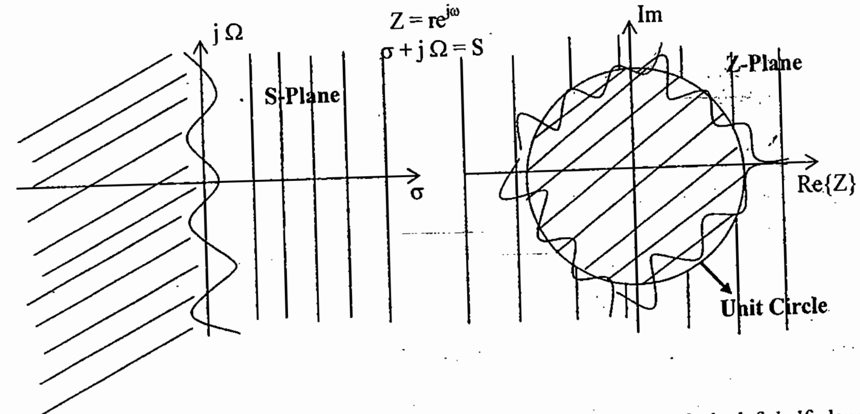
P7.3.4 The impulse response h(n) of a LTI system is real. The transfer function H(z) of the system has only one pole and it is at z = 4/3. The zeros of H(z) are non-real and located at |z|=3/4. The system is
(a) stable & causal (b) unstable & anticausal
(c) unstable & causal (d) stable & anticausal

P7.3.5 Find the first 3 samples of $X(Z) = \frac{1}{1-1.5Z^{-1}+0.5Z^{-2}}$

for (a) |Z| > 1 (b) |Z| < 0.5 using power series expansion method?

P7.3.6 Given $H(z)=\frac{z^{-1}}{1-z^{-1}-z^{-2}}$ if the ZT converges on the unit circle find impulse response?

7.4 RELATION BETWEEN S-PLANE & Z – PLANE:-



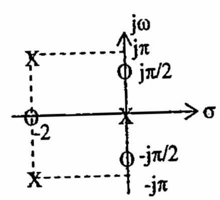
The j Ω axis in the S-plane should map into the unit circle in the Z-plane & the left-half plane of the S-plane should map into the inside of the unit circle in the Z-plane. Thus a stable analog filter will be converted to a stable digital filter.

$$Z = e^{ST} \Rightarrow \omega = \Omega T$$

Table : Mapping of frequencies from the S-plane to Z-plane

S – Plane	Z – Plane
0	0
$\Omega_S/4$	$\pi/2$
$\Omega_S/2$	π
$3\,\Omega_S/4$	$1.25\,\pi$
$1.25\,\Omega_S$	$\pi/2$
$1.5\,\Omega_S$	π
$1.75\,\Omega_S$	$1.25\,\pi$
$2\,\Omega_S$	$2\,\pi$

Example : The pole-zero plot of the transfer function $[H_a(s)]$ of a LTI system in the S-plane is shown in fig. The corresponding impulse response $h_a(t)$ is sampled at 2 Hz to get the discrete impulse response h(n). Find the equivalent pole-zero plot in the z-plane.

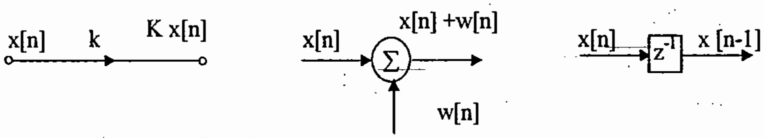


7.5 Realisation Of Digital Systems

Realization structures for IIR digital filters:

Realization involves converting a given transfer function H(z) into a suitable filter structure. Block diagrams are used to depict filter structures & they show the computational procedure for implementing the digital filter. The basic elements of realization structures are

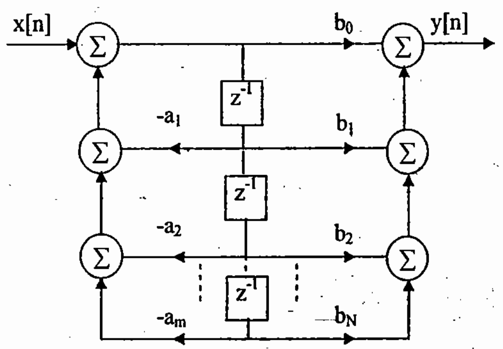
(1) Multiplier (2) Adder/Accumulator (3) Delay



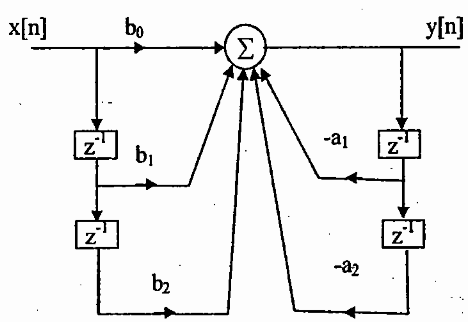
General form of IIR filter is

$$y(n)=\sum_{k=0}^{N} b_k x[n-k]-\sum_{k=1}^{M} a_k y[n-k],\ M \ge N \quad \Rightarrow \quad H(z)=\frac{\sum_{K=0}^{N} b_k z^{-k}}{1+\sum_{k=1}^{M} a_k z^{-k}}$$

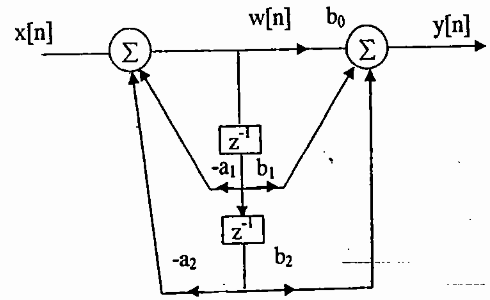
Direct form realization of IIR filter



Suppose $H(z)=\dfrac{b_0+b_1z^{-1}+b_2z^{-2}}{1+a_1z^{-1}+a_2z^{-2}}$



Canonic II order section:-



$$w(n)=x[n]-\sum_{k=1}^{2} a_k w[n-k] \qquad y[n]=\sum_{k=0}^{2} b_k w[n-k]$$

Canonic section is the most popular because it has a good round off noise property & requires minimum number of storage elements but it is susceptible to internal overflow.

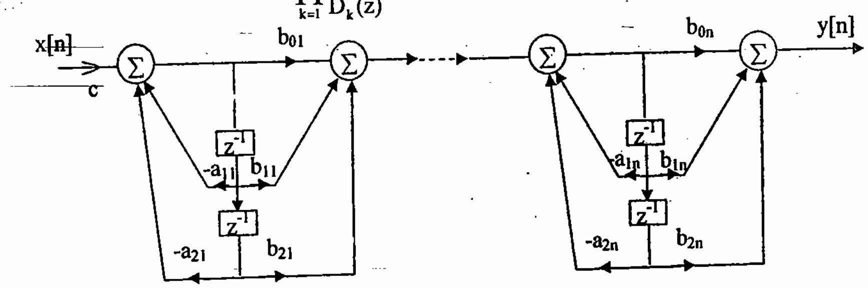
Cascade & parallel realization structures:

To implement higher order filters we are using cascade & parallel structures. In cascade realization the transfer function is factored into $\frac{N}{2}$ II order factors.

$$H(z)=\prod_{k=1}^{N/2}\left[\frac{b_{0k}+b_{1k}z^{-1}+b_{2k}z^{-2}}{1+a_{1k}z^{-1}+a_{2k}z^{-2}}\right]$$

$$=\prod_{k=1}^{N/2}\frac{N_k(z)}{D_k(z)}$$

N is assumed to be even



$$H(z)=C\prod_{k=1}^{n}\left[\frac{b_0k+b_1kz^{-1}+b_2kz^{-2}}{1+a_1kz^{-1}+a_2kz^{-2}}\right]$$

Three difficulties arise with cascade realizations:

(1) How to pair the numerator factors with denominator factors

(2) Order in which individual sections should be connected

(3) Need to scale the signal levels at various points within the filter to avoid the levels becoming too large or too small.

In parallel realization an Nth order transfer H(z) is expanded using partial fractions as

$$H(z) = C + \sum_{k=1}^{N/2} H_k(z)$$

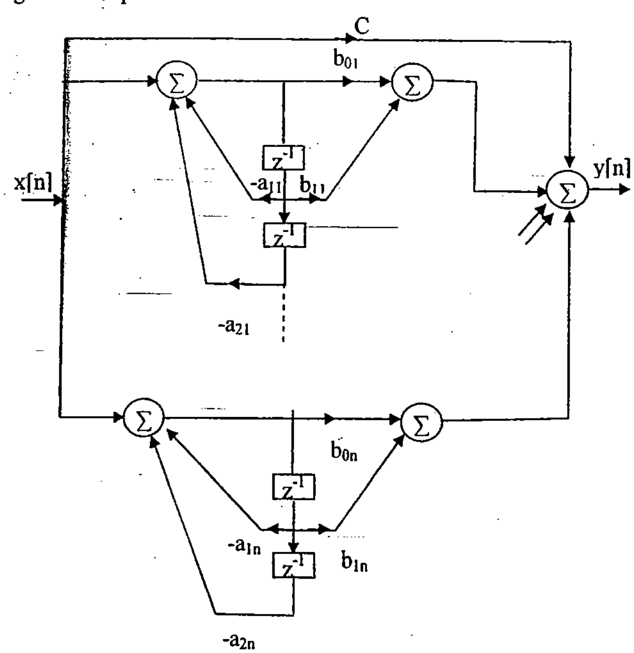
$$C = b_N / a_N$$

$$H_k(z) = \frac{b_{0k} + b_{1k} z^{-1}}{1 + a_{1k} z^{-1} + a_{2k} z^{-2}}$$

In parallel realization, Numerator coefficient for Z^{-2} is zero and order in which the section is connected is not important. Scaling is easier & can be carried out for each block independently zeros of parallel structures are more sensitive to coefficient quantization errors. Most available software packages produce coefficients for cascade realization but not for parallel structure.

- From a practical viewpoint, parallel & cascade forms using low order filters minimize the effects of finite word length.
- In the canonic direct form structure with large N, a small change in a filter coefficient due to parameter quantization results in a large change in location of poles & zeros of the system.
- In a parallel realization, a change in coefficient will effect only a localized segment.

- In cascade realization, higher – order filters are designed using string of II order filters because these lower order filters are easier to design, are less susceptible to coefficient quantization errors & stability problems & their implementations allow easier data word scaling to reduce potential overflow effects of data word size.

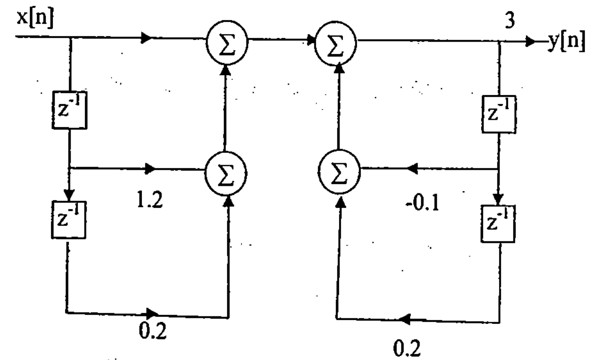


Example: - Obtain DF 1, DF II, cascaded & parallel structures for the system

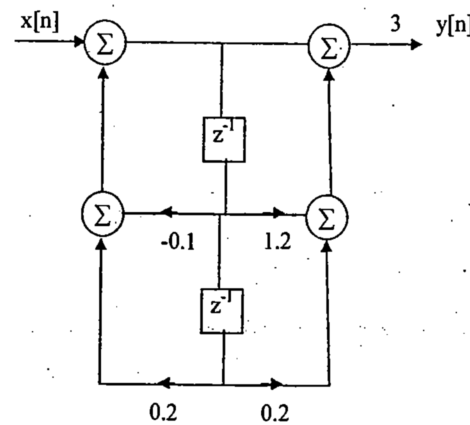
$$y[n] = -0.1y[n-1] + 0.2y[n-2] + 3x[n] + 3.6x[n-1] + 0.6x[n-2]?$$

Solution:- $H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{3(1+1.2z^{-1}+0.2z^{-2})}{1+0.1z^{-1}-0.2z^{-2}}$

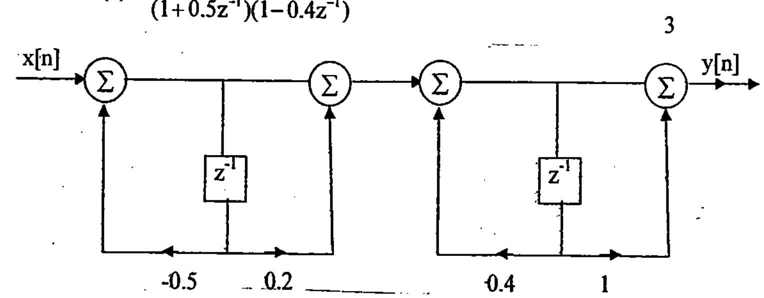
DF1:-



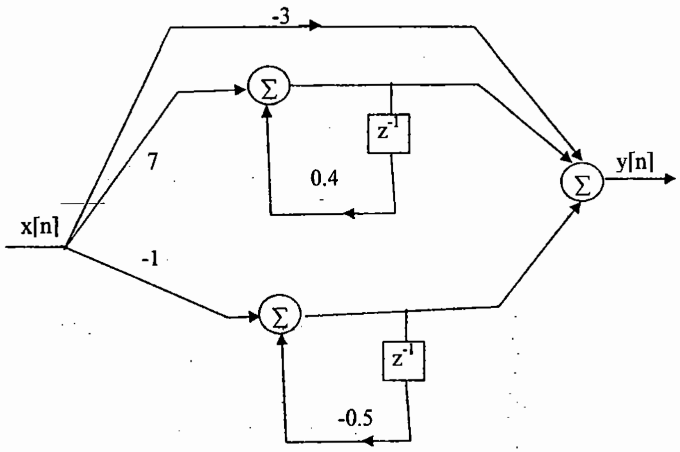
DF11:-



Cascaded:- $H(z) = \dfrac{3(1+0.2z^{-1})(1+z^{-1})}{(1+0.5z^{-1})(1-0.4z^{-1})}$



Parallel form:- $H(z) = -3 + \frac{7}{1-0.4z^{-1}} - \frac{1}{1+0.5z^{-1}}$



7.6 PREVIOUS QUESTIONS:-

(1) Z.T. of a signal is $C(z) = \frac{z^{-1}(1-z^{-4})}{4(1-z^{-1})^2}$. Its final value is ____________ GATE

(a) 1/4 (b) 0 (c) 1 (d) ∝

(2) If I.R. of a D.T. system is $h(n) = -5^{-n}u[-n-1]$, then the system function H(z) is equal to__ GATE

(a) $\frac{-z}{z-5}$ & is stable (b) $\frac{z}{z-5}$ & is stable

(c) $\frac{-z}{z-5}$ & is unstable (d) $\frac{z}{z-5}$ & is unstable

(3) A casual LTI system is described by the D.E $2y(n) = \alpha y[n-2] - 2x[n] + \beta x[n-1]$ The system is stable only if GATE

(a) $|\alpha| = 2, |\beta| < 2$ (b) $|\alpha| > 2, |\beta| > 2$

(c) $|\alpha| < 2$, any β (d) $|\beta| < 2$, any α

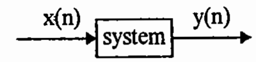
(4) The R.O.C of Z.T. of the sequence $\left(\frac{5}{6}\right)^n u(n) - \left(\frac{6}{5}\right)^n u(-n-1)$ must be GATE

(a) $|z| < 5/6$ (b) $|z| > 5/6$ (c) $5/6 < |z| < 6/5$ (d) $6/5 < |z| < \infty$

(5) If $X(z) = \frac{z+z^{-3}}{z+z^{-1}}$, then x(n) series has... IES

(a) alternate 0's (b) alternate 1's (c) alternate 2's (d) alternate -1's

(6) For the system shown, $x(n) = K\delta(n)$, and y(n) is related to x(n) as $y(n) - \frac{1}{2}y(n-1) = x(n)$. What is y(n)? IAS



(7) If $x(n) \leftrightarrow X(z)$ then Z.T. of x(n-2) & x[n/2] will be ____________ DRDO

(a) $z^{-2}X(z), 2X(2z)$ (b) $z^{-2}X(z), X(2z)$

(c) $X(z-2), X(z/2)$ (d) $z^{-2}X(z), X(z^2)$

(8) Given $X(z) = 4z^{-3} + 3z^{-1} + 2 - 6z^{2} + 2z^{3}$. It is applied to a system, with a transfer function $H(z) = 3z^{-1} - 2$. GATE

Let the output be y(n). Which of the following is TRUE?

(a) y(n) is non Causal with finite support

(b) y(n) is Causal with infinite support

(c) $y(n) = 0; |n| > 3$

(d) $\text{Re}\{Y(z)\}_{z=e^{j\theta}} = -\text{Re}[Y(z)]_{z=e^{-j\theta}}$
$\text{Im}\{Y(z)\}_{z=e^{j\theta}} = \text{Im}\{Y(z)\}_{z=e^{-j\theta}}; -\Pi \le \theta \le \Pi$

(9) The R.O.C of Z.T. of $x(n) = (1/3)^n u(n) - (1/2)^n u(-n-1)$ is ________ GATE

(a) $\frac{1}{3} < |z| < \frac{1}{2}$ (b) $|z| > \frac{1}{2}$ (c) $|z| < \frac{1}{3}$ (d) $2 < |z| < 3$

(10) The unit impulse response of the system described by $y(n) - y(n-1) = x(n) - x(n-1)$ is IAS

(a) $\delta(n)$ (b) u(n) (c) 1 (d) 0

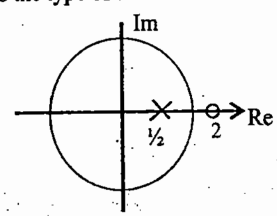
(11) Pole-zero plot of a DF is shown below. What is the type of the filter? ISRO

(A) LPF

(B) HPF

(C) BPF

(D) APF



(12) H(z) is a discrete rational TF. To ensure that both H(z) and its inverse are stable its

(A) Poles must be inside the unit circle and zeros must be outside the unit circle.

(B) Poles must be outside the unit circle and zeros must be inside the unit circle.

(C) Poles and zeros must be inside the unit circle.

(D) Poles and zeros must be outside the unit circle. GATE

(13) A DF having T.F. $H(z) = \frac{P_0 + P_1 z^{-1} + P_3 z^{-3}}{1 + d_3 z^{-3}}$ using DFI and DFII realizations of IIR the number of delay units required in DFI and DFII are, respectively .... GATE

(A) 6 & 6 (B) 6 & 3 (C) 3 & 3 (D) 3 & 2

(14) Consider $X(z) = 5z^2 + 4z^{-1} + 3; 0 < |z| < \infty$. The I.Z.T. x(n) is GATE

(A) $5\delta(n+2) + 3\delta(n) + 4\delta(n-1)$ (B) $5\delta(n-2) + 3\delta(n) + 4\delta(n+1)$

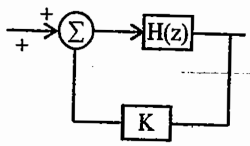
(C) $5u(n+2) + 3u(n) + 4u(n-1)$ (D) $5u(n-2) + 3u(n) + 4u(n+1)$

(15) An analog signal is sampled at 9 KHz. The sequence so obtained is filtered by an FIR filter with T.F. $H(z) = 1 - z^{-6}$. One of the analog frequencies for which the magnitude response of the filter is zero is …….. **GATE**

(A) 0.75 KHz (B) 1 KHz (C) 1.5 KHz (D) 2 KHz

(16) Consider a DTS shown in fig. where I.R. of H(z) is h(0)=0 , h(1)= h(2)=1, h(3)=h(4)=………..=0. The system is stable for range of values of k is **GATE**

(A) [-1, 1/2]
(B) [-1, 1]
(C) [-1/2, 1]
(D) [-1/2, 2]



7.7 PRACTICE PROBLEM SET:-

(1) Find the Z.T. R.O.C, and the location of poles & zeros of the following signals?

(a) $x(n) = \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{3}\right)^n u(n)$

(b) $x(n) = \left(\frac{-1}{3}\right)^n u(n) - \left(\frac{3}{4}\right)^n u(-n-1)$

(c) $x(n) = (2/3)^{|n|}$

(d) $x(n) = \left(\frac{1}{4}\right)^n \{u(n) - u(n-5)\}$

(e) $x(n) = \left(\frac{1}{2}\right)^{n+1} u(n+3)$

(f) $x(n) = \left(\frac{1}{4}\right)^n u(3-n)$

(2) Using properties of Z.T., find the Z.T. of the following signals?

(a) $x(n) = \left(\frac{1}{2}\right)^n u(n) * 2^n u(-n-1)$

(b) $y(n) = \left(\frac{-1}{2}\right)^n u(n) * \left(\frac{1}{4}\right)^{-n} u(-n)$

(c) $x(n) = a^n \cos\omega_0 n u(n), |a| < 1$

(3) Find the time –domain signals corresponding to following Z.T.?

(a) $X(z) = \dfrac{1 + \frac{7}{6}z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{3}z^{-1}\right)}, |z| > \frac{1}{2}$

(b) $X(z) = \dfrac{z^2 - 3z}{z^2 + \frac{3}{2}z - 1}, \frac{1}{2} < |z| < 2$

(c) $X(z) = z^6 + z^2 + 3 + 2z^{-3} + z^{-4}, 0 < |z| < \infty$

(4) If $H(z) = \dfrac{1}{1 - \frac{1}{4}z^{-2}}$ & $h(n) = A_1\alpha_1^n u(n) + A_2\alpha_2^n u(n)$, find the values of A_1, A_2, α_1 and α_2?

(5) The system function of causal LTI system is $H(z) = \dfrac{1 - z^{-1}}{1 + \frac{3}{4}z^{-1}}$ if the input to the system is $x(n) = \left(\frac{1}{3}\right)^n u(n) + u[-n-1]$ Find I.R & output. Is the system stable?

(6) A causal LTI system has the system function $H(z) = \dfrac{1 + 2z^{-1} + z^{-2}}{\left(1 + \frac{1}{2}z^{-1}\right)\left(1 - z^{-1}\right)}$

(a) Find impulse Response (b) Find out if input is $x[n] = e^{jn\pi/2}$?

(7) If the intput of an LTI system is x[n]=u[n], output is $y[n] = \left(\frac{1}{2}\right)^{n-1} u[n+1]$
(a) Find H(z) & h[n], (b) Is the system causal & stable?

(8) When the input to an LTI system is $x[n] = \left(\frac{1}{2}\right)^n u[n] + (2)^n u[-n-1]$ the output is $y(n) = 6\left(\frac{1}{2}\right)^n u[n] - 6\left(\frac{3}{4}\right)^n u[n]$.

(a) Find H(z) & indicate ROC (b) find impulse Response
(c) Write the difference equation that characterizes the system
(d) Is the system stable & causal?

(9) Compute the response of the system y[n] = 0.7y [n-1] – 0.12y [n-2] +x [n-1] +x [n-2] to the input x[n] = n u[n]. Is the system stable?

(10) Determine the impulse & step response of the following causal systems & determine which of the systems are stable.

(a) $y[n] = \frac{3}{4}y[n-1] - \frac{1}{8}y[n-2] + x[n]$ (b) $y[n] = 0.6y[n-1] - 0.08y[n-2] + x[n]$

(11) The step response of an LTI system is $s[n] = \left(\frac{1}{3}\right)^{n-2} u[n+2]$

(a) Find system function H (z) & h[n]? Check if the system is causal & stable?

(12) The input to a causal LTI system is $x(n) = u[-n-1] + \left(\frac{1}{2}\right)^n u(n)$

The Z.T. of the output is $Y(z) = \dfrac{-\frac{1}{2}z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + z^{-1}\right)}$

(a) Find system function with R.O.C? (b) What is R.O.C of Y(z) & find y(n)?

(13) Find the signal x(n) with Z.T. $X(z) = \dfrac{3}{1 - \frac{10}{3}z^{-1} + z^{-2}}$ if X(z) converges on the unit circle?

(14) A causal L.T.I system is described by the D.E. $y(n) = y[n-1] + y[n-2] + x[n-1]$
(a) Find the T.F. with R.O.C?
(b) Find the unit sample response?
(c) You should have found the system to be unstable. Find a stable (noncausal) unit sample response that satisfies the difference equation?

(15) Consider the Z.T. X(z) whose pole – zero plot is shown in fig. 15
(a) Determine the R.O.C of X(z) if it is known that the F.T. exists.
(b) How many possible two – sided sequences have the pole – zero plot shown in figure.
(c) Is it possible for the pole – zero plot to be associated with a sequence that is both stable & causal?

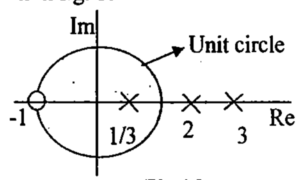


Fig 15

(16) Explain the condition of BIBO stability in z – domain. A LTI system is characterized by the system function $H(z) = \dfrac{3 - 4z^{-1}}{1 - 3.5z^{-1} + 1.5z^{-2}}$ specify the ROC and determine h(n), when (i) system is stable (ii) system is causal

(17) Consider the digital filter structure shown in **fig.17**
(a) Find transfer function for this causal filter
(b) For what values of 'k' the system is stable?
(c) Find y(n) if k=1 & $x(n) = (2/3)^n\ \forall n$.

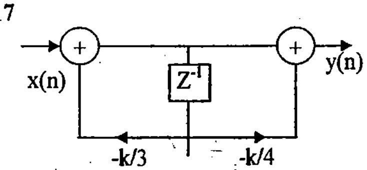


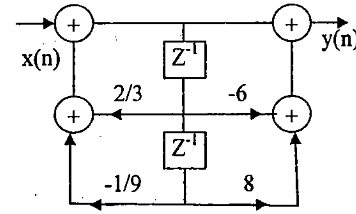
Fig. 17

(18) Show that the following systems are equivalent?
(a) $y(n) = 0.2y[n-1] + x(n) - 0.3x[n-1] + 0.02x[n-2]$ (b) $y(n) = x(n) - 0.1x[n-1]$

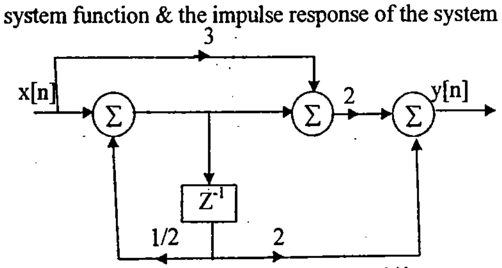
(19) Determine the I.Z.T. of the following?
(a) $X(z) = \dfrac{1 - \frac{1}{3}z^{-1}}{1 + \frac{1}{3}z^{-1}}$, x(n) is right – sided ⇒ long Division
(b) $X(z) = \dfrac{3}{z - \frac{1}{4} - \frac{1}{8}z^{-1}}$, x(n) is stable ⇒ p.f. expansion
(c) $X(z) = \ln(1 - 4z), |z| < \frac{1}{4}$ ⇒ Power series
(d) $X(z) = \dfrac{1}{1 - \frac{1}{3}z^{-3}}, |z| > 3^{-1/3}$

(20) Find y[n], $n \geq 0$ for the following difference equation?
(a) $y[n] = \frac{1}{2}y[n-1] + x[n],\ x[n] = \left(\frac{1}{3}\right)^n u[n], y[-1] = 1$
(b) $y[n] + \frac{1}{2}y[n-1] + \frac{1}{4}y[n-2] = 0,\ y[-1] = y[-2] = 1$

(21) Consider a causal L.T.I. system whose input x(n) & output y(n) are related through the block diagram shown in figure?
(a) Determine a difference equation relating y(n) & x(n)
(b) Is this system stable?



(22) Determine the system function & the impulse response of the system shown in figure?



(23) Consider a sequence x[n] for which $X(z) = \dfrac{1/3}{1 - \frac{1}{2}z^{-1}} + \dfrac{1/4}{1 - 2z^{-1}}$ and for which the R.O.C includes the unit circle. Find x[0] using initial value theorem?

(24) Determine a sequence x[n] whose Z.T. is $X(z) = e^z + e^{1/z}, z \neq 0$

(25) Find the R.O.C of Z.T. of output Y(z), given X(z) & H(z)?
(a) $X(z) = \dfrac{1}{1 + \frac{1}{2}z^{-1}}; |z| > \frac{1}{2}$ & $H(z) = \dfrac{1}{1 - \frac{1}{4}z^{-1}}; |z| > \frac{1}{4}$
(b) $X(z) = \dfrac{1}{\left(1 - \frac{1}{3}z^{-1}\right)\left(1 + 3z^{-1}\right)}; \frac{1}{3} < |z| < 3$ $H(z) = \dfrac{1 + 3z^{-1}}{1 + \frac{1}{3}z^{-1}}; |z| > \frac{1}{3}$

(26) Find the I.Z.T. of $X(z) = \dfrac{1}{(1 + 0.2z^{-1})(1 - 0.2z^{-1})^2}$

(27) Given $x[n] \leftrightarrow \dfrac{4z}{(z + 0.5)^2}$, |z|>0.5. Find Z.T. of the following signals & specify ROC?
a) $y_1(n) = x(n-2)$ b) $y_2(n) = 2^n x(n)$ c) $y_3(n) = nx(n)$
d) $y_4(n) = n^2 x(n)$ e) $y_5(n) = (n-2)\,x(n)$ f) $y_6(n) = x(-n)$

(28) The transfer function of 2 cascaded systems $H_1(z)$ & $H_2(z)$ is known to be $H(z) = \frac{z^2+0.25}{z^2-0.25}$.

It is also known that the unit step response of first system is $[2-(0.5)^n]u(n)$. Find $H_1(z)$ & $H_2(z)$?

(29) Apply I.V.T & F.V.T for (i) $X_1(z) = \frac{1+z^{-1}}{(1-0.25z^{-2})}$

(ii) $X_2(z) = \frac{z(z-2)}{(z-1)(z-0.5)}$

REVIEW NOTES

Chapter 8. DFT & FFT

8.1 Introduction To DFT

→ The Fourier series describes periodic signals by discrete spectra, whereas the DTFT describes discrete signals by periodic spectra. As a result, signals that are both discrete and periodic in one domain are also periodic & discrete in the other. This is the basis for the formulation of the DFT.

→ Sampled version of D.T.F.T. spectrum is D.F.T.

→ The N point DFT of a signal $x(n) = X[k] = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\Pi kn}{n}}, K = 0,1,\ldots(N-1)$

IDFT of $X[k] = x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X[k] e^{\frac{j2\Pi kn}{N}}, n = 0,1,\ldots(N-1)$

→ The DFT & its IDFT are also periodic with period N, and it is sufficient to compute the results for only one period (0 to N-1).

$$X[k] = \sum_{n=0}^{N-1} x(n) W_N^{kn} \;|\; x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X[k] W_N^{-kn} \text{ where } W_N = e^{-j2\Pi/N} \text{ is the phase factor.}$$

Periodicity: - $W_N^{K+N} = W_N^{K}$ | **Symmetry:-** $W_N^{K+\frac{N}{2}} = -W_N^{K}$

P8.1.1 Analog data to be spectrum analyzed are sampled at 10 kHz & DFT of 1024 samples are computed. Find the frequency spacing between spectral samples?

P8.1.2 Find the 4 point DFT of x (n) = {0, 1, 2, 3}?

Note: - For direct calculation of N point, we require N^2 complex multiplications and N (N-1) additions.

P8.1.3 Let $x(n) \xleftrightarrow{N pt} X[k]$, then prove the following statements.

i) If $x(n) = -x[N-1-n]$, then $X(0)=0$

ii) If $x(n) = x[N-1-n]$, with N even, $X\left[\frac{N}{2}\right] = 0$

P8.1.4 Fig.8.1.4 shows a finite length sequence x (n). Sketch the signals

(a) $x([n-2])_4$

(b) $x([n+1])_4$

(c) $x([-n])_4$

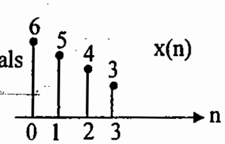


FIG (8.1.4)

P8.1.5 The first five points of an 8 point DFT of a real – valued sequence are {0.25, 0.125 – j0.3018, 0, 0.125 – j0.0518, 0}. Find other 3 points?

8.2) PROPERTIES OF DFT:-

(1) Circular shift:- $x[n-n_0]_N \longleftrightarrow e^{\frac{-j2\pi}{N}Kn_0} X[k]$

$$x[n-\frac{N}{2}] \longleftrightarrow (-1)^k X[k]$$

(2) Modulation:- $x(n)e^{\frac{j2\Pi}{N}k_0 n} \longleftrightarrow X[k-k_0]$

$$(-1)^n x(n) \longleftrightarrow X[k-\frac{N}{2}]$$

(3) Circular Convolution: $x(n) \, N \, h(n) \leftrightarrow X(k)H(k)$

(4) Central ordinates:-

$$X[0]=\sum_{n=0}^{N-1} x(n) \qquad x[0]=\frac{1}{N}\sum_{k=0}^{N-1} X(k)$$

$$X[\frac{N}{2}]=\sum_{n=0}^{N-1}(-1)^n x(n), N \text{ even} \qquad x[\frac{N}{2}]=\frac{1}{N}\sum_{k=0}^{N-1}(-1)^k X(k)$$

(5) Parseval's relation:-

$$\sum_{n=0}^{N-1}|x(n)|^2=\frac{1}{N}\sum_{k=0}^{N-1}|X(k)|^2$$

P8.2.1 Given x (n) = {1, -2, 3, -4, 5, -6}, without calculating DFT, find the following quantities.

(a) X(0) (b) $\sum_{k=0}^{5} X(k)$ (c) X(3) (d) $\sum_{k=0}^{5}|X(k)|^2$ (e) $\sum_{k=0}^{5}(-1)^K X(k)$

P8.2.2 The two 8-point sequences $x_1(n)$ & $x_2(n)$ shown in fig(8.2.2) have DFTs $X_1(k)$ & $X_2(k)$ respectively.

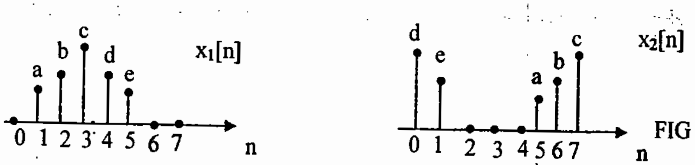


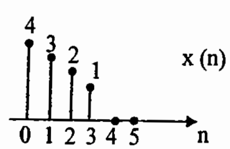
FIG (8.2.2)

Find the relation between $X_1(k)$ & $X_2(k)$?

P8.2.3 Consider the sequence x(n) shown in fig 8.2.3. Find y(n) whose six – point DFT is

$Y(k) = W_6^{4k} X(k)$, where X(k) is the six-point DFT of x(n).

Fig. 8.2.3



P8.2.4 Two finite length signals $x_1(n)$ & $x_2(n)$ are defined as $x_1(n)$ ={1, 2, 1, 1, 2, 1, 1, 2} and $x_2(n)$={0, 1, 3, 2}. Let $x_3(n)$ be eight point circular convolution of $x_1(n)$ & $x_2(n)$ i.e., $x_3(n)=x_1(n) \; 8 \; x_2(n)$. Find $x_3(2)$?

P8.2.5 Figure 8.2.5 shows two finite – length sequences. Sketch their N point circular convolution for N=6 and for N=10.

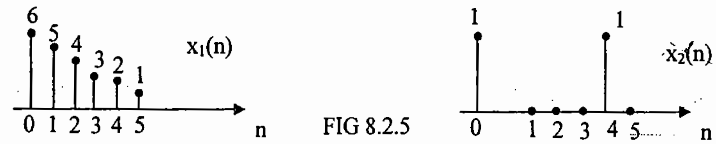
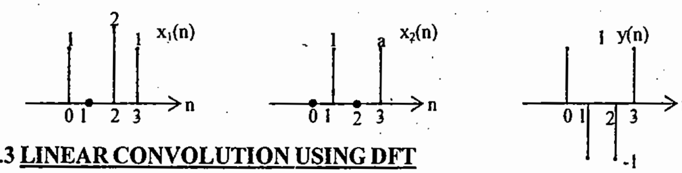


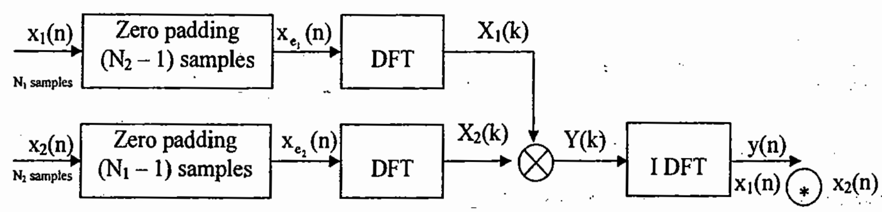
FIG 8.2.5

P8.2.6 2 sequences $x_1[n]$ & $x_2[n]$ are shown in figure and their resultant 4 point circular convolution is y[n]. The value of $x_2[n]$ at n = 3 is unknown, find 'a'?



8.3 LINEAR CONVOLUTION USING DFT

The circular convolution of 2 sequences, of lengths N_1 & N_2, respectively, can be made equal to the linear convolution of the 2 sequences by zero padding both sequences so that they both consist of $(N_1 + N_2 - 1)$ samples.



Let us consider the computational efficiency of calculating a convolution using the DFT rather than the direct method. In calculating the convolution of two N – element sequences using DFT method, we required $3N\log_2{}^{2N} + 2N$ Complex multiplications where as direct convolution of 2 sequences requires N^2 complex multiplication. ∴DFT method is more efficient for $N \geq 32$.

P8.3.1 Find the linear convolution of x (n) = {1, 2} & h (n) = {1, 2, 3} using DFT method?

8.4) CONVOLUTION OF LONG SEQUENCES:-

(1) Overlap – Add method

(2) Overlap – Save method

Some times we have to process a long stream of incoming data by a filter whose impulse response is much shorter than that of incoming data. The convolution of a short sequence h(n) of length N with a very long sequence x(n) of length L > > N can involve large amount of computation & memory.

(1) Overlap – Add method:-

Suppose h(n) is of length N, and the length of x(n) is **L = mN** (if not, we can always zero pad it to this length). We partition x[n] into m segments $x_0(n)$, $x_1(n)$ …$x_{m-1}(n)$, each of length N. We find the regular convolution of each section with h(n) to give partial results $y_0(n)$, $y_1(n)$, … $y_{m-1}(n)$.

$$y(n) = y_0[n] + y_1[n-N] + y_{m-1}[n-(m-1)N]$$

Since each regular convolution contains (2N - 1) samples, we zero – pad h(n) and each section $x_k[n]$ with [N - 1] zeros before finding $y_k[n]$ using the FFT. Splitting x(n) into equal – length segments is not a strict requirement.

Example: - Let x(n) = {1, 2, 3, 3, 4, 5} and h(n) = {1, 1, 1} using Overlap – add method find y(n)?

Solution:- L=6 & N=3. $x_0(n) = \{1, 2, 3\}$ h(n) = {1, 1, 1}

$x_1(n) = \{3, 4, 5\}$

$y_0(n) = x_0(n) * h(n) = \{1, 3, 6, 5, 3\}$

$y_1(n) = x_1(n) * h(n) = \{3, 7, 12, 9, 5\}$

Shifting & superposition results in the required convolution

$y(n) = y_0(n) + y_1[n-3]$

$y_0(n)\rightarrow$	1	3	6	5	3			
$y_1[n-3]\rightarrow$				3	7	12	9	5

y(n) = {1, 3, 6, 8, 10, 12, 9, 5}

(2) Overlap – Save method:-

If L > N and we zero – pad the second sequence to length L, their periodic convolution has (2L – 1) samples. Its first (N –1) samples are contaminated by wraparound, and the rest correspond to the regular convolution. eg. Let L=16 & N=7. If we pad N by 9 zeros, their regular convolution has 31(or 2L–1) samples with 9 trailing zeros (L–N=9). For periodic convolution 15 samples (L –1=15) are wrapped around. Since the last nine [or L – N] are zeros, only the first 6 samples of the periodic convolution are contaminated by wraparound, which is the basis idea of this method.

First, we add (N-1) leading zeros to the longer sequence x(n) & section it into k overlapping {by N - 1} segments of length M. Typically, we choose M ≈ 2N.

Next, we zero-pad h(n) { with trailing zeros} to length M, and find the periodic convolution of h(n) with each section of x(n). Finally, we discard the first (N-1) [contaminated] samples from each convolution & glue(concatenate) the results to give the required convolution.

Example: - Find convolution of x(n) = {1, 2, 3, 4, 5} & h(n) = {1, 1, 1} using Overlap Save method?

Solution: - First add(N - 1) = 2 zeros to x(n) = {0, 0, 1, 2, 3, 3, 4, 5}

Take M = 2N – 1 = 5, section into K overlapping segments of length M(5)

$x_0(n) = \{0, 0, 1, 2, 3\}$ Zero pad h(n) to length M = 5 samples

$x_1(n) = \{2, 3, 3, 4, 5\}$ h(n) = {1, 1, 1, 0, 0}

$x_2(n) = \{4, 5, 0, 0, 0\}$

$x_0(n) \circledast h(n) = \{5, 3, 1, 3, 6\}$

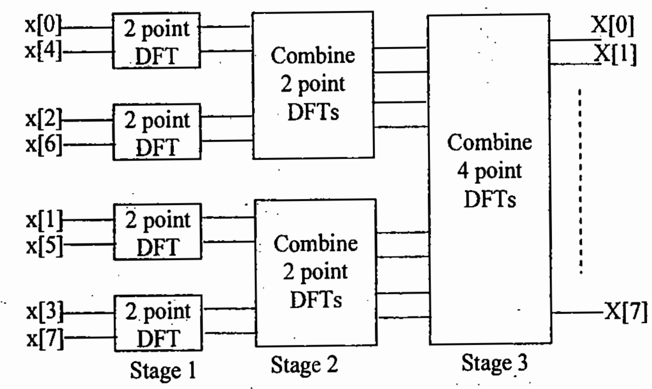
$x_4(n) \circledast h(n) = \{11, 10, 8, 10, 12\}$

$x_2(n) \circledast h(n) = \{4, 9, 9, 5, 0\}$

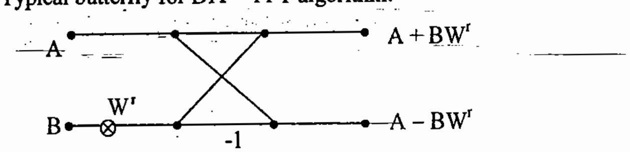
We discard the first 2 samples from each convolution & give the results to obtain y(n) = {1, 3, 6, 8, 10, 12, 9, 5, 0}

8.5 FFT:- Fast algorithm reduce the problem of calculating an N-point DFT to that of calculating many smaller-size DFTs. The computation is carried out separately on even – indexed and odd-indexed samples to reduce the computational effort. All algorithms allocate for computed results. The less the storage required, the more efficient is the algorithm. Many FFT algorithms reduce storage requirements by performing computations in place by storing results in the same memory locations that previously held the data.

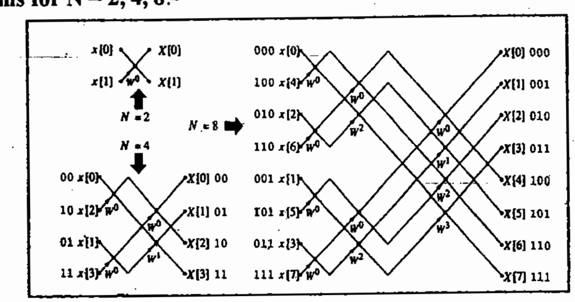
3 stages in an 8-point DIT-FFT :-



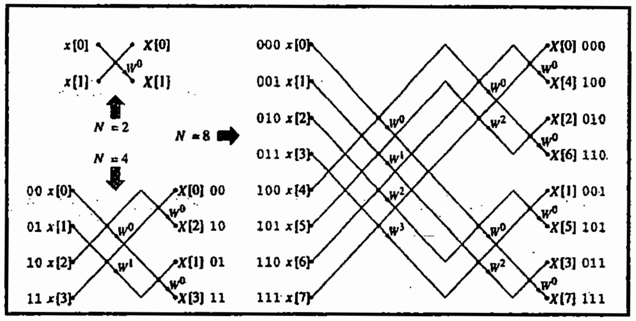
Typical butterfly for DIT – FFT algorithm:-



DIT algorithms for N = 2, 4, 8:-



DIF algorithm for N = 2, 4, 8:-



Feature	N-point DFT	N-point FFT
Algorithm	Solution of N equations in N unknowns	N/2 butterflies/stage m stages Total butterflies=m(N/2)
Multiplication	N per equation	1 per butterfly
Addition	N-1 per equation	1 per butterfly
Total multiplication	N^2	$N/2 \log^N_2$
Total additions	N(N-1)	$N \log^N_2$

Bit – reversal using Bruneman's algorithm:-

(1) Start with {0, 1} multiply by 2 to get {0, 2}
(2) Add 1 to the list of numbers obtained above. Now it is {1, 3}.
(3) Append the list in step2 to that in step1 to get {0, 2, 1, 3}
(4) The list obtained in step3 now becomes the starting list in step1. The steps are repeated until the desired length of list is obtained.

$$\underbrace{0 \quad 1}$$
$$\downarrow \times 2$$
$$\underbrace{0 \quad 2 \xrightarrow{+1} 1 \quad 3}$$
$$\downarrow \times 2$$
$$\underbrace{0 \quad 4 \quad 2 \quad 6 \xrightarrow{+1} 1 \quad 5 \quad 3 \quad 7}$$
$$\downarrow \times 2$$
$$\overbrace{0 \; 8 \; 4 \; 12 \; 2 \; 10 \; 6 \; 14} \xrightarrow{+1} 1 \; 9 \; 5 \; 13 \; 3 \; 11 \; 7 \; 15$$

P8.5.1 Let x[n] be a real 8 point sequence & let X(k) be its 8 point DFT

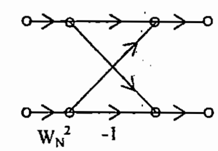
(A) Evaluate $\frac{1}{8}\sum_{k=0}^{7} X(k)e^{j(2\pi/8)kn} \mid n = 9$ in terms of x(n)

(B) Let w(n) be a 4 point sequence for $0 \le n \le 3$ & W(k) be its 4 point DFT. If W(k) = X(k) + X(k+4), express w(n) in terms of x(n)

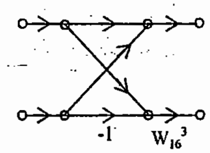
(C) Let y(n) be an 8 point sequence for $0 \le n \le 7$ and Y(k) be 8 point DFT. If $Y(k) = \begin{cases} 2X(k) & k = 0,2,4,6 \\ 0 \; for & k = 1,3,5,7 \end{cases}$, express y(n) in terms of x(n).

P8.5.2 Consider the butterfly shown in fig. which was extracted from a SFG implementing an FFT algorithm. Choose the most accurate statement from the following

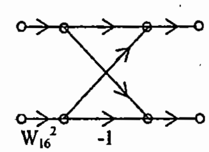
(A) The butterfly was extracted from DIT-FFT
(B) The butterfly was extracted from DIF-FFT
(C) It is not possible to say from the fig. which kind of FFT algorithm the butterflycame from.



P8.5.3 The butterfly was taken from DIF-FFT with N=16, where the input sequence was arranged in normal order. A 16 point FFT will have 4 stages (m=1,2,3,4). Which of the stages have butterflies of this form. Justify your answer.



P8.5.4 The butterfly was taken from DIT-FFT with N=16. Assume that 4 stages of the SFG are indexed by m=1,2,3,4. Which of the 4 stages have butterflies of this form?



PREVIOUS QUESTIONS

01. How many adders are required to realize a 256 point radix-2 FFT using DIT?
(a) 256 (b) 1024 (c) 4096 (d) 2048

02. 4 point DFT of real D.T. signal x(n) of length 4 is X[k], k = 0, 1, 2, 3. If X[0] = 5, X[1] = 1+j1, X[2] = 0.5. Then X[3] and x[0] respectively are
(a) 1–j, 1.875 (b) 1–j, 1.5 (c) 1+j, 1.875 (d) 0.1–j 0.1, 1.5

03. For an N point FFT algorithm with $N = 2^m$, which one of the following statements is TRUE?
(a) It is not possible to construct a SFG with both I/p & o/p in normal order
(b) The no. of butterflies in the m^{th} stage is N/m
(c) In-place computation requires storage of only 2N node data
(d) Computation of a butterfly requires only one complex multiplication

04. The 4 point DFT of a DT sequence {1,0,2,3} is
(a) [0, -2+2j, 2, -2-2j] (b) [2, 2+2j, 6, -2-2j]
(c) [6, 1-3j, 2, 1+3j] (d) [6, -1+3j, 0, -1-3j]

05. x[n] is a real-valued periodic sequence with period N and its DFT is X[k]. The DFT Y[k] of the sequence $y(n) = \frac{1}{N}\sum_{r=0}^{N-1} x(r)\,x(n+r)$ is

(a) $|X(k)|^2$ (b) $\frac{1}{N}\sum_{r=0}^{N-1} X(r)\,X^*(k+r)$ (c) $\frac{1}{N}\sum_{r=0}^{N-1} X(r)\,X(k+r)$ (d) 0

06. x[n] is a real – valued sequence given by {x[0], x[1], x[2], x[3], x[4], x[5], x[6]}. If first 4 DFT coefficient of x[n] are X[0], X[1], X[2] & X[3] the coefficient X[4] is equal to
(a) X[2] (b) $X^*[2]$ (c) $X^*[3]$ (d) X[3]

07. Suppose N = 32 FFT algorithm has a "twiddle" factor of W_{32}^2 for one of the butterflies in its fifth (last) stage. Is the FFT a DIT (or)DIF algorithm

8.6 PRACTICE PROBLEM SET:-

(1) Calculate the DFT of the following sequences?
(a) $x(n) = \delta(n)$
(b) $x(n)=\delta[n-n_0]$, $0 \le n_0 \le N-1$

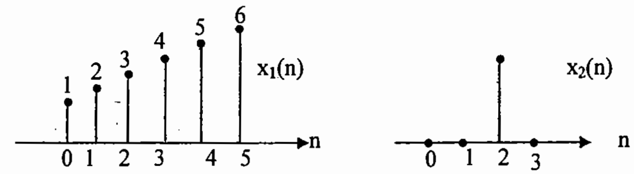
(c) $x(n) = \begin{cases} 1 & n \text{ even } 0 \le n \le N-1 \\ 0 & n \text{ odd} \end{cases}$

(d) $x(n)=a^n$, $0 \le n \le N-1$

(e) $x(n) = \begin{cases} 1 & 0 \le n \le \frac{N}{2}-1 \\ 0 & \frac{N}{2} \le n \le N-1 \end{cases}$

(2) Let $X(e^{j\omega})$ denote the DTFT of $x(n)=(1/2)^n u(n)$. Let y (n) denote a finite - duration sequence of length 10 i.e., y(n)=0 for n < 0 & n ≥ 10. The 10 point DFT of y(n) is Y(k). Which is $Y(k)=X[e^{j2\Pi k/10}]$ find y(n)?

(3) Sketch 6 point circular convolution of the following sequences?

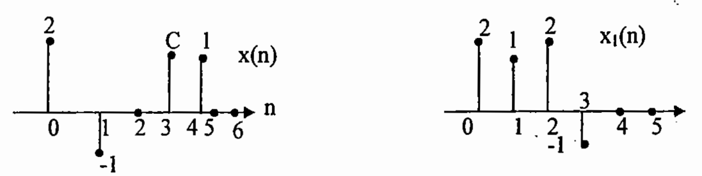


(4) Find the circular convolution of $x_1(n)$={1, 2, 3, 1} & $x_2(n)$={4, 3, 2, 2}

(5) Consider the sequence x(n)={2, 1, 1, 2}. The five point DFT of x(n) is X(k). Find the sequence y(n) whose DFT is $Y(k) = W_5^{-2k}X[k]$?

(6) Let x(n) be an N - point real sequence with N – point DFT X(k) (N even) and x[n] satisfies the symmetry $x[n+\frac{N}{2}] = -x(n)$ $n = 0,1,....\frac{N}{2}-1$ property i.e., upper half of the sequence is the negative of the lower half show that X[k] =0 for even k.

(7) Figure shows a sequence x(n) for which the value of x(3) is an unknown constant C. Let $X_1(k)=e^{j2\Pi 3k/5}$ where X(k) is 5 point DFT of x(n). Find the value of 'C'?



(8) Given x(n)={2, 3, 2, 1}. Find the 12-point signal described by y(n)={x(n), x(n), x(n)} and 12-point zero - interpolated signal h(n)=x[n/3]?

HINT:- Replication in one domain corresponds to zero interpolation in other. If a signal is replicated by M, its DFT is zero interpolated & scaled by M.

$$\underbrace{\{x(n), x(n), \ldots\ldots x(n)\}}_{\text{M-fold replication}} \leftrightarrow MX[k/M]$$

$$x[n/M] \leftrightarrow \underbrace{\{X(k), X(k), \ldots\ldots X(k)\}}_{\text{M-fold replication}}$$

Ans:- Y(k) = {24, 0, 0, -j6, 0, 0, 0, 0, 0, j6, 0, 0}

H(k) = {8, -j2, 0, j2, 8, -j2, 0, j2, 8, -j2, 0, j2}

(9) Given two four – point sequences $x[n] = [1, 0.75, 0.5, 0.25]$ and $y[n] = [0.75, 0.5, 0.25, 1]$ express DFT $Y(k)$ in terms of DFT $X(k)$

(10) For each DFT pair shown, compute values of the boxed quantities, using properties.
$\{\boxed{x_0}, 3, -4, 0, 2\} \leftrightarrow \{5, \boxed{X_1}, -1.28, -j4.39, \boxed{X_3}, 8.78 - j1.4\}$

(11) Let $x(n)=\{1, 2, 1\}$ & $h(n)=\{1, 2, 1, 3, 2, 2, 3, 0, 1, 0, 2, 2\}$ find their convolution using
(a) Overlap-add method
(b) overlap-save method

REVIEW NOTES

CHAPTER 9. ADDITIONAL QUESTIONS

01. Which of the following could not be the Fourier series expansion of a periodic signal?
(a) 2 cost + 3 cos 3t (b) 2 cos 0.5t + 3 cos 3.5 t
(c) 2 cos 0.25t + 3 cos 0.00054t (d) 2 cosπ t + 3 cos2π t

02. The value of $\int_{-\infty}^{+\infty} \frac{\sin 3\tau}{\tau} \delta(t-\tau) d\tau$ is -------------
(a) sin3t **(b) (sin3t)/t** (c) sin3 (d) 0

03. The value of the integral $\int_{-\infty}^{+\infty} \delta(1-t)(t^3+4)dt$ is ---------------
(a) 0 (b) 3 **(c) 5** (d) not defined

04. Given the signal x(t) = 2 cost + 3 cos3t + 4 cos4t. Then power in x(t) is -------- watts
(a) 10 **(b) 14.5** (c) $\sqrt{29}$ (d) none

05. The signal $x(t) = te^{-t} u(t)$ is -----------
(a) Energy Signal (b) Power Signal (c) neither energy nor power (d) none

06. The integral value $\int_{-\infty}^{t} [\delta(\tau+2) - \delta(\tau-2)] d\tau$ is.............
(a) rect(t/4) (b) rect(t/2) (c) rect(t) (d) 4

07. Consider an energy signal x(t), over the range $-3 \le t \le 3$, with energy E = 12 J. Then the signal energy in 2x(t) is ---------
(a) 24 J (b) 12 J **(c) 48 J** (d) 6 J

08. The fundamental period of $x(n) = e^{j0.2n\pi} + e^{-j0.3n\pi}$ is ----------
(a) 10 **(b) 20** (c) non periodic (d) none

09. The signal $x(n) = (j)^{n/2}$ is..............
(a) Periodic with fundamental period N = 8 (b) Periodic with fundamental period N=4
(c) Non periodic (d) None

10. The differential equation $y^1(t) - 4y(t)y(2t) = x(t)$ is ____________
(a) linear & time – invariant (b) linear & time – variant
(c) nonlinear & time – invariant **(d) nonlinear & time – variant**

11. Consider the sinusoidal signal $x(n) = 10\cos\left[\frac{4n\pi}{31} + \frac{\pi}{5}\right]$. Then fundamental period of x(n) is
(a) 31 (b) 31/2 (c) $4\pi/31$ (d) none

12. The system $y(n) = 2x(2^n)$ is
(a) Time – invariant & causal (b) Time – variant & causal
(c) Time – invariant & causal **(d) Time – variant & non causal**

13. The value of $\delta(t) * 2\delta(t-1) * 3\delta(t-2)$ is
(a) $2\delta(t-2)$ (b) $6\delta(t-2)$ (c) $\delta(t-3)$ **(d) $6\delta(t-3)$**

14. Convolution of $e^{-2t}u(t-2)$ with $\delta(t+2)$ is
(a) $e^{-2(t+2)}u(t)$ (b) $e^{-2t}u(t)$ (c) $e^{-2(t+2)}u(t+2)$ (d) $e^{-2t}u(t+2)$

15. An LTI system with impulse response $h(n) = 5^n u(3-n)$ is
(a) Causal & stable **(b) non causal & stable**
(c) Causal & unstable (d) non causal & unstable

16. The impulse response of a system is $h(n) = a^n u(n)$. The condition for the system to be BIBO stable is
(a) 'a' is real & positive (b) 'a' is real & negative (c) $|a|>1$ **(d) $|a|<1$**

17. When impulse response of an LTI system is integrated over whole time – domain, it gives
(a) Steady – stable response of system **(b) d.c response of system**
(c) transient response of system (d) steady – stable error of system

18. A periodic signal which can be expanded in Fourier series
(a) is power signal
(b) is neither signal
(c) is neither a power nor an energy signal
(d) can be either a power or an energy signal depending on the nature

19. The convolution of u(n) with u(n–4) at n = 5 is
(a) 5 **(b) 2** (c) 1 (d) 0

20. Given $x(n) = x_1(n) + x_2(n)$ where $x_1(n) = x_1(n+18)$ & $x_2(n) = x_2(n+54)$, then x(n) is
(a) periodic with periodic of 54 (b) periodic with periodic of 27
(c) periodic with periodic of 18 (d) non periodic with periodic of 18

21. The ideal LPF is not realizable because it is
(a) non-causal (b) non linear (c) unstable (d) time-varying

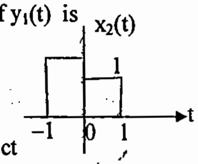
22. The inverse L.T. of e^{-s}/s is
(a) e^{-t} **(b) u(t–1)** (c) δ(t-1) (d) (t–1) u(t–1)

23. An analog ECG signal contains frequency upto 100 Hz. This signal is sampled at a rate 250 samples/sec. The highest frequency that can be represented uniquely at this sampling rate is
(a) 125 Hz (b) 200 Hz **(c) 100 Hz** (d) 250 Hz

24. If the result of convolution of 2 infinite duration signals is causal signal, then
(a) both the signals are anticausal (b) one signal is causal & other is anticausal
(c) both the signals are causal (d) one signal is causal & other is non-causal

25. Assume $z(t) = \int_{-\infty}^{+\infty} x(-\tau+a)\,h(t+\tau)d\tau$. Given $y(t) = x(t) * h(t)$ then z(t) in terms of y(t) is
(a) y(a–t) (b) y(t–a) **(c) y(t+a)** (d) y(–t–a)

26. For an LTI system, the response of the system to an input $x_1(t) = u(t)-u(t-1)$ is $y_1(t)$. Then response of the system to the input $x_2(t)$ shown in fig. in terms of $y_1(t)$ is



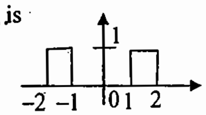
(a) $2y_1(t)+y_1(t+2)$ (b) $y_1(t+1)+2y_1(t)$
(c) $y_1(t-2)+y_1(t)$ **(d) $2y_1(t+1)+y_1(t)$**

27. If $y(t) = x(t) * h(t)$ then which of the following statement is correct
(a) $y(3t) = 3x(3t) * h(3t)$ (b) $y(3t) = x(3t) * h(3t)$
(c) $y(3t) = 1/3\ x(3t) * h(3t)$ (d) $y(3t) = 1/9\ x(3t) * h(3t)$

28. Given an LTI system with impulse response $h(t) = \alpha e^{-\alpha t} u(t) > 0$, and signal is $x(t) = \sin^2 2t$. Then the output signal has d.c. value as
(a) 1/2 (b) 0 (c) 1 (d) α

29. Given the periodic signal $x(t) = \sum_{n=-\infty}^{+\infty} u(t-3n) - u(t-3n-1)$ (n even). Then d.c. component of this signal is
(a) 1/6 (b) 1/3 (c) ½ (d) None

30. Given $\Pi(t/T) \leftrightarrow T\ \text{sinc}(fT)$. Then F.T. of the signal shown in fig is



(a) 2sincf cos(3πf) (b) sinc2f cos(3πf)
(c) sinc3f cos(πf) (d) None

31. The value of the integral $\int_0^{\infty}[\cos(t-T)\delta(\tau+3)d\tau]$
(a) cos(t–3) (b) cos(t+3) **(c) 0** (d) None

32. The system $y(t) = \sqrt{x(t)}$ with input x(t) & output y(t) is
(a) nonlinear, time-invariant (b) nonlinear, time-variant
(c) linear, time-invariant (d) linear, time-variant

33. The system described by the equation $y(t) = \int_{t_0}^{t} \tau x(\tau(\tau) + y(t_0)$ is--------- ($t_0 < t$)
(a) memory & causal (b) memory & noncausal
(c) static & causal (d) static & noncausal

34. Consider a CTs whose input & output are related by $y(t) = \begin{cases} x(t), & t \le b \\ 0, & t > b \end{cases}$ for a fixed b. This system is with ______
(a) memoryless & time-invariant **(b) memoryless & time-variant**
(c) memory & time-variant (d) memory & time-variant

35. Consider a DTs described by the advanced form difference equation 2y(n+3)–y(n+2)+3y(n+1=x(n+3)+2x(n+1). If we simplify the equation then its order is
(a) 3 **(b) 2** (c) 1 (d) None

36. The difference equation y(n)+2y(n–1)=x(n–1)+3x(n–2)-x(n–3) is ______
(a) FIR filter **(b) IIR filter** (c) either FIR or IIR (d) None

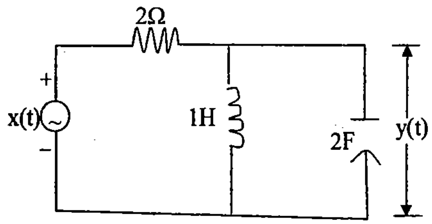
37. The D.E. describing the Network shown in Fig. is

(a) $4y^{11}(t) + y^{1} + 2y(t) = x^{1}(t)$

(b) $y^{11}(t) + 4y^{1}(t) + 2y(t) = x^{1}(t)$

(c) $y^{11}(t) + 2y(t) = x(t)$

(d) None



38. Given a periodic signal $x(t) = 3 + \sin 6t - 2\cos 6t + \pi \sin 9t - \cos 12t$. The amplitude of II harmonic of positive exponent term is

(a) 1.57 **(b) 1.12** (c) 0.5 (d) 3

39. Consider the transfer function $H(s) = \dfrac{s-2}{s^2+s+100.25}$ Then zero frequency response of the system is

(a) $0.02e^{j\pi}$ (b) $0.01e^{-j\pi/2}$ (c) $0.07e^{j\pi/3}$ (d) None

40. Given the signal $x(t)=16\cos(20\pi t+\pi/4)+6\cos(30\pi t+\pi/6)+4\cos(40\pi t+\pi/3)$ the power contained in the frequency interval 12 Hz to 22Hz is

(a) 22W **(b) 26W** (c) 10W (d) 20W

41. Given $x(t)=2\sin^2(2500\pi t)\cos(2\times10^4\,\pi t)$, then the average power is

(a) 0.75 watts (b) 0.5 watts (c) 1 watt (d) 2 watts

42. If the input to a system is $\cos 10\pi t + 2\cos 20\pi t$, then the type of distortion (if any) introduced by the system if output is $y(t)= \cos(10\pi t-\pi/4)+2\cos(20\pi t-\pi/4)$ is

(a) Amplitude **(b) Phase** (c) Both (a) and (b) (d) None

43. The initial value of $X(s) = \dfrac{s^2+5s+7}{s^2+3s+2}$ is

(a) 1 (b) 2 (c) 3.5 (d) 0

44. Consider an LTI system with input & output related through the equation

$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 2)\, d\tau$. The impulse response of the system is

(a) $e^{-2t}u(t)$ (b) $e^{-2t}u(t-2)$ **(c) $e^{-2(t-2)}u(t-2)$** (d) $e^{-2(t-2)}u(t)$

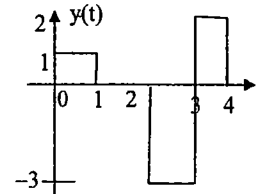
45. Define the area under a continuous time signal v(t) is

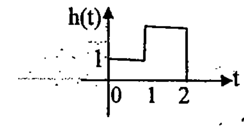
$A_v = \int_{-\infty}^{+\infty} V(t)dt$ if $y(t) = x(t) * h(t)$, then A_y =

(a) A_x+A_h **(b) A_xA_h** (c) A_x-A_h (d) $(A_x+A_h)^2$

46. Given input of system is $x(t) = \delta(t) - 2\delta(t-1)+\delta(t-2)$ and impulse response h(t) is shown in fig(1). Then output of the system is

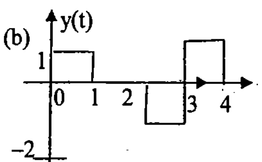
(a)



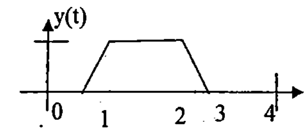


Fig(1)

(b)



(c)



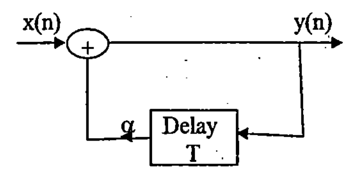
(d) None

47. The impulse response $h(t) = e^{t}u(-t-1)$ is

(a) Causal & stable **(b) Non Causal & Stable**
(c) Causal & Unstable (d) Non Causal & Stable

48. For the system shown in figure System stability is guaranteed when α is

(a) $\alpha > 1$ (b) $\alpha < 0$
(c) $0 < \alpha < 1$ (d) None



figure

49. Given $y(t) = |x(t)|$ where $x(t) = \cos t$. Then amplitude of d.c. component of y(t) is

(a) 0 **(b) 2** (c) 1/2 (d) None

50. Consider an LTI system with impulse response $h(t) = e^{-4t}u(t)$. Then output y(t) for input

$x(t) = \sum_{n=-\infty}^{+\infty} \delta(t-n)$ Is

(a) $1/(4+jn\pi)$ (b) $1/(4-jn\pi)$ (c) $1/(4+jn(\pi)/2)$ (d) $1/(2+jn\pi)$

51. The Fourier transform of $x(t) = \sum^{+\infty} \alpha^k \delta(t-kT), |\alpha|<1$ is

(a) $1/1+\alpha e^{jwT}$ (b) $e^{-jwT}/1-\alpha e^{-jwT}$ (c) $1/1-\alpha e^{-jwT}$ (d) $1/1+\alpha e^{-jwT}$

52. The Fourier transform of $x(t) = u_1(t) + 2\delta(3-2t)$ is
{where $u_1(t)$ is the differentiation of an impulse }

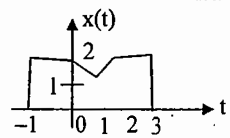
(a) $1 + e^{-jw/2}$ (b) $2 + 3e^{-jw}$ **(c) $jw + e^{-j3w/2}$** (d) $jw + 2e^{-j2w/3}$

53. The inverse fourier transform of $X(\omega) = \frac{2\sin[3(\omega - 2\pi)]}{(\omega - 2\pi)}$ is

(a) $x(t) = e^{-2\pi t}$; $|t| < 3$ (b) $x(t) = 1$; $|t| < 3$
(c) $x(t) = e^{j2\pi t}$; $|t| < 3$ (d) Data is not sufficient

54. A real, continuous time function x(t) has a F.T. X(w) Whose magnitude obeys the relation $\ln|X(w)| = -|w|$. Then the signal x(t) is an even function of time is
(a) $t/1+t^2$ (b) $-\pi/1+t^2$ **(c) $1/\pi(1+t^2)$** (d) $-1/\pi(1+t^2)$

55. Let X(w) denotes F.T. of the signal X(t) shown in fig.
The phase of X(w) is $\angle x(w) =$
(a) $-w$ (b) $+w$ (c) e^{-jw} (d) 0 (or) $\pm z$



56. In the above problem (55), the value of $\int_{-\infty}^{\infty} X(w)\,dw$ is

(a) 2π (b) $1/2\pi$ **(c) 4π** (d) $1/4\pi$

57. The output y(t) of a causal LTI system is related is the input x(t) by D.E. $dy/dt + 2y(t) = x(t)$. If $x(t) = e^{-t}u(t)$ then output of the system is
(a) $e^{-t}u(t) + 1/2\, e^{-2t}u(t)$ **(b) $e^{-t}u(t) - 1/2\, e^{-2t}u(t)$**
(c) $-e^{-t}u(t) - 1/2\, e^{-2t}u(t)$ (d) $2e^{-t}u(t) - e^{-2t}u(t)$

58. The Hilbert transform of $x(t) = \cos 3t$ is
(a) $-\cos 3t$ **(b) $\sin 3t$** (c) $-\sin 3t$ (d) none

59. Consider the Differential Equation of an LTI system is $d^2y(t)/dt^2 + 6dy(t) + 9y(t) = d^2x(t)/dt^2 + 3dx(t)/dt + 2x(t)$. Then impulse response is
(a) $\delta(t) - 3e^{-3t}u(t) + 2e^{-3t}u(t)$ (b) $\delta(t) + 3e^{-3t}u(t) - 2e^{-3t}u(t)$
(c) $\delta(t) + 3e^{-3t}u(t) - 2e^{-3t}u(t)$ (d) none of these

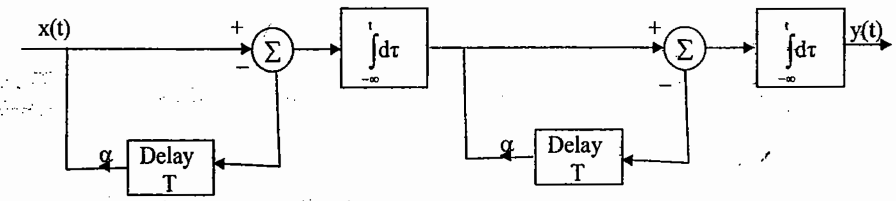
60. A signal x(t) of finite energy is applied to a square –law device whose output is y(t). The spectrum of x(t) is limited in frequency interval $-w \le f \le w$. Then limits of spectrum of y(t) is
(a) $-2w \le f \le 2w$ (b) $-w \le f \le w$ (c) $-w \le f \le 2w$ (d) $-2w \le f \le w$

61. The Fourier Transform of a signal $g(t) \leftrightarrow G(f) = \begin{cases} 1, f > 0, \\ 1/2, f = 0 \\ 0, f < 0 \end{cases}$ Then signal g(t) is

(a) $\frac{-1}{j2\pi t} + \frac{1}{2}\delta(t)$ (b) $\frac{-1}{j2\pi t} - \frac{1}{2}\delta(t)$

(c) $\frac{-1}{j2\pi t} + \frac{1}{2}\delta(t)$ (d) None of these

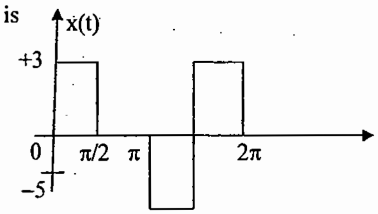
62. The frequency response of the linear system shown in Fig(4) is



Fig

(a) $T^2\,\mathrm{SinC}(fT)e^{-2\pi fT}$ (b) $T^2\,\mathrm{SinC}^2(fT)e^{-2\pi fT}$
(c) $T^2\,\mathrm{SinC}^2(fT)e^{-j2\pi fT}$ (d) $T^2\,\mathrm{SinC}(fT)e^{-2\pi fT}$

63. The average value of the signal shown if Fig. is
(a) −1.75
(b) 0
(c) 13
(d) None

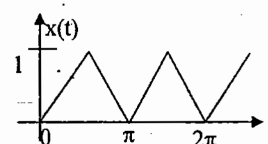


64. A harmonic signal $x(t) = 3\sin(4t+20^0) - 4\cos(12t+40^0)$. Then the amplitude of II harmonic is
(a) 0 (b) 3 (c) −4 (d) −1

65. The exponential fourier series coefficient for the signal shown in Fig, is

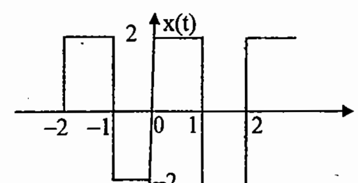
(a) $\frac{-4}{n^2\pi^2}$(odd n) (b) $\frac{-2}{n^2\pi^2}$(odd n)

(c) $\frac{-1}{n^2\pi^2}$(odd n) (d) None of these



Fig

66. If energy of first 4 harmonics of the signal shown in fig. is equal to 1.28 Joules. What is the energy contained in the rest of harmonics?
(a) 2.72 J (b) 4 J
(c) 1.28 J (d) 1 J



67. Consider a filter with frequency response $H(\omega) = 1/(1+j\omega)$ & input is $x(t) = e^{-2t}u(t)$. Then Energy in output is
(a) 1/3 (b) 2 (c) 1/2 (d) Data is not sufficient

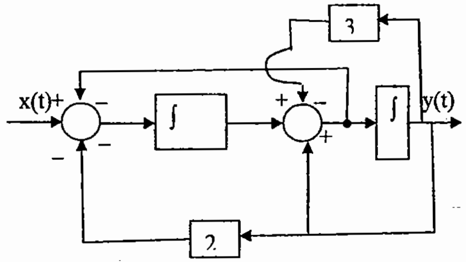
68. For the system shown in fig., the relation between input & output is described by D.E.

(a) $\frac{d^2y(t)}{dt^2} - \frac{3dy(t)}{dt} + 2y(t) = x(t)$

(b) $\frac{d^2y(t)}{dt^2} + \frac{3dy(t)}{dt} + 2y(t) = x(t)$

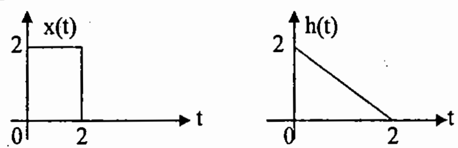
(c) $\frac{d^2y(t)}{dt^2} + \frac{3dy(t)}{dt} + 2y(t) = x(t)$

(d) $\frac{d^2y(t)}{dt^2} + \frac{3dy(t)}{dt} + 2y(t) = x(t)$

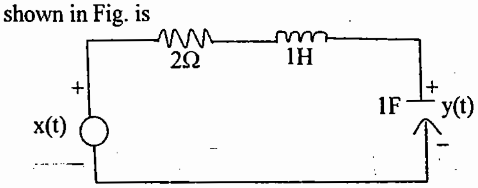


69. A discrete –time system is described by the II order differential equation $y[k+2]-1.5y[k+1]+y[k] = 2x[k]$. Given $y[-1] = 1$, $y[-2] = 2$. If a unitstep sequence is applied as input then $y[k]$ at $k = 2$ is
(a) –0.125 **(b) 0.125** (c) –1.75 (d) 1.25

70. For an LTI system the input signal x(t) & impulse response h(t) are shown in Fig. the output observed at t = 3 sec is
(a) 0
(b) 1
(c) –1
(d) –32



71. The impulse response of the system shown in Fig. is
(a) te^{-t} u(t)
(b) te^{t} u(t)
(c) $t^2 e^{-t}$ u(t)
(d) (t/2) e^{-t} u(t)



72. The laplace transform of $\sin^2 \omega t$ u(t) is
(a) $\omega/[s^2 + \omega^2]$ (b) $\omega^2/[s(s^2 + \omega^2)^2]$ (c) $2\omega^2/[(s^2 + 4\omega^2)]$ **(d) $2\omega^2/[s(s^2 + 2\omega^2)]$**

73. If $X(s) = (-3s^2 + 2) / (s^3 + s^2 + 3s + 2)$. Then the initial value is
(a) –3 (b) 3 (c) 0 (d) 1

74. Given $X(s) = (5s - 1) / (s^3 - 3s - 2)$. Then inverse Laplace transform is
(a) $(e^{-t} + 2te^{-t} + e^{3t})$ u(t) (b) $(-e^{-t} + 2te^{-t} + e^{-2t})$ u(t)
(c) $(-e^{-t} + 2te^{-t} + e^{2t})$ u(t) (d) $(3e^{-t} + 2te^{2t})$ u(t)

75. A LTI continuous – time system has the impulse response cos2t + 4 sin 2t. Then the response to the input $\left[\frac{5e^{-t}}{7} - \frac{12e^{-8t}}{7}\right]$u(t) is
(a) $(-\cos 2t + e^{-t})$ u(t) (b) $(-\cos 2t - e^{-t})$ u(t) (c) $(\sin 2t - e^{-t})$ u(t) (d) $(-\sin 2t + e^{-t})$ u(t)

76. A LTI continuous – time system has the transfer form $H(s)=[s(s+1) / (s^2 + 2s + 4)$. If h(t) * v(t) = δ(t). Then v(t) is
(a) $(-\delta(t)-4-3e^{-t})$ u(t) (b) $(4-3e^{-t})$ u(t) **(c) $(\delta(t) + 4-3e^{-t})$ u(t)** (d) None of these

77. A discrete time signal x(n) has Z – transform $X(z) = (z + 1) / [z(z-1)]$. The value of x(10,000) is
(a) 1 (b) –1 **(c) 2** (d) 0

78. If x(n) = u(n) & h(n) = nu(n) then x(n) * h(n) will be
(a) n^2 u(n) (b) (n+1) u(n) **(c) $\frac{n(n+1)\,u(n)}{2}$** (d) $(n^2/2)$ (u(n))

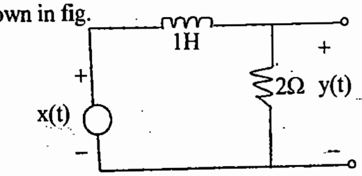
79. The signal $x(n) = \sin^2 [\pi n/12]$ is
(a) Periodic with period = 1/6 **(b) Periodic with period = 12**
(c) A periodic (d) None of these

80. The Fourier transform of $\delta[t-t_0] / a]$ is
(a) $|a|\, e^{j\omega t_0}$ (b) $(1/|a|)\, e^{j\omega t_0}$ (c) $\delta(\omega-\omega_0)\, e^{-j\omega_0 t_0}$ (d) $e^{-j\omega t_0}$

81. The Fourier transform of $f\left(\frac{t}{a}+b\right)$ is
(a) $(1/|a|e^{jab\omega}\, F(a\omega)$ **(b) $|a|\, e^{jab\omega}\, F(a\omega)$** (c) $|a|e^{-jab\omega}\, F(a\omega)$ (d) $|a|\, e^{-jab\omega}\, F(a\omega)$

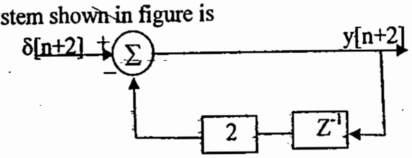
82. If frequency response of the system is $-3e^{-j2\omega}$, then the response if the input is u(t) + δ(t–3) is
(a) 3u(t–2) + 3δ(t–5) **(b) –3u(t–2) –3δ(t–5)**
(c) –3u(t–5) + 3δ(t–2) (d) –3u(t–2) + 3δ(t–5)

83. The output of the system shown in fig. If the input is te^{-2t} u(t) is
(a) $t^2 e^{-2t}$ u(t)
(b) $t e^{-2t}$ u(t)
(c) $(t^2/2)\, e^{-2t}$ u(t)
(d) $(t/2)\, e^{-2t}$ u(t)



84. The Z – transform of $e^{jn\theta}$ is
(a) 1∀ Z (b) $Z / (Z-e^{-j\theta})$; |Z| > 1 **(c) $Z/(Z-e^{j\theta})$; |Z| > 1** (d) $Z/(Z+e^{j\theta})$; |Z| > 1

85. The Z – Transform of the system shown in figure is
(a) $1 / 1 + 2z^{-1}$
(b) $1 / 1 - 2z^{-1}$
(c) $z^{-1} / 1 - 2z^{-1}$
(d) $z^{-1} / 1 + 2z^{-1}$



86. A LTI continuous – time system has transfer function. H(s) = 1 / (s + 1). The output when input applied is $\cos(t + 45^0)$ u(t) is
(a) $(1/\sqrt{2})(\cos t - e^{-t})u(t)$ (b) $(\cos t - e^{-t})u(t)$
(c) $(1/\sqrt{2})(\cos t + e^{-t})u(t)$ (c) $(1/\sqrt{2})(-\cos t - e^{-t})u(t)$

87. A LTI continuous – time system has frequency response $H(\omega)$, it is known that the input $x(t) = 1 + 4\cos(2\pi t) + 8\sin(3\pi t - 90^0)$ produces the response $y(t) = 2 - 2\sin(2\pi t)$. Then $H(\omega)$ at $\omega = 3\pi$ is
(a) 0 (b) 1 (c) $(1/2)\, e^{-j\pi/2}$ (d) None of these

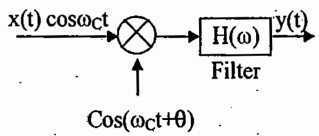
88. A continuous time signal x(t) has F.T. $X(\omega)=1/(j\omega+b)$ where 'b' is Constant. Then F.T. of $1/(jt-b)$ is
(a) $2\pi e^{b\omega}\, u(-\omega)$ (b) $2\pi e^{-b\omega}\, u(\omega)$ **(c) $-2\pi e^{-b\omega}\, u(\omega)$** (d) None of these

89. The Nyquist rate for the signal $x(t) = Sa(4t) + 2Sa^2(3t)$ is ————
(a) 6 rad/sec (b) 8 rad/sec **(c) 12 rad/sec** (d) 24 rad/sec

90. Given $h(n) = \{1, 2, \underset{\uparrow}{1}, 0, 2\}$ Then system is
(a) Causal & stable **(b) Non – Causal & stable**
(c) Causal & Unstable (d) Non – causal & unstable

91. A signal x(t) with band width B is put on a carrier $\cos\omega_c t$ with $\omega_c >> B$. The modulated signal $x(t)\cos\omega_c t$ is then applied to the system shown in figure. The frequency response of the filter in the system is given by

$$H(\omega)=\begin{cases} 2, & -B<\omega<B \\ 0 & ;\text{else where}\end{cases}$$

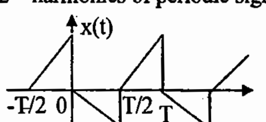


(a) x(t)
(b) (1/2) x(t)
(c) 2x(t)
(d) –x(t)

92. A recursive filter is described by $y(n) = 0.7y[n-1] - 0.3\, y(n-2) - 6x[n-1]$. The static gain of the filter is ——— HINT: Static gain is H(z) at z = 1
(a) 10 (b) 1 (c) 0 (d) None of these

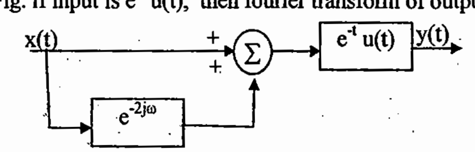
93. The difference equation is described by $y(n) = (1/4)\, x(n+2) + (1/2)\, x(n+1) + (1/2)\, x(n) + (1/2)\, x(n-1) + (1/4)\, x(n-2)$ where input is x(n) & output y(n) is
(a) Causal & stable (b) Causal & unstable
(c) Non causal & Stable (d) Non Causal & Unstable

94. The power contained in the first 2 – harmonics of periodic signal shown if Fig. are.
(a) 0.3083 watts
(b) 0.33 watts
(c) 0.67 watts
(d) 0.0308 watts



Fig

95. For the system shown in Fig. if input is $e^{-t}\, u(t)$, then fourier transform of output is
(a) $1-e^{-i2\omega}/(1+j\omega)$
(b) $1+e^{-i2\omega}/(1+j\omega)^2$
(c) $1/(1+j\omega)^2$
(d) $1+e^{-2\omega}/(1+j\omega)^2$



96. Let $x(t) \leftrightarrow X(\omega) = \begin{cases} 1, & |\omega|<1 \\ 0, & |\omega|>1.\end{cases}$ Consider $y(t) = \dfrac{d^2x(t)}{dt^2}$. Then value of $\int_{+\infty}^{-\infty} |y(t)|^2 dt$ is
(a) $3/\pi$ (b) 2/3 **(c) $1/3\pi$** (d) $1/6\pi^2$

97. Consider a discrete – time LTI system whose system function is $H(z) = z/(z-1/2)$; $|z| > 1/2$. Then step response of system is
(a) $[2 + (1/2)^n]\, u(n)$ **(b) $[2 - (1/2)^n]\, u(n)$**
(c) $[1-(1/2)^n]\, u(n-1)$ (d) $[1+(1/2)^{n-1}]\, u(n)$

98. The initial & final values of $x(z) = [2z(z-5/12)] / [(z-1/2)(z-1/3)]$; $|z| > (1/2)$ is respectively
(a) 2 & 0 (b) 0 & 2 (c) 0 & 1 (d) 1 & 0

99. The Z.T. of $n(n-1)\, a^{n-2}\, u(n)$ is
(a) $2z/(z+a)^3$; $|z| > |a|$ (b) $z/(z+a)^3$; $|z| > a$
(c) $2z/(z-a)^3$; $|z| > |a|$ (d) $z/(z-a)^3$; $|z| > a$

100. Consider a continuous – time signal $x(t) = e^{-at}$, $t \geq 0$. Let the sequence x(n) be obtained by uniform sampling of x(t) such that $x(n) = x(nTs)$. Then Z.T. of x(n) is
(a) $1/(1-e^{aTs} z^{-1})$ **(b) $1/(1-e^{-aTs} z^{-1})$** (c) $z^{-1}/(1 - e^{aTs} z^{-1})$ (d) None of these

ASSERTION AND REASON

In questions, two statements A(Assertion) & R(Reason) are given mark the answer as
(a) If A & R both are true & R is the correct explanation of A.
(b) If A & R both are true and R is not the correct explanation of A.
(c) If A is true & R is false.
(d) If A is false & R is true.

1. Assertion (A): The L.T of $e^{-at} \sin\omega t$ is $\omega/(s+a)^2 + \omega^2$
Reason (R): If the L.T of f(t) is F(s), then L.T of f(t) e^{-at} is F(s+a) [B]

2. Assertion (A): All memory less systems are causal
Reason (R): All causal systems are memory less [C]

3. Assertion (A): An LTI discrete system represented by the difference equation
$y(n+2) - 5y(n+1) + 6y(n) = x(n)$ is unstable
Reason(R) : A system is unstable if the roots of the characteristic equation lie outside the unit Circle. [B]

4. Assertion (A): The value of $4t^2\, \delta(2t-4)$ is $8\, \delta(t-2)$
Reason(R) : Area under impulse function is one.

5. Assertion (A): The average power in the signal $x(n) = 6\, e^{j2\pi n/4}$ is 36 W.
Reason(R) : The average power in the signal x(n) is $P_{av} = \underset{N\to\infty}{Lt} \dfrac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$

6. Assertion (A): The differential equation $y'(t)+2y^2(t)=2x(t)-x(t)$ is nonlinear but time invariant.
Reason(R) : An LTI system is causal if $h(t) = 0;\ t<0$

7. Assertion (A) : The periodic signal $x(t) = 2+4\sin(6\pi t)$ is having exponential F.S. coefficients $\{j2, 2, -j2\}$
Reason(R) : An odd periodic signal contains only sine terms

8. Assertion(A): Truncation in time- domain causes ripples in frequency domain.
Reason(R) : For a periodic signal to have F.S. it should satisfy Dirichet condition [A]

9. Assertion(A): In the exponential Fourier representation of a real-valued periodic function f(t) of frequency fo, the coefficients of the terms $e^{j2\pi n f_0 t}$ & $e^{-j2\pi n f_0 t}$ are negatives of each other.
Reason(R) : The discrete magnitude spectrum of f(t) is even & the phase spectrum is odd [D]

10. Assertion(A): The stability of the system is assured if the ROC includes the unit circle in the z-plane.
Reason(R) : For a causal stable system all the poles should be outside the unit circle in z-plane [C]

11. Assertion(A): The Laplace transforms of the signal $e^{-2t}u(t)$ and $-e^{-2t}u(-t)$ is $1/(S+2)$
Reason(R) : The ROC for $e^{-2t}u(t)$ ROC is $\sigma > -2$ and for $-e^{-2t}u(-t)$ is $\sigma < -2$. [B]

12. Assertion (A): For a periodic signal $x(t) = 3\sin(4t+20^0)-4\cos(12t+40^0)$, the amplitude of II harmonic is 3.
Reason(R) : For signal x(t), fundamental period is $\pi/2$ [D]

13. Assertion(A): F.T. of unit step function is $1/j\omega + \pi\delta(\omega)$.
Reason(R) : L.T. evaluated along the imaginary axis becomes F.T. [B]

14. Assertion(A): For an LTI system to be stable $\int |h(\tau)d\tau < \infty$.
Reason(R) : For an LTI system with system function H(s), ROC must include the imaginary axis. [A]

15. Assertion(A): F.T. of $1/\pi t$ is $-j\text{sgn}(w)$
Reason(R) : If $x(t) \leftrightarrow X(w)$ then $X(t) \leftrightarrow 2\pi x(-w)$ [B]

16. Assertion(A): Ideal filters are non causal
Reason(R) : The condition for distortionless transmission is magnitude spectrum must be linear function of frequency [C]

17. Assertion(A): Linear phase characteristics is must condition for FIR filter.
Reason(R) : IIR filters are generally stable [C]

Match the following

1) List1 (F(s)) — List II (f(t))

A. $\dfrac{10}{s(s+100)}$ — 1. $10\delta(t)$

B. $\dfrac{10}{s^2+100}$ — 2. $e^{-10t}\cos 10t\, u(t)$

C. $\dfrac{s+10}{(s+10)^2+100}$ — 3. $\sin 10t\ u(t)$

D. 10 — 4. $(1-e^{-10t})\, u(t)$

	A	B	C	D
a)	3	4	1	2
b)	4	3	1	2
c)	3	4	2	1
d)	4	3	2	1

[d]

2) **List 1 (Input- output relation)**
A) $y(n) = x(n)$
B) $yn) = x(n^2)$
C) $y(n) = x^2(-n)$
D) $y(n) = x^2(n)$

List II (property of system)
1. Nonlinear, noncausal
2. Linear, nonlcausal
3. Linear, causal
4. Nonlinear, causal

	A	B	C	D
a)	1	4	3	2
b)	3	2	1	4
c)	1	2	3	4
d)	3	4	1	2

[b]

3) **List1 (function in time – domain)**
A) Delta function
B) Gate function
C) Normalised Qaussian function
D) Sinusoidal function

List II (F.T. of the function)
1) Delta function
2) Gaussian function
3) Constant function
4) Sampling function

	A	B	C	D
a)	1	2	4	3
b)	3	4	2	1
c)	1	2	2	3
d)	3	4	4	1

[b]

4) The following table gives some time functions and L.T.

f(t)	F(s)
1. $\delta(t)$	s
2. u(t)	1/s
3. tu(t)	$2/s^2$
4. t^2 u(t)	$2/s^3$

Of the correctly matched pair is [a]

(a) 2 & 4 (b) 1 & 4 (c) 3 & 4 (d) 1 & 2

5) **List 1 (x(n))**

A) $a^n u(n)$

B) $a^{n-2} u(n-2)$

C) $a^{jn} u(n)$

List II (Z.T) X(z)

1) $\frac{az}{(z-a)^2}$

2) $\frac{ze}{Ze^{-j}-a}$

3) $\frac{z}{Z-a}$

4) $\frac{z^{-1}}{z-a}$

	A	B	C	D
a)	3	2	4	1
b)	2	3	4	1
c)	3	4	2	1
d)	2	4	2	3

[c]

6) **List 1 (properties)**

A) f(t) + f(–t) = 0

B) f(t) – f(–t) = 0

C) f(t) + f(t–t/2) = 0

D) f(t) –f(t–t/2) = 0

List II (characteristic of trigonometric form)

1. even harmonics can exist
2. Odd harmonics can exist
3. dc & cosine teerms can exist
4. sine terms can exist
5. cosine terms of even harmonics can exist

	A	B	C	D
a)	4	5	3	1
b)	3	4	1	2
c)	5	4	2	3
d)	4	3	2	1

[d]

7) **List I (signals)**

A) g(t–2)

B) t g(t)

C) g(–t)

D) g(3t + 1)

List II (transform)

1) jd /dw G(ω)

2) 1/3 G(ω/3) $e^{+j\omega/3}$

3) $e^{-j2\omega}$ G(ω)

4) G(–ω)

	A	B	C	D
a)	1	4	3	2
b)	3	1	4	2
c)	3	1	2	4
d)	1	3	4	2

[b]

8) **List I (Z.T)**

A) 1

B) $\frac{z}{z-2}$ right sided

C) log $(1+az^{-1})$, right sided

D) $\frac{3}{z-1/4-1/8^{z-1}}$, stable

List II (ROC)

1) |z| < 2

2) |z| > 2

3) |z| > 1/a

4) |z| > a

5) |z| > ½

6) entire Z – plane

	A	B	C	D
a)	6	1	3	5
b)	6	3	5	1
c)	6	2	4	5
d)	6	5	4	2

[c]

9) **List I (signal transform)**

A) $\frac{s}{(s+1)(s+2)}$

B) $\frac{s}{(s-1)(s+2)}$

C) $\frac{s}{(s-1)(s-2)}$

D) $\frac{s}{S(s+1)}$

List II (characteristic)

1. causal, unstable
2. noncausal, stable
3. causal, marginally stable

	A	B	C	D
a)	4	2	1	3
b)	4	1	3	2
c)	2	4	3	1
d)	1	4	2	3

[a]

Notes for Important Points

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